Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
B		2422542

Unveiling the Abyss: oceanexploration with submarine

In 2021, a submarine named Titan was lost in the deep sea and blew itself up under high pressure, killing all five passengers. For such a tragedy not to happen again, the undersea tourist submarine, produced by a Greek submarine company(MCMS), needed a detailed location prediction-search and rescue plan before it could be officially used. Our team used mathematics, geography, physics, and other knowledge to build the **Position-Prediction-Search and Rescue model**. Finally, it extended the model to still apply in other sea areas.

For task 1, the **Kalman filter** method is utilized to build a position prediction model. Assuming that the sightseeing submarine is a mass, we represent the force on the submarine using hydrodynamics and take the velocity vector and position vector at the moment of loss of connection (① Loss of propulsion (small perturbation by uncertainties) ② Maintenance of propulsion (no small perturbation)) as the initialized state. At the same time, we assume that the uncertainties, such as terrain, do not change dramatically in a short period, and then using the Kalman filter method, the average value of the distance error between the two cases and the exact position is **50.23cm(for case** ①**)** and **42.85cm (for case** ②**)** respectively, given the initialization state, which can predict the position of the submarine better.

For task 2, We prepared **seven types** of **detection equipment** for the main ship. Using a **multi-objective planning method**, we designed a **cost-effective and high-performance** way of purchasing the equipment. After scoring the performance using the entropy weighting method, we concluded that installing **3 side-scan sonar** and **7 magnetometer detections** is the best option. Additionally, we recommend adding search and rescue devices like underwater robots for timely rescue activities.

For task 3, The **rescue devices** on the main ship are given a search pattern to locate the lost power submarine. The underwater robot is used to perform the rescue and its shortest-moving path problem is solved using **genetic algorithms**. The maximum path found was **569.188m** and the minimum path was **300.781m**. The initial deployment point of the main ship was determined based on the predicted position obtained from task one.

For task 4, To extend the model and locate multiple submarines, the parameters of uncertainty factors need to be changed. An optimal path can be obtained using the **Traveler's Problem** of the Third Question, with a maximum path of **1117.34m** and a minimum path of **505.12m** when two submarines are around.

Finally, a sensitivity analysis is performed. In the prediction model, the prediction value obtained still has a small error by adjusting the force of the current on the submarine. In the equipment selection optimization model, a new optimal solution can be calculated by adjusting the equipment price and the maximum depth of the equipment detection. The model is robust and can be self-adjusted according to the results.

Keywords: Prediction-Search and Rescue Model, Multi-objective planning, genetic algorithms, Kalman filter

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1 Introduction

1.1 Problem Background

With the development of science and technology, mankind's exploration of nature does not only stop at land and sky, but also leaves human footprints in the ocean. Submersibles can carry human beings to the deep sea area for exploration, which faces a problem that cannot be ignored: safety.

In 2021, the Indonesian Navy submarine KRI Nanggala 402 was lost during a torpedo firing exercise, and the missing submarine was eventually found with all 53 crew members dead. This incident serves as a reminder to emphasize the importance of predicting and searching for submarine trajectories in order to prevent irreversible consequences if the submarine loses communication with the main ship or suffers from mechanical defects, including loss of propulsion of the submersible.

However, the underwater environment is complex and changeable, unlike search and rescue on land or on the surface, the position of the submersible is not only subject to its own speed, acceleration, rotational angular velocity and other factors, but also may be affected by the ocean currents, ocean density, and the geography of the seafloor, which makes it more difficult to predict the position of the submersible, and the time of search and rescue has been delayed as a result.

Therefore, we built a model to predict the position of the submersible over time to address safety issues for the company "Mini-Cruise Marine Submarine" (MCMS).

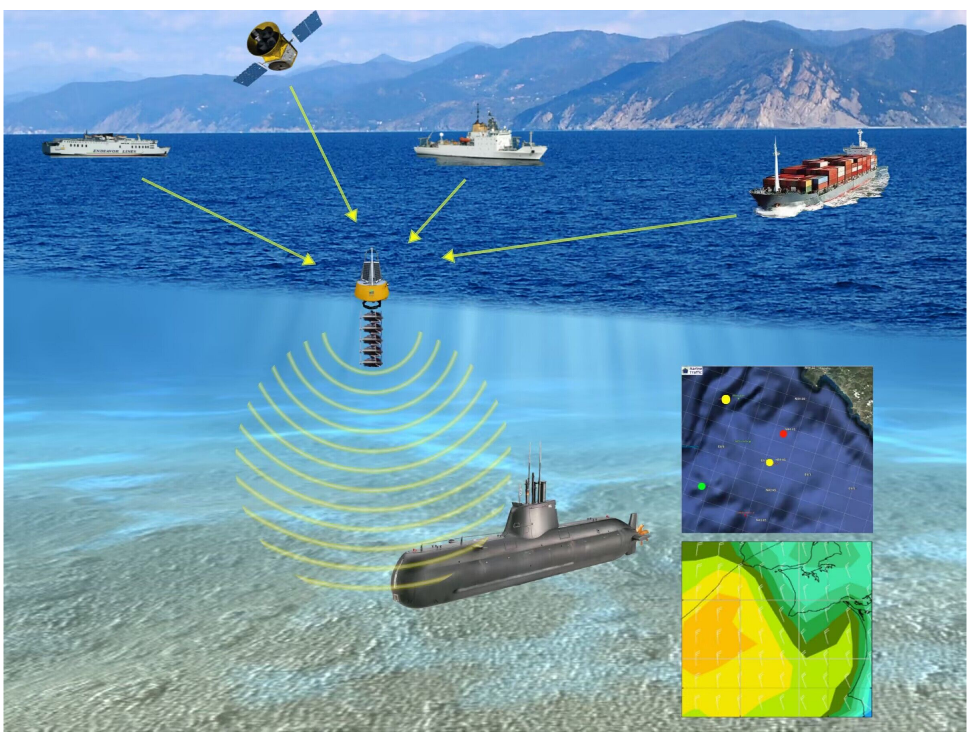


Figure 1: **Elevation relative to sea level [1]**

1.2 Restatement of the Problem

Task 1: Building a model to predict the position of a submersible.

- Position of the submersible over time.
- Consideration of environmental changes within the sea and identification of uncertainties affecting forecasting results.
- Finding the right equipment to regularly send information to the main ship that reduces uncertainty

Task 2:Deployment of equipment on board the main ship.

- Search equipment. Recommendations for MCMS companies are made for different types of equipment, taking into account the availability of the equipment, the costs associated with its maintenance, preparation and use.
- Rescue equipment. Rescue vessels carry suitable rescue equipment to assist in the search and rescue of submersibles.

Task 3: Development of a search and rescue model.

- Based on the information in the predictive position model, the location and search and rescue mode of the initially deployed equipment is recommended to find the missing submersible in the fastest possible time.
- Determine the probability of finding the submersible, which is a function of time and cumulative search results.

Task 4: Model Expansion.

- Location Expansion. Change the parameters to expand the model to other tourist destinations and analyze the changes in the results.
- Number of submersibles expansion. When multiple submersibles are moving in the same vicinity, factors such as the current environment will be affected and model changes at this point will be analyzed.

