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The efficient modelling of high frequency transaction data: A new application of estimating functions in financial economics

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HIGHLIGHTS

- Parameter estimation of ACD models using the Estimating Functions (EF) approach.
- Study the finite sample behaviour of corresponding new estimators.
- Investigate the asymptotic behaviours of these proposed estimators.

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ABSTRACT

This paper investigates the Estimating Function method in the context of ACD modelling and appraises the properties of these estimates. A simulation study is conducted to demonstrate that these estimates are more efficient than the corresponding ML and QML estimates.

1. Introduction

In recent years with the rapid developments in computing power and storage capacity, it is possible to record every single transaction together with its characteristics (such as price, volume, etc.) in finance. The availability of these intraday datasets has aided in evolving a new area of financial research based on high frequency data analysis. A distinctive feature of intraday data is that observations are irregularly time-spaced and these irregular time intervals may convey important information. Motivated by these considerations, Engle and Russell (1998) developed the class of Autoregressive Conditional Duration (ACD) models for such irregular spaced data. Engle and Russell's ACD models show that they can successfully illustrate the progression of time durations for heavily traded or high frequency stocks. In the ACD specification, the mean of the distribution of inter-trade durations is assumed to depend on past durations.

In Engle and Russell's study of the unexplained structure in ACD residuals for the International Business Machines (IBM) stock they found evidence supporting the existence of nonlinear effects of recent durations on the conditional mean. In particular, these effects seem to be lower than the ones predicted by the linear specification for both very long and very short durations.

Following the findings of Engle and Russell (1998), several substantive extensions to the basic model with nonlinear specifications for studying the behaviour of irregularly time-spaced

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financial data have been put forward. These extensions are aimed at providing some additional flexibility to the original ACD model so that some of its empirical and statistical drawbacks can be addressed. These extensions include Bauwens and Giot's (2000) Logarithmic ACD (Log-ACD) model, Dufour and Engle's (2000) Box–Cox ACD (BCACD) model and Exponential ACD (EXACD) model, and Bauwens and Veredas (2004) Stochastic Conditional Duration Model. A good review of the various ACD models can be found in Pacurar (2008).

Another issue in ACD modelling is the choice of a suitable distribution for the errors. Engle and Russell (1998) used the theory of monotonic hazard functions such as the Exponential and Weibull distributions and successfully applied these to model the data on transactions of IBM stocks. Due to the fact that these distributions have restrictions and weak performances in practice, many authors have proposed a number of alternative flexible distributions in applications such as the Burr distribution (Grammig and Maurer, 2000), Generalized F distribution (Hautsch, 2001) and the Generalized Gamma distribution (see, Lunde, 1999 and Bauwens et al., 2004).

The most common methods of parameter estimation for ACD models are the Maximum Likelihood (ML) and Quasi Maximum Likelihood (QML) methods. Engle and Russell (1998) use the ML method to estimate the parameters of ACD models. Also see Bauwens and Giot (2000), and Dufour and Engle (2000) for details. Applications of ACD models are discussed by Allen et al. (2008, 2009) using the QML. These methods do not work well unless we can identify the distribution of the error.

In this paper, we use the theory of Estimating Function (EF) as an alternative method in the parameter estimation of nonlinear specifications and various popular distributions of errors including BCACD(p, q) and EXACD(p, q) models with Exponential, Weibull and Generalized Gamma (G.Gamma) distributions. This EF method has been successfully applied in many time series models including the class of ACD models. For example, David and Turtle (2000) applied the EF method in the context of autoregressive conditional heteroscedasticity (ARCH) models. Peiris et al. (2007) compared the performance of the EF and ML estimates of basic ACD models with linear specifications using a large scale simulation study. Pathmanathan et al. (2009) have obtained further simulation results based on different non-negative distributions for errors. Allen et al. (2012) considered the class of ACD models with errors from the standard Weibull distribution to develop the EF estimation procedure.

The remainder of this paper is organized as follows. Section 2 reviews the general class of ACD models including BCACD(p, q) and EXACD(p, q) models. Section 3 discusses the methodologies adopted for assessing estimating performances, namely the EF ML and QML methods. Section 4 illustrates the parameter estimation results. Section 5 concludes with some significant remarks.

2. A review of general ACD(p, q) models

Let t_i be the time of the i-th transaction and let x_i be the i-th adjusted duration such that $x_i = t_i - t_{i-1}$. Let

$$\psi_i = E[x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1] = E[x_i \mid F_{i-1}], \tag{1}$$

where F_{i-1} is the information set available at $(i-1)$-th trade.