1.3 Our Work

MCMS is a company that builds submersibles that they intend to use to take tourists exploring shipwrecks on the Ionian seabed. In order for the MCMS company to obtain regulatory approval, we helped model and make recommendations to predict the location of the submersible over time. Our main work is described below:

- **Modeling the predicted positions.** We categorize the states of the submarine into two: a submarine with thrust and a submarine without thrust. Separately, we analyze the force for the two cases and randomly generate the motion path. Then we predict the submarine trajectory with Kalman filtering and perform uncertainty analysis.

- **Develop an optimization model for equipment selection.** We compared six metrics related to cost and performance for seven different devices, and finally selected three sweep sonars and seven magnetometers with good performance and low cost under a certain budget.
- **Modeling the fastest search and rescue.** We viewed finding the search path as a traveler's problem and used a genetic algorithm to find the optimal path.
- **Model Extension.** We extend the above model to other tourist destinations and to cases where multiple submersibles occur near the same location.
- **Sensitivity analysis.** We performed a sensitivity analysis of the model for predicting submarine locations. In response to the fluctuation of price and the up and down errors in performance, we conducted a sensitivity analysis on the selection of equipment, and the results show that the optimal solution remains unchanged within a certain error range.

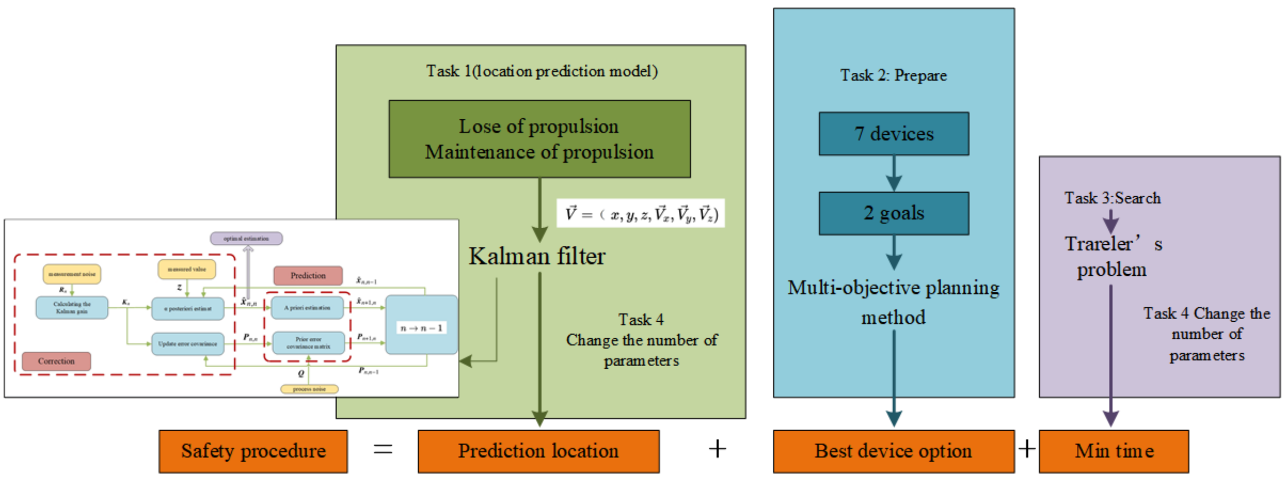


Figure 2: **The Framework of Our Work**

2 Assumptions and Notations

2.1 Assumptions

- **Ignore the shape of the submarine.**
The drag force on a submarine is related to the size of the area of the surface perpendicular to the direction of velocity, and a change in the direction of velocity during navigation will result in a change in the drag force, which, for simplicity of computation, is taken to be constant.
- **Ocean currents do not vary with time.**
Ocean currents are large-scale movements of seawater that are relatively constant throughout the year and have a certain direction. When modeling to predict the position of a submarine over time, we assume that the submarine is subjected to currents that do not vary over time.

- **It is assumed that the measurement noise and process noise caused by uncertainties such as ocean currents on submarine trajectory prediction follow a normal distribution.**
  This is the key assumption in Kalman filtering.

2.2 Notations

Table 1: **Symbols and their meanings**

Symbol	Meaning	Units
$\overrightarrow{F_g}$	the gravitational force on the submarine, directed vertically downward	N
$\overrightarrow{F_{buo}}$	the buoyancy force on the submarine,directed vertically upward	N
$\overrightarrow{F_c}$	the force generated by the effect of ocean currents	N
$\overrightarrow{F_{res}}$	the drag force on the submarine, which is in the opposite direction of the speed	N
$\overrightarrow{F}$	The submarine's own thrust.	N

3 Model I:Predictive Position Model

3.1 Establishment of the basic model

step1. analysis of forces

In order to predict the position of a submarine over time, the forces on the submarine in the water need to be analyzed. There are two types of submarine motion underwater:

(1). Submarines with mechanical problems or running out of fuel

Unlike a typical search and rescue conducted on land or at the surface, a defective submarine will find itself on the seafloor or at some point of neutral buoyancy underwater, where its position may be further influenced by ocean currents. The forces on the submarine are analyzed as shown in Figure 3. According to Newton's laws of motion we have that

$$m\overrightarrow{a} = \overrightarrow{F_g} + \overrightarrow{F_{buo}} + \overrightarrow{F_c} - \overrightarrow{F_{res}} \tag{1}$$

The position of the submarine is $\overrightarrow{r} = (x_t, y_t, z_t)$,then

$$\overrightarrow{a} = \frac{\partial^2 \overrightarrow{r}}{\partial t^2} \tag{2}$$

$$\overrightarrow{F_{buo}} = \rho_{water} g V_{dra} \tag{3}$$

$$\overrightarrow{F_{res}} = \frac{1}{2}\rho A C_d v^2 \tag{4}$$

Where C_d is the coefficient of drag and A is the area of the surface perpendicular to the direction of velocity. And v is the speed of submarine, which is determined by $\overrightarrow{v} = (\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t})$.

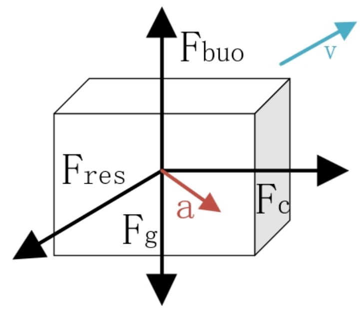


Figure 3: **Analysis of forces on a submarine in the water when there is no thrust**

(2). The submarine is lost, but still has thrust and can move normally

At this point, the submarine is subjected to an additional thrust as compared to the case without thrust above. This thrust will maintain the motion of the submarine so that it can maintain the trajectory of the spiral descent, i.e., downward uniform motion in the vertical direction, and circular motion on the surface parallel to the horizontal plane.

step2. Kalman filtering

Kalman filtering is an algorithm for optimally estimating the state of a linear system using its state equations with input and output observations of the system. Since the observed data includes the effects of noise and disturbances in the system, the optimal estimation can also be viewed as a filtering process. Kalman filtering will take into account the joint distribution of the values of each measurement at different times and then produce an estimate of the unknown variables, so it will be more accurate than an estimation based only on a single measurement. Figure 4 shows the flow chart of Kalman filtering method.