Then, the basic ACD model for the variable x_i is defined as

$$x_i = \psi_i \varepsilon_i, \tag{2}$$

where ε_i is a sequence of independently and identically distributed (i.i.d.) non-negative random variables with a known density $f(\cdot)$ and ε_i is independent of F_{i-1}.

This paper considers the following ACD specifications based on BCACD(p, q) and EXACD(p, q) due to Dufour and Engle (2000). They have discussed two main drawbacks of linear ACD or LINACD (p, q) models with constraints on the parameters to ensure non-negative durations and the assumption of linearity being inappropriate in many applications.

Now consider the following nonlinear ACD specifications:

$$\text{(i) BCACD}(p, q) : \ln \psi_i = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{i-j}^{\delta} + \sum_{j=1}^{q} \beta_j \ln \psi_{i-j}, \tag{3}$$

where ω, α_j, β_j and δ are parameters.

$$\text{(ii) EXACD}(p, q) : \ln \psi_i = \omega + \sum_{j=1}^{p} \left[\alpha_j \varepsilon_{i-j} + \delta_j \left|\varepsilon_{i-j} - 1\right|\right] + \sum_{j=1}^{q} \beta_j \ln \psi_{i-j}, \tag{4}$$

where ω, α_j, β_j and δ_j are parameters.

The main problem that remains is the estimation of parameters. With that view in mind Section 3 reviews the EF, ML and QML estimation methods for ACD modelling.

3. Parameter estimation

This section considers the estimation of parameters using the EF method and compares the results via the ML and QML methods.

3.1. The EF method

Let $\{x_1, x_2, \ldots, x_n\}$ be a discrete-time stochastic process and we are interested in fitting a suitable model based on this sample of size n. Let Θ be a class of probability distributions F on R^n and $\boldsymbol{\theta} = \theta(F)$, $F \in \Theta$, be a vector of real parameters.

Suppose that the real valued function $h_i(\cdot)$ of $x_1, x_2, \ldots, x_i$ and $\boldsymbol{\theta}$ satisfy

$$E_{i-1,F}[h_i(\cdot)] = 0, \quad (i = 1, 2, \ldots, n, F \in \Theta) \tag{5}$$

and

$$E(h_i h_j) = 0, \quad (i \neq j) \tag{6}$$

where $E_{i-1,F}(\cdot)$ denotes the expectation holding the first $i-1$ values $x_1, x_2, \ldots, x_{i-1}$ fixed, $E_{i-1,F}(\cdot) \equiv E_{i-1}$, $E_F(\cdot) \equiv E(\cdot)$(unconditional mean) and $h_i(\cdot) = h_i$.

Any real valued function $g(\mathbf{x}; \boldsymbol{\theta})$, of the random vector $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ and the parameter $\boldsymbol{\theta}$, that can be used to estimate $\boldsymbol{\theta}$ is called an estimating function. Under standard regularity conditions (see e.g. Godambe, 1985), the function $g(\mathbf{x}; \boldsymbol{\theta})$ satisfying $E[g(\mathbf{x}; \boldsymbol{\theta})] = 0$ is called a regular unbiased estimating function. Following Godambe (1960) and Godambe and Thompson (1978, 1984), an optimal estimate of $\boldsymbol{\theta}$ must satisfy the following:

(i) the values of $g(\mathbf{x}; \boldsymbol{\theta})$ are clustered around 0, as much as possible (i.e. $E[g^2(\mathbf{x}; \boldsymbol{\theta})]$ should be as small as possible);
(ii) it is desirable that $E[g(\mathbf{x}; \boldsymbol{\theta} + \delta\boldsymbol{\theta})]$, $\delta > 0$, should be as far away from 0 as possible. This is conveniently translated as $E\left(\left[\frac{\partial g(\mathbf{x};\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right]\right)$ should be as large as possible.

Therefore, among all regular unbiased EFs $g(\mathbf{x}; \boldsymbol{\theta})$, $g^*(\mathbf{x}; \boldsymbol{\theta})$ is said to be optimum if

$$E[g^2(\mathbf{x}; \boldsymbol{\theta})] \Big/ \left\{E\left(\left[\frac{\partial g(\mathbf{x}; \boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right]\right)\right\}^2 \tag{7}$$

is minimized for all $F \in \Theta$ at $g(\mathbf{x}; \boldsymbol{\theta}) = g^*(\mathbf{x}; \boldsymbol{\theta})$. An optimal estimate $\boldsymbol{\theta}$ is obtained by solving the optimum estimating equation(s) so that $g^*(\mathbf{x}; \boldsymbol{\theta}) = 0$.