When predicting the position of submarines over time, we need to use Kalman filtering to improve the accuracy of the estimation due to the uncertainty of the environment and the limitation of the equipment. The multidimensional Kalman filter satisfies the following five equations [2]:Extrapolation equation of state, Covariance extrapolation equation, State update equation, Covariance update equation, Kalman gain equation.

$$\begin{cases} \hat{\boldsymbol{x}}_{n+1,n} = \boldsymbol{F}\hat{\boldsymbol{x}}_{n,n} + \boldsymbol{G}\boldsymbol{u}_n + \boldsymbol{w}_n \\ \boldsymbol{P}_{n+1,n} = \boldsymbol{F}\boldsymbol{P}_{n,n} + \boldsymbol{F}^T + \boldsymbol{Q} \\ \hat{\boldsymbol{x}}_{n,n} = \hat{\boldsymbol{x}}_{n,n-1} + \boldsymbol{K}_n(z - \boldsymbol{H}\hat{\boldsymbol{x}}_{n,n-1}) \\ \boldsymbol{P}_{n,n} = (\boldsymbol{I} - \boldsymbol{K}_n\boldsymbol{H})\boldsymbol{P}_{n,n-1}(\boldsymbol{I} - \boldsymbol{K}_n\boldsymbol{H})^T + \boldsymbol{K}_n\boldsymbol{R}_n\boldsymbol{K}_n^T \\ \boldsymbol{K}_n = \boldsymbol{P}_{n,n-1}\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{P}_{n,n-1}\boldsymbol{H}^T + \boldsymbol{R}_n)^{-1} \end{cases} \tag{5}$$

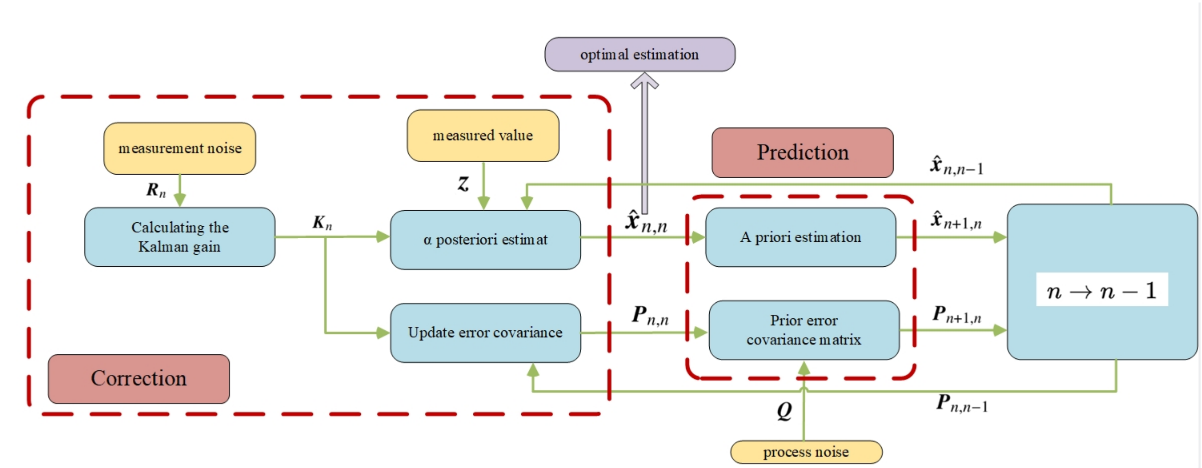


Figure 4: **Flow chart of Kalman filtering method**

Where:
$\hat{\boldsymbol{x}}_{n+1,n}$ is a predicted system state vector at time step $n + 1$;
$\hat{\boldsymbol{x}}_{n,n}$ is an estimated system state vector at time step n;
$\boldsymbol{u}_n$ is a control variable or input variable;

$\boldsymbol{w}_n$ is a process noise or disturbance - an unmeasurable input that affects the state. In the predictive position model, $\boldsymbol{w}_n$ is the **prediction error caused by the uncertainties** faced by the submarine while operating in the water, including the forces and moments exerted on the submarine by ocean currents, the errors caused by the complexity of the seafloor topography, and the errors caused by inaccuracies in the equipment, etc.

$\boldsymbol{F}$ is a transition matrix;
$\boldsymbol{G}$ is a control matrix or input transition matrix (mapping control to state variables);
$\boldsymbol{P}_{n,n}$ is the uncertainty of an estimate - covariance matrix of the current state;
$\boldsymbol{P}_{n+1,n}$ is the uncertainty of a prediction - covariance matrix for the next state;
$\boldsymbol{Q}$ is the process noise matrix;
$\hat{\boldsymbol{x}}_{n,n-1}$ is a predicted system state vector at time step$n - 1$;
$\boldsymbol{K}_n$ is a Kalman Gain;
z_n is a measurement;
$\boldsymbol{H}$ is an observation matrix;
$\boldsymbol{P}_{n,n-1}$ is the prior estimate uncertainty (covariance) matrix of the current state;
$\boldsymbol{R}_n$ is the Measurement Uncertainty;
$\boldsymbol{I}$ is an ldentity Matrix.

In the Kalman filtering method, our goal is to find an optimal Kn such that the estimated value $\hat{\boldsymbol{x}}_{n,n}$ is closest to the actual value $\boldsymbol{x}_{n,n}$, i.e., to find the most probable next position of the submarine based on the previous state of the submarine, in the presence of both process noise (effect of currents on the submarine's trajectory) and measurement noise (equipment error).

3.2 Consideration of uncertainties and selection of devices

1. Uncertainty factor

Accurate analysis of the submarine's motion state, force, etc. is the key to predicting the submarine's position, and in reality, the submarine's motion is affected by many factors, which will lead to the prediction results of the deviation. Among them, the most influential is **the environment of the ocean, tides, sea surface waves, seabed sedimentary structures**, etc. will affect the propagation of sonar signals, resulting in inaccurate positioning [2]. Moreover, the submarine's dynamics model in the sea is highly nonlinearly coupled, which makes it difficult to predict the accurate motion trajectory.

- **Environmental uncertainties**

When a submarine is **in the water**, the **thermocline** has a significant impact on the submarine's stealthiness and may also affect sound propagation leading to inaccurate sonar measurements. **Density stratification** has a great impact on the operation of submarine surfacing and diving, when the submarine passes through the density stratification may therefore suddenly sink, jeopardizing safety.

When a submarine is **maneuvering on the seabed**, mountain peaks may block the propagation of hydroacoustic sound. In addition, due to the complexity and variability of the seabed surface, as well as the **"Venturi" effect**, the submarine will be subjected to forces and moments related to the seabed surface, which will cause great difficulties in the submarine's near-sea maneuvering operations [2].

- **Equipment Errors**

With the continuous progress of science and technology, deep-sea submersibles are becoming more and more advanced, capable of carrying all kinds of electronic devices, mechanical equipment and personnel to reach all kinds of deep-sea environments quickly and accurately. However, the equipment will inevitably have certain errors, for example, **the inertial navigation system is still lack of accurate and reliable speed sensor**, which is not accurate enough in long-distance operation due to the existence of cumulative errors [3].

- **Submersible kinematic state uncertainty**

The external forces and moments on the submersible include propeller thrust, hydrodynamic forces, gravity and buoyancy and moments, etc., while the environmentally induced disturbance forces have to be analyzed according to the specific operating environment [4].

2. Equipment Selection

In order to minimize the impact of the uncertainty factors proposed above on the submarine's position prediction, the submarine should send information to the main ship at regular intervals to inform the main ship of the **seawater flow rate, temperature, depth and other indicators**, so as to enable the main ship to understand the environmental information in the vicinity of the submarine, and thus to improve the accuracy of the position prediction. The main equipment used includes the following:

- **localization sonar**

The localization sonar tells the divers where they are now, the mother ship is fitted with a localization sonar base array, and the submersible is fitted with a transponder, and they call out to each other with the help of sound waves.

- **Doppler velocimeter**

Doppler velocimeters use the **Doppler effect** to measure the **speed** at which the submersible is traveling, as well as the speed of the currents below the submersible, and after some arithmetic work, they are able to derive the trajectory of the submersible, letting both the submersible and the mother ship know each other's position.

- **inertial navigation equipment**

The inertial navigation system **utilizes inertial elements (accelerometers) to measure the acceleration of the carrier itself, and then obtains the velocity and position after integration and operation**, so as to achieve the purpose of navigation and positioning of the carrier. However, the positioning error of inertial navigation will disperse with the growth of time. In order to make up for this shortcoming of inertial navigation, a **combination of navigation** is usually used, i.e., with other navigation methods (e.g., odometer, heading projection), **Kalman filtering** is applied for information fusion, so as to achieve complementary effects.

- **Deep-sea sensors**

There is a wide range of deep sea sensors with various functions. For example, **dissolved oxygen meters** are used to assess water quality and measure the amount of dissolved oxygen in liquids. **Imaging sensors** are used for underwater photography. **Pressure sensors** are used to monitor seismic activity underwater.

3.3 Analysis of the results

(1).The submarine has thrust. The trajectory spirals down.

The actual motion of a submarine in the water can be viewed as a spiral descent. Firstly we randomly generated a trajectory of the submarine's motion, assuming that the submarine moves vertically downward in a uniform linear motion while moving in a circular motion in a plane parallel to the horizontal plane. Considering the error caused by the uncertainty factor, we obtained the topography and current data of the Ionian seabed on the software Panoply and exported them in Python. Then we used Kalman filter to predict the position of the submersible based on the state and position of the submersible at the previous moment.

As in Figure 5, the x-axis and y-axis indicate latitude and longitude, and the z-axis indicates the diving depth of the submersible. The background section plots a scatterplot of the topography of the Ionian seabed with different colors indicating the actual elevation. A local zoom has been applied to make the image more intuitive and concise. The blue trajectory indicates the observed trajectory of the submersible, i.e., the actual motion trajectory; the red trajectory indicates the predicted trajectory of

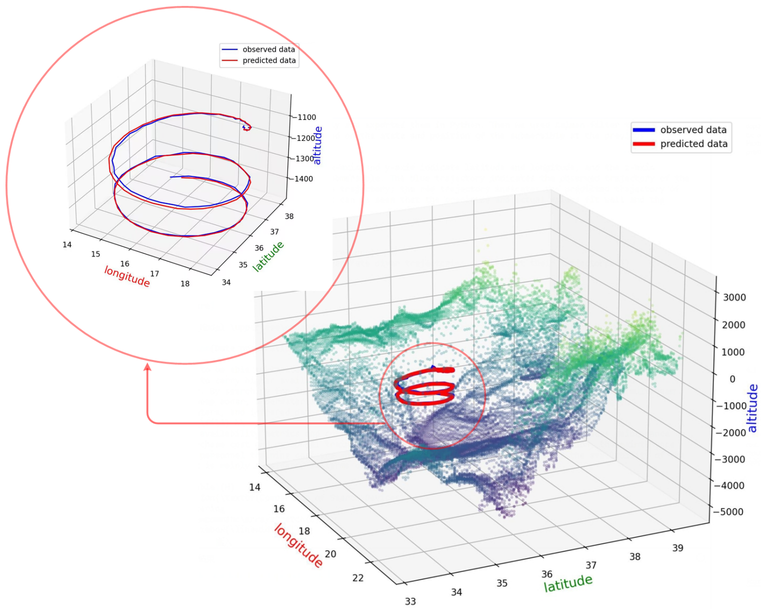


Figure 5: **Simulation to generate diver trajectories and predict their trajectories**

the submersible. From the figure, it can be seen that the predicted and actual values fit well and the prediction error is small.

(2).The submarine has no thrust and is moving roughly in the direction of the current.

When the submarine has no thrust, the submarine's trajectory is roughly a straight line. As shown in the Figure 6, we first randomly generated the actual motion trajectory of the submarine; the blue points are observation points indicating the position of the submarine at different times. Then we used Kalman filtering to predict the motion trajectory of the submarine, as shown by the red points. As a result, the red points and the blue points almost completely overlap, i.e., the prediction accuracy is high. The error between predicted and actual position is only 0.3%

4 Model II: Equipment Selection Optimization Model

4.1 Data preparation

In order to be able to locate a submarine quickly and accurately after it has been lost, it is necessary for MCMS to carry higher availability search equipment within a sufficient budget to be deployed if necessary. By searching for information, we selected the most commonly used equipment: **multibeam sonar, sweep sonar, underwater autonomous vehicles, underwater robots, buoys and locator tags, magnetometers, and infrared detectors**.

Specific availability metrics and costs are shown in Table 2. Among them, the cost includes maintenance cost, purchase cost and personnel training cost, and for the convenience of calculation, the maintenance cost and personnel training cost are annual average values. In the search process, the availability of the equipment is mainly considered in terms of both **depth and accuracy.**

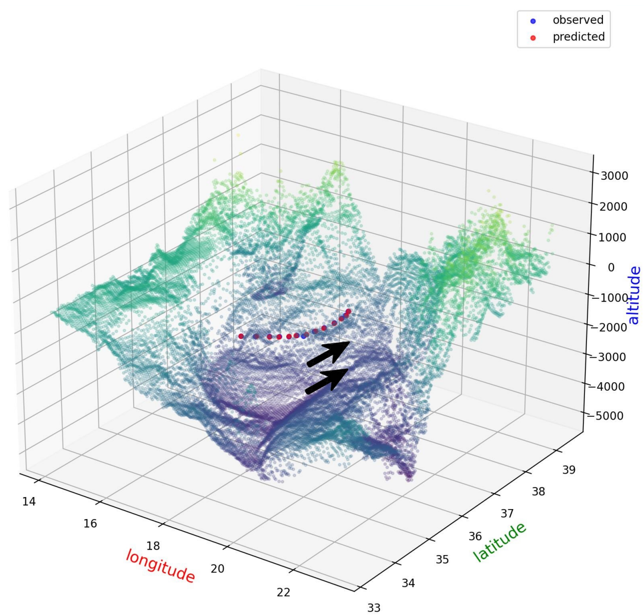


Figure 6: **Plot of actual and predicted trajectories when the submarine is not thrusting**