Table 1
Estimation results for the standardized Exponential BCACD(1, 1) model with various distributions obtained from sample size $n = 500$ with $N = 2000$ simulation runs ($\omega = 0.30, \alpha_1 = 0.20, \delta = 2.0, \beta_1 = 0.50$ and $\psi_1 = 0.50$). Data are generated from various distributions as given in column 1. Values in parentheses are obtained from sample size $n = 2000$.

True standardized distribution	$\hat{\omega}$		$\hat{\alpha}_1$		$\hat{\delta}$		$\hat{\beta}_1$	
Exponential	ML	EF	ML	EF	ML	EF	ML	EF
Mean	0.3016 (0.2996)	0.3064 (0.2991)	0.2012 (0.2002)	0.2024 (0.2012)	1.9996 (1.9992)	1.9987 (1.9964)	0.4994 (0.5002)	0.4952 (0.5004)
Bias	0.0016 (−0.0004)	0.0064 (−0.0009)	0.0012 (0.0002)	0.0024 (0.0012)	−0.0004 (−0.0008)	−0.0013 (−0.0036)	−0.0006 (0.0002)	0.0048 (0.0004)
SE	0.0425 (0.0164)	0.0643 (0.0295)	0.0283 (0.0113)	0.0452 (0.0203)	0.0845 (0.0251)	0.1488 (0.0640)	0.0250 (0.0081)	0.0401 (0.0186)
MSE	0.0018 (0.0003)	0.0042 (0.0009)	0.0008 (0.0001)	0.0020 (0.0004)	0.0071 (0.0006)	0.0221 (0.0041)	0.0006 (0.0001)	0.0016 (0.0003)
Weibull with $\gamma = 1.5$	QML	EF	QML	EF	QML	EF	QML	EF
Mean	0.3043 (0.3015)	0.3051 (0.3028)	0.2018 (0.2001)	0.2062 (0.2013)	2.0116 (2.0013)	1.9977 (1.9993)	0.4959 (0.4989)	0.4927 (0.4968)
Bias	0.0043 (0.0015)	0.0059 (0.0028)	0.0018 (0.0001)	0.0062 (0.0013)	0.0116 (0.0013)	−0.0023 (−0.0007)	−0.0041 (−0.0011)	−0.0073 (−0.0032)
SE	0.0732 (0.0336)	0.1007 (0.0345)	0.0472 (0.0202)	0.0898 (0.0259)	0.2182 (0.0900)	0.2623 (0.1176)	0.0538 (0.0242)	0.0546 (0.0274)
MSE	0.0054 (0.0011)	0.0101 (0.0012)	0.0022 (0.0004)	0.0081 (0.0007)	0.0478 (0.0081)	0.0688 (0.0138)	0.0029 (0.0006)	0.0030 (0.0008)
G.Gamma with $\kappa = 2.0,\ \gamma = 1.0$	QML	EF	QML	EF	QML	EF	QML	EF
Mean	0.3040 (0.3004)	0.3047 (0.3009)	0.2004 (0.2006)	0.2046 (0.2019)	2.0129 (1.9977)	1.9952 (1.9949)	0.4978 (0.4991)	0.4946 (0.4980)
Bias	0.004 (0.0004)	0.0047 (0.0009)	0.0004 (0.0006)	0.0046 (0.0019)	0.0129 (−0.0023)	−0.0048 (−0.0051)	−0.0022 (−0.0009)	−0.0054 (−0.0020)
SE	0.0666 (0.0295)	0.0671 (0.0332)	0.0410 (0.0177)	0.0508 (0.0236)	0.1764 (0.0710)	0.2128 (0.0960)	0.0476 (0.0205)	0.0513 (0.0251)
MSE	0.0045 (0.0009)	0.0045 (0.0011)	0.0017 (0.0003)	0.0026 (0.0006)	0.0313 (0.0050)	0.0453 (0.0092)	0.0023 (0.0004)	0.0027 (0.0006)

Linear unbiased estimating functions

Consider the class of linear unbiased estimating functions L formed by

$$g(\mathbf{x}; \boldsymbol{\theta}) = \sum_{i=1}^{n} h_i a_{i-1},$$

where h_i are as defined in (5) and (6), and a_{i-1} is a suitably chosen function of the random variates $x_1, x_2, \ldots, x_{i-1}$ and the parameter $\boldsymbol{\theta}$ for all $i = 1, 2, \ldots, n$.