Table 2: **Comparison of Search Devices**

Devices	Equipment cost price (dollar/year)	Maintenance time(year)	Equipment purchase cost (dollar)	Personnel training cost (dollar/year)	Water depth (m)	Accuracy (mm)
Multibeam sonar	314.28	50	15714	500	500	6
Sweep sonar	71.42	50	3571	400	100	12.5
Underwater autonomous vehicles	1000	10	15714	750	1000	3
Underwater robots	1200	10	17142	1000	100	1
Buoys and locator tags	214.2	10	2142	200	500	10
Magnetometers	1833.3	30	55000	900	300	10
Infrared detectors	65.7	50	3285	200	300	0.1

4.2 Model construction

We categorize the evaluation of equipment into two areas: performance and cost. The goal is to select better performing equipment with a given budget. First, we evaluated the performance of the seven devices using the entropy weighting method, and the two reference indexes for device availability were the depth of water detected and the detection accuracy. Then we conducted a multi-objective planning to comprehensively evaluate the goodness of the seven devices.

1. Evaluating the performance of seven devices using the entropy weight method

- step1. Indicator positivization[5]

There are $m = 7$ objects to be evaluated, denoted by $i = 1, 2, \cdots, 7$ for multibeam sonar, sweep sonar, underwater autonomous vehicles, underwater robots, buoys and locator tags, magnetometers, and infrared detectors. There are $n = 2$ evaluation metrics, denoted by $j = 1, 2$ for depth and accuracy. They form the data matrix:

$$X = (x_{ij})_{m\times n} = \begin{bmatrix} 500 & 0.006 \\ 100 & 0.0125 \\ 1000 & 0.003 \\ 100 & 0.001 \\ 500 & 0.01 \\ 300 & 0.01 \\ 300 & 0.0001 \end{bmatrix} \tag{6}$$

In this case, the greater the water depth the better, the smaller the accuracy the better, and for the convenience of evaluation, we processed the accuracy index so that the result is greater and better:

$$x'_{ij} = max(x_{ij}) - x_{ij} \tag{7}$$

In this case, the greater the water depth the better, the smaller the accuracy the better, and for the convenience of evaluation, we processed the accuracy index so that the result is greater and better:$x'_{ij} = max(x_{ij}) - x_{ij}$
We get the normalization indicator:

$$X' = \begin{bmatrix} 500 & 0.0065 \\ 100 & 0 \\ 1000 & 0.0095 \\ 100 & 0.0115 \\ 500 & 0.0025 \\ 300 & 0.0025 \\ 300 & 0.0124 \end{bmatrix} \tag{8}$$

- step2. Data standardization

Since the water depth and accuracy are not of the same order of magnitude, they cannot be compared directly and should be kept within the same size range. Let the data matrix after normalization be $R = (r_{ij})m \times n$ where

$$r_{ij} = \frac{x'_{ij} - min(x'_j)}{max(x'_j) - min(x'_j)} \tag{9}$$

Then,

$$R = \begin{bmatrix} 0.4456 & 0.5251 \\ 0.0020 & 0.0020 \\ 1 & 0.7666 \\ 0.0020 & 0.9276 \\ 0.4456 & 0.2032 \\ 0.2238 & 0.2032 \\ 0.2038 & 1.0000 \end{bmatrix} \tag{10}$$

- step3. Calculate the information entropy

For the two metrics r_1 (depth) and r_2 (accuracy), the information entropy is

$$E_j = -\frac{1}{\ln m} \sum_{i=1}^{m} p_{ij} \ln p_{ij} \tag{11}$$

Where $p_{ij} = \frac{r_{ij}}{\sum_{j=1}^{n} r_{ij}}$ The result is $E = \begin{bmatrix} 0.7479 & 0.8424 \end{bmatrix}$ It can be seen that the information entropy Ej of the bathymetry index is smaller, indicating that its degree of variation is greater and the amount of information provided is also greater, so the bathymetry plays a greater role in the comprehensive evaluation.

- step4. Calculate the weights as well as the scores

The weight is

$$w_j = \frac{(1 - E_j)}{\sum_{j=1}^{n} (1 - E_j)} \tag{12}$$

The score is

$$S_j = \sum_{j=1}^{n} w_j r_{ij} \tag{13}$$

And the results is

$$W = \begin{bmatrix} 0.6152 & 0.3848 \end{bmatrix}$$

$$S = \begin{bmatrix} 52.3158 \\ 0.2197 \\ 100 \\ 39.3443 \\ 38.7073 \\ 23.7162 \\ 57.3974 \end{bmatrix}$$

From the results, it can be seen that underwater autonomous vehicles have the highest score i.e. optimal performance while sweep sonar has the lowest score i.e. worst performance.

2. multi-objective planning

In order to use a more efficient way of combining equipment within a certain budget, we use the multi-objective planning method to solve the problem. The two objectives are: 1. make the performance as much as possible greater than or equal to e. 2. make the cost as much as possible less than or equal to m. Each of the two objectives corresponds to the objective function $f_i(i = 1.2)$. By reviewing the data, we find that performance has a slightly higher priority level than cost, so the priority factors $P1 = 0.6, P2 = 0.4$.

- step1. Introduction of positive and negative bias variables

Let d_1^0 and d_2^0 denote the target values of performance and cost, respectively, i.e., $d_1^0 = e$ and $d_2^0 = q$. We introduce positive and negative bias variables, which have the following signs and meanings:

Deviation variables	**Notation**	**Hidden meaning**
Performance Positive Bias Variables	$d_1^+ = max(f_1 - d_1^0, 0)$	Excess of actual performance over target performance e
Negative performance bias variables	$d_1^- = -min(f_1 - d_1^0, 0)$	Portion of actual performance below target performance e
Cost Positive Bias Variables	$d_2^+ = max(f_2 - d_2^0, 0)$	Excess of actual performance over target performance q
Negative cost bias variables	$d_2^- = -min(f_2 - d_2^0, 0)$	Portion of actual performance below target performance q

- step2. List absolute and goal constraints

Absolute constraint: the total number of all devices should be less than n, whereupon $\sum_{i=1}^m n_i \le n$. Suppose the firm's total budget for search and rescue equipment is w, then $\sum_{i=1}^m n_i x_i \le w$. And fulfilled: $x_i, d_i^-, d_i^+ \le 0, i = 1, 2, \cdots, m$.

Objective constraints: In order to make the calculation more simple and easy to understand, the use of positive and negative deviation variables, the principle of "more backward, less complementary", we will turn the objective function into an equation constraints:

$$\begin{cases} \sum_{i=1}^m n_i u_i + d_1^- - d_1^+ = e \\ \sum_{i=1}^m n_i x_i + d_2^- - d_2^+ = q \end{cases} \tag{14}$$

- step3. Determine the objective function

The performance objective requires not less than the target value, then d_1^- the smaller the better; the cost objective requires not exceeding the target value, then d_2^+ the smaller the better, so the objective function is set to $f = min(P_1 d_1^- + P_2 d_2^+)$

In summary, our goals is to get $f = min(P_1 d_1^- + P_2 d_2^+)$

$$s.t. \begin{cases} \sum_{i=1}^{m} n_i \leq n \\ \sum_{i=1}^{m} n_i x_i \leq w \\ \sum_{i=1}^{m} n_i u_i + d_1^- - d_1^+ = e \\ \sum_{i=1}^{m} n_i x_i + d_2^- - d_2^+ = q \\ x_i, d_i^-, d_i^+ \leq 0, i = 1, 2, \cdots, m \end{cases} \tag{15}$$

4.3 Analysis of the results

When the expected efficiency $e = 400$, the expected cost $m = 20,000$, the upper limit on the number of equipment $n = 10$, and the upper cost limit $m = 30,000$, we have:

$$N = \begin{bmatrix} 0 \\ 3.1720 \\ 0 \\ 0 \\ 0 \\ 6.8280 \\ 0 \end{bmatrix}$$

Rounding the results yields: $n2 = 3, n6 = 7$, i.e., 3 sweep sonars, 7 magnetometers, which satisfies the need to keep the cost within the budget while getting more practical equipment.

5 Model III:Fastest Search and Rescue Model

5.1 Model construction

During the search and rescue process, we first detect several possible locations of the submarine using sonar, and then search for the optimal route to minimize the time to find the missing submarine. Assuming that the speed of search and rescue vessel is constant, our goal is transformed from a shortest time problem to a shortest path problem (Traveler's Problem (TSP)): starting from a point of origin, it is required to pass through all possible locations without repetition, and finally return to the point of origin with the shortest total path.

We choose genetic algorithm to solve the optimal path. The starting point of the genetic algorithm is to simulate the reproduction behavior of populations in nature, which will be more adapted to the environment from generation to generation after natural selection. Its basic idea is to simulate the concept of genes to encode the answer to the problem to be solved, then evaluation, superiority and inferiority, and finally decoding to reduce the genes to a feasible solution. The flowchart is shown in Figure 7.

- step1. Encoding

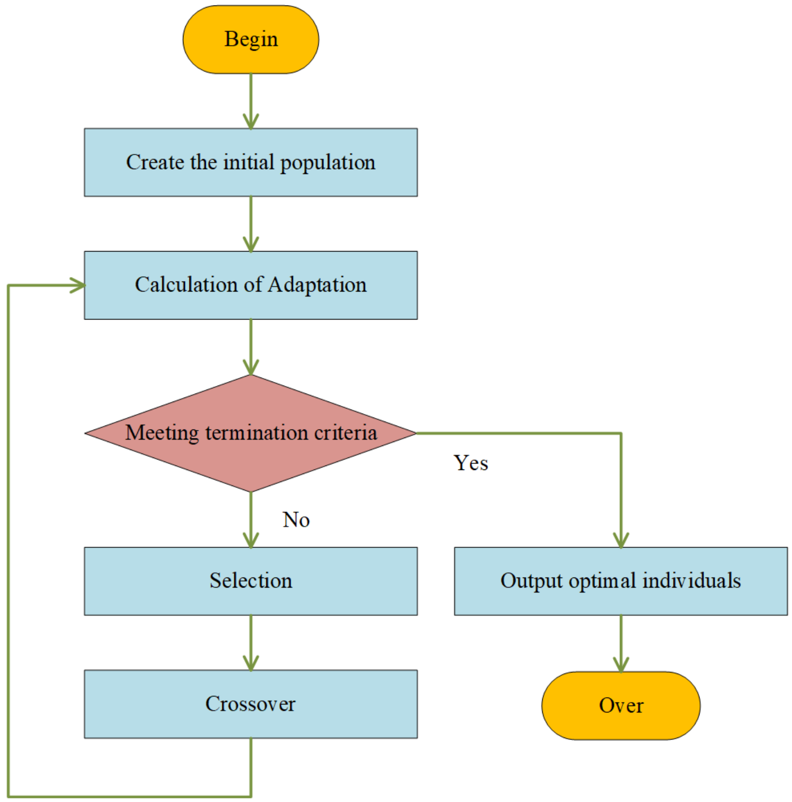


Figure 7: **Flowchart of genetic algorithm**

Represent the problem requiring a solution as a chromosome or individual in genetic space. Role: genes are segregated and uniformly expressed by all individuals. For the problem of searching for submarines, we use a real number encoding where each individual is a set of path sequences. The initial population, the search order, is generated randomly.

- step2. Calculating Adaptation

The fitness function, also called the evaluation function, is an indicator used to determine the degree of merit of individuals in a group. In general, the higher the fitness, the easier it is to be selected. We want the path traveled by the search and rescue ship to be as short as possible, so we define the fitness as the reciprocal of the total distance traveled. Let there are n possible locations for the submarine, and represent the distance by an $n \times n$ matrix, where D_{ij} is the distance from the ith location to the jth location, and the objective function is:

$$f = min \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} D_{ij} x_{ij} \tag{16}$$

Where, x_{ij} is the decision variable, $x_{ij} = 1$ if position i to position j is in the solution and 0 otherwise.

- step3. Option

Selection of individuals from the parent generation that enter into subsequent reproduction. The fitness values of all individuals in the population need to be summed up and then normalized, and ultimately the individuals corresponding to the region on which the random number falls are selected by random numbers.

- step4. Crossover

A new individual is generated by replacing and reorganizing parts of the structure of two parent individuals. First fix the crossover position, then find the corresponding position of the paired data, and finally disrupt the order and exchange the positions.

- step5. Variation

Changes in the gene values of certain loci of individual strings in the population are made to randomly search the entire space where the solution may exist, and to a certain extent, to find the global optimal solution.

5.2 Analysis of the results

The search process consists of three main steps:

1. Positioning. Firstly, the possible position of the submarine is detected by devices such as sonar or magnetic detector and indicated by coordinate points. Within the range of sonar detection, the search and rescue equipment searches according to an approximate spiral to make the shortest path, and the search pattern is as shown in the Figure 8, and the points of the possible positions searched for are as shown by the black dots in the Figure 10. The x-axis, y-axis and z-axis in the Figure 10 indicate the longitude, dimension and altitude respectively.

2. Find the optimal path. Then using a genetic algorithm, iterates the shortest path that can pass through all locations without repetition and eventually return to the starting point, forming a closed loop, the black connecting line in the Figure 10(a) is the shortest search path. The longest path in the search for a submersible is 569.188m and the shortest is 300.781m. The path convergence diagram is shown in Figure 9(a)

3. Search and Rescue. Along the search path, select the appropriate detection and rescue equipment to find the lost submersible.

In addition to this, we acquired remote sensing maps of bedrock in the vicinity of ionia, as shown in the Figure 11, to facilitate search and rescue efforts.