It is well known that the function $g^*(\mathbf{x}; \boldsymbol{\theta})$ minimizing (7) is given by

$$g^*(\mathbf{x}; \boldsymbol{\theta}) = \sum_{i=1}^{n} h_i a^*_{i-1},$$

where $a^*_{i-1} = E_{i-1}\left[\frac{\partial h_i}{\partial \boldsymbol{\theta}}\right] / E_{i-1}[h_i^2]$ (See Godambe, 1985 for details).

An optimal estimate of $\boldsymbol{\theta}$ (in the sense of Godambe, 1985) can be obtained by solving the equation(s) $g^*(\mathbf{x}; \boldsymbol{\theta}) = 0$.

Below we discuss the EF method for estimating the parameters of BCACD(p, q) and EXACD(p, q) models.

3.1.1. *Estimation of ACD models using the EF method*

Consider the ACD models given by $x_i = \psi_i \varepsilon_i$ with specifications given by (3) and (4) where $\{\varepsilon_i\}$ is a sequence of i.i.d. random variables with $E(\varepsilon_i) = 1$ and $\mathrm{Var}(\varepsilon_i) = \sigma_\varepsilon^2$.

Suppose that $\{\varepsilon_i\}$ follows a certain standardized distribution, that is a distribution with unit expectation. It can be seen that the mean and variance of the conditional distribution of x_i given F_{i-1} are $\mu_i = \psi_i$ and $\sigma_i^2 = \psi_i^2 \sigma_\varepsilon^2$ respectively.

To find the EF estimates for BCACD(p, q) and EXACD(p, q) models, we find the function h_i to satisfy the conditions in (5) and (6). An obvious choice for h_i is clearly $h_i = \psi_i - x_i$.

Following the theorem due to Godambe (1985), we construct a linear unbiased estimating function such that $g^*(\mathbf{x}; \boldsymbol{\theta}) = \sum_{i=1}^{n} h_i a^*_{i-1}$ where n is the number of observations.

It can be seen that the optimal value of a_i in the sense of Godambe (1985) is given by

$$a^*_{i-1} = E_{i-1}\left[\frac{\partial h_i}{\partial \boldsymbol{\theta}}\right] \Big/ E_{i-1}[h_i^2] = E_{i-1}\left[\frac{\partial h_i}{\partial \boldsymbol{\theta}}\right] \Big/ \psi_i^2 \sigma_\varepsilon^2,$$

where $\boldsymbol{\theta}$ is a parameter vector for the above model specifications (3) and (4).

The corresponding optimal EFs for the model specifications of BCACD(p, q) and EXACD(p, q) are given by

$$g^*(\mathbf{x}; \boldsymbol{\theta}) = \sum_{i=1}^{n} \frac{1}{\psi_i^2 \sigma_\varepsilon^2} E_{i-1}\left[\frac{\partial h_i}{\partial \boldsymbol{\theta}}\right] (\psi_i - x_i),$$

where $E_{i-1}\left[\frac{\partial h_i}{\partial \boldsymbol{\theta}}\right]$ are the conditional expectations of the partial derivatives of h_i with respect to the parameters. The estimates $\hat{\boldsymbol{\theta}}$ can be obtained by solving the nonlinear equations of $g^*(\mathbf{x}; \boldsymbol{\theta}) = 0$. We can use the method reported in Allen et al. (2012) to obtain the partial derivatives of h_i with respect to the parameters for these two models.

Below, we state the associated results for ML and QML estimations for comparison.

3.2. *Maximum Likelihood (ML) and Quasi Maximum Likelihood (QML) methods*

In the literature, the parameters of the ACD models with various specifications are estimated by ML and QML methods. In order to

Table 2
Estimation results for the standardized Exponential EXACD(1, 1) model with various distributions obtained from sample size $n = 500$ with $N = 2000$ simulation runs ($\omega = 0.20$, $\alpha_1 = 0.30$, $\delta_1 = 0.40$, $\beta_1 = 0.30$ and $\psi_1 = 0.50$). Data are generated from various distributions as given in column 1. Values in parentheses are obtained from sample size $n = 2000$.