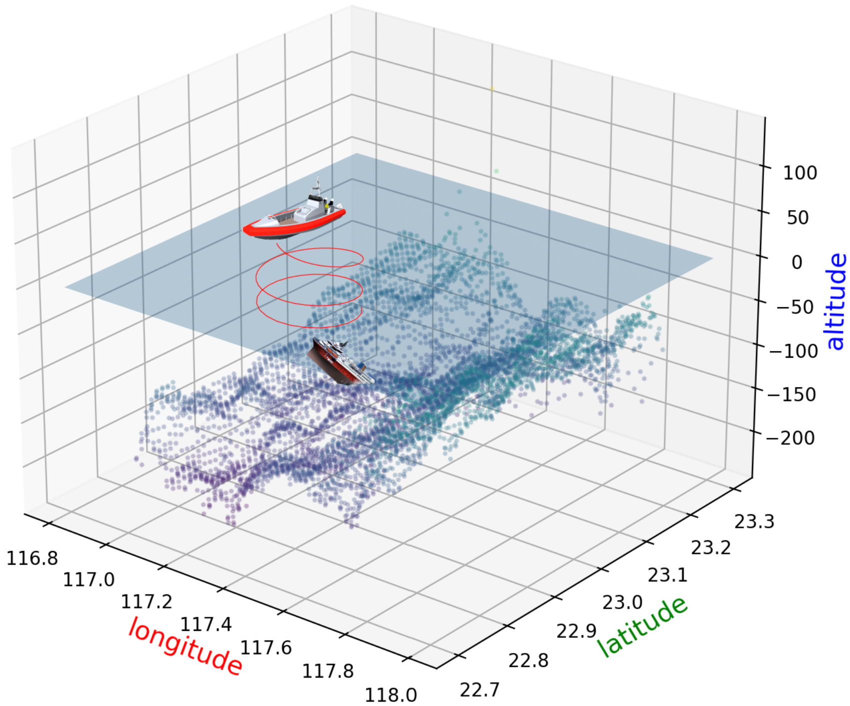


Figure 8: **Initial deployment point and search pattern of the main ship**

6 Model Expansion

6.1 Expansion to other regions

When extending the model to other destinations, water temperature, seafloor topography, current direction and pattern, and seawater density will be quite different, and we will mainly consider changes in topography and currents. We need to consider the geotransfer current to correct the behavior of the submersible. The basic formula for the geotransfer current is:

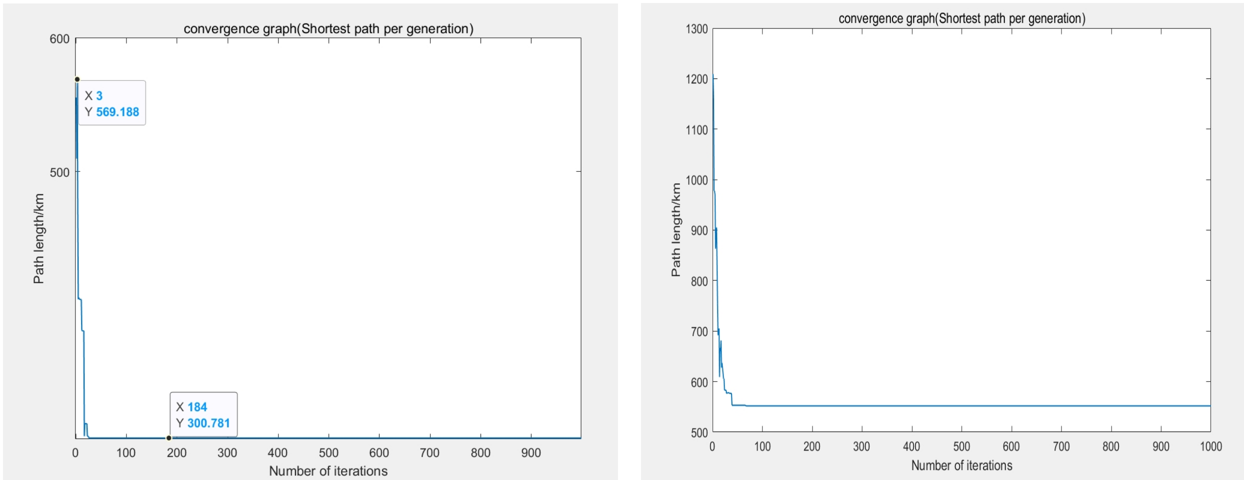
$$V = \frac{-g}{f} \times \frac{\triangle h}{\triangle x} \tag{17}$$

where v is the flow velocity, g is the gravitational acceleration, f is the Coriolis parameter (related to latitude), $\triangle h$ is the difference in sea surface height (usually proportional to the difference in underwater pressure), and $\triangle x$ is the horizontal distance.

The Coriolis parameter f is a parameter related to the rotation of the Earth to describe the effect of the Coriolis force, which is directly related to latitude. The formula for the Coriolis parameter is:

$$f = 2\Omega \sin \varphi \tag{18}$$

Where Ω is the angular velocity of the Earth's rotation, which is approximately $7.2921 \times 10^(- 5)$ arc degrees per second, and φ is the latitude.



(a) One submarine (b) Two submarines

Figure 9: **The shortest path convergence diagram**

6.2 Expansion to multiple submersibles

When multiple submersibles are active near the same position, the impact on the submersibles includes two main aspects:

- **communication.** Nearby submersibles will block or reflect the sonar signals sent and received, resulting in inaccurate positioning and the inability to send accurate position information to the main vessel at regular intervals.
- **Motion status.**Nearby submersibles may cause seawater disturbance, increasing the uncertainty of the force on the submersible.

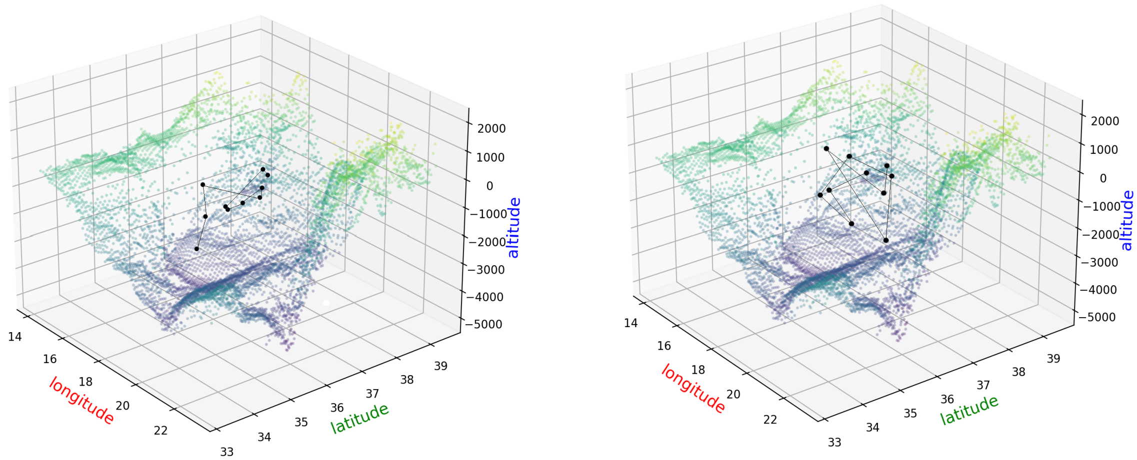
For both the primary vessel and the rescue vessel, the loss of multiple submersibles means an increase in the amount of rescue tasks and an increase in the difficulty of localization.

- Multiple submersibles operating near the same location, the disturbance of the sea water and the obstruction of other submersibles will affect the accuracy of the localization.
- Multiple objectives need to be considered in the search for the shortest possible rescue path to minimize the total rescue time and maximize the success rate of the rescue. The longest path in the search for the two submersibles was calculated to be 1117.34m and the shortest to be 505.12. The shortest path convergence diagram for the two yachts is shown in Figure 9(b)

7 Sensitivity Analysis:

7.1 Predictive Position Model

When predicting the position of a submarine, it faces environmental uncertainty, with different positions being affected by different currents and possibly different forces at different times. These will affect



(a) Shortest path (b) Longest path

Figure 10: **Searching for the longest and shortest route paths of a submarine**

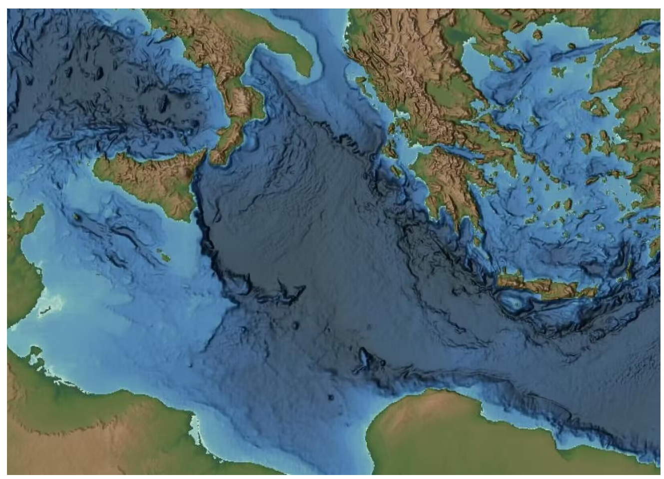


Figure 11: **Sensing maps of bedrock in the vicinity of ionia**

the trajectory of the submarine and increase the difficulty of prediction. Within the acceptable range, we modify the force of the ocean current on the submarine, and the modified predicted trajectory still almost coincides with the actual trajectory. The model passed the sensitivity test.

7.2 Equipment Selection Optimization Model

In determining the best option for carrying equipment, we have listed price and performance indicators for each type of equipment. In reality, there is a price difference between different models of the same equipment and performance varies. Therefore, we vary the price of each device up or down by \$10, and the depth up or down by 10 m. With the same budget and objectives, the optimal solution we get is still to buy 3 sweep sonars, 7 magnetometers. the model passes the sensitivity test.

8 Strengths and Weaknesses

8.1 Strengths

- **Extensible and flexible**
  We downloaded and exported a topographic map of the Ionian seafloor from the software Panoply, and considered the effect of ocean currents on the submarine's trajectory by gathering information. The method can be easily extended to other regions by simply importing the seafloor topography and current distribution of different regions.

- **High predictive accuracy**
  In the predictive position model, we use Kalman filtering, at which time the predicted position of the submarine depends only on the motion state of the previous moment, and the accuracy is greatly improved.

- **Visualization**
  We did a good job of visualization, including a topographic map of the Ionian seafloor, a remote sensing map of the bedrock near the Ionian, a flowchart of the Kalman Filter method, a flowchart of the Genetic Algorithm, a table comparing the various metrics of the seven different devices, and so on.

- **Synthesis, scientific**
  We have taken into account the effects of seafloor topography and ocean currents on the submarine's position, and we have searched a large amount of literature for scientific and reliable data sources.

8.2 Weaknesses

- **Genetic Algorithm Flaws**
  In the fastest search and rescue model, we use genetic algorithm, but here the optimal solution is a local optimal solution and there is no guarantee that the resulting solution is a global optimal solution.

- **Simplified Submarine Shape**
  When analyzing the forces on a submarine, the submarine is assumed to be a rectangular body for ease of calculation. In reality, the shape of the submarine is more complex, which makes the force analysis more complicated and variable.

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MEMO to CEO of MCMS, Inc.

To: MCMS, Inc. CEO
From: Team 2422542
Date: February 5, 2024

We were aware that MCMS wanted to use the submersible to take tourists to explore the wreckage of a shipwreck on the Ionian seabed, and that safety procedures would need to be put in place to gain regulatory approval before departure, to prevent loss of communication with the main vessel and mechanical failures, including loss of power to the submersible. We therefore developed predictive position models to predict the position of the submersible over time. We have also developed equipment selection optimisation models and fastest search and rescue models so that if a submersible is lost to the host vessel it can be found the fastest with the best performing equipment at the least cost. I am honoured to have the opportunity to present our findings to you.

After the loss of a submersible, a prediction of the position of the submersible over time is first made, and two scenarios are possible:

1. No malfunctions occurred and there was sufficient thrust to maintain normal travel. At this time, the trajectory of the submersible is spiralled downwards, ignoring the perturbations brought about by ocean currents to the trajectory of the submersible.

2. A malfunction occurs and there is no thrust. At this time, the movement of the submersible is greatly affected by ocean currents and terrain, and the trajectory is linear.

We first simulated the motion trajectories of the submersible in the two cases respectively, and then predicted them with the position prediction model. The difference between the actual position and the predicted position is only 0.032%, which makes the prediction results very accurate, and therefore it is recommended to use this model to predict the possible position of the lost submersible.

In order to search for lost submersibles faster and at a lower cost, the choice of equipment is crucial. The seven most common and highly used types of equipment on the market are: multibeam sonar, sweep sonar, underwater autonomous vehicles, underwater robots, buoys and locator tags, magnetometers, and infrared detectors. Considering the depth of detection and detection accuracy as the two most important performance metrics, we scored the performance of these seven devices in the following order: 52.3158, 0.2197, 100, 39.3443, 38.7073, 23.7173, 23.7173, 39.3443, 38.7073, 23.7162, and 57.3975.

Considering the cost of manual training, purchase costs, and annual maintenance costs, in order to select 10 devices with a target performance score of 400 and a target cost of $20,000withinabudgetof$30,000, we recommend purchasing three Sweep Sonars and seven Magnetometers.Additionally. We recommend that the rescue vessel be assisted by underwater robots that are flexible and easy to operate, so that rescue can be carried out in a timely manner upon detection of the submersible, reducing casualties and property damage.

In order to find the lost submarine faster, we use the fastest search and rescue model to plan the shortest search and rescue route. We propose to predict the course of action of the submarine after the loss of the submarine, and find the most probable position of the submarine at this moment, which will be the initial deployment point of the main ship search equipment. It is recommended that the search pattern be centred on the initial deployment point in a circular pattern for faster and wider search. After determining the likely location of the submersible, we recommend using the path provided in the fastest search and rescue model to pass through each possible location without repetition to improve search and rescue efficiency and reduce the time to find the missing submersible.The calculated path length for searching one submersible is $569.188m$ and the shortest is $300.781m$, while the path length for searching two submersibles is $1,117.34m$ and the shortest is 505.12. From this, we can see that our planning greatly reduces the search paths and greatly shortens the search and rescue time.

It is important to note that our model can be extended to any area and multiple submersibles. We sincerely hope that our models and suggestions will help you to develop reasonable and efficient safety procedures for the safety of your visitors. Thank you very much for this valuable opportunity!