True standardized distribution	$\hat{\omega}$		$\hat{\alpha}_1$		$\hat{\delta}_1$		$\hat{\beta}_1$	
Exponential	ML	EF	ML	EF	ML	EF	ML	EF
Mean	0.2189 (0.2046)	0.2204 (0.2038)	0.2933 (0.2980)	0.2937 (0.2985)	0.4024 (0.3997)	0.4012 (0.3997)	0.2842 (0.2956)	0.2834 (0.2976)
Bias	0.0189 (0.0046)	0.0204 (0.0038)	−0.0067 (−0.0020)	−0.0063 (−0.0015)	0.0024 (−0.0003)	0.0012 (−0.0003)	−0.0158 (−0.0044)	−0.0166 (−0.0024)
SE	0.0926 (0.0435)	0.0948 (0.0467)	0.0607 (0.0300)	0.0614 (0.0306)	0.0897 (0.0445)	0.0917 (0.0453)	0.0759 (0.0367)	0.0782 (0.0383)
MSE	0.0089 (0.0019)	0.0094 (0.0022)	0.0037 (0.0009)	0.0038 (0.0009)	0.0081 (0.0020)	0.0084 (0.0021)	0.0060 (0.0014)	0.0064 (0.0015)
Weibull with $\gamma = 1.5$	QML	EF	QML	EF	QML	EF	QML	EF
Mean	0.2185 (0.2029)	0.2171 (0.2067)	0.2947 (0.2986)	0.2967 (0.2983)	0.4022 (0.4010)	0.3952 (0.3994)	0.2839 (0.2976)	0.2878 (0.2951)
Bias	0.0185 (0.0029)	0.0171 (0.0067)	−0.0053 (−0.0014)	−0.0033 (−0.0017)	0.0022 (0.0010)	−0.0048 (−0.0006)	−0.0161 (−0.0024)	−0.0122 (−0.0049)
SE	0.0878 (0.0431)	0.0877 (0.0426)	0.0498 (0.0246)	0.0493 (0.0250)	0.0801 (0.0402)	0.0804 (0.0406)	0.0824 (0.0403)	0.0847 (0.0415)
MSE	0.0081 (0.0019)	0.0080 (0.0019)	0.0025 (0.0006)	0.0024 (0.0006)	0.0064 (0.0016)	0.0065 (0.0016)	0.0070 (0.0016)	0.0073 (0.0017)
G.Gamma with $\kappa = 2.0$, $\gamma = 1.0$	QML	EF	QML	EF	QML	EF	QML	EF
Mean	0.2147 (0.2036)	0.2153 (0.2051)	0.2949 (0.2997)	0.2967 (0.2990)	0.4000 (0.3997)	0.3980 (0.3998)	0.2899 (0.2962)	0.2888 (0.2953)
Bias	0.0147 (0.0036)	0.0153 (0.0051)	−0.0051 (−0.0003)	−0.0033 (−0.0010)	0.0000 (−0.0003)	−0.0020 (−0.0002)	−0.0101 (−0.0038)	−0.0122 (−0.0039)
SE	0.0862 (0.0413)	0.0861 (0.0430)	0.0527 (0.0252)	0.0541 (0.0257)	0.0824 (0.0410)	0.0824 (0.0414)	0.0833 (0.0294)	0.0824 (0.0413)
MSE	0.0076 (0.0017)	0.0077 (0.0019)	0.0028 (0.0006)	0.0029 (0.0007)	0.0068 (0.0017)	0.0068 (0.0017)	0.0070 (0.0009)	0.0070 (0.0017)

estimate the parameter, let $l(\boldsymbol{\lambda})$ be the log-likelihood function with parameter vector $\boldsymbol{\lambda}$ so that

$$l(\boldsymbol{\lambda}) = \sum_{i=1}^{n} \log f(\varepsilon_i, \boldsymbol{\lambda}). \tag{8}$$

Then the ML estimator, $\hat{\boldsymbol{\lambda}}$, of $\boldsymbol{\lambda}$ is given by

$$\hat{\boldsymbol{\lambda}} = \arg\max_{\boldsymbol{\lambda} \in \Lambda} l(\boldsymbol{\lambda}). \tag{9}$$

Note that the log-likelihood function in each case is given by (8) with $f(\varepsilon_i, \boldsymbol{\lambda})$ replaced by the appropriate density function. However, in practice, the true distribution of ε_i is seldom known, and the corresponding estimator $\hat{\boldsymbol{\lambda}}$, as defined in (9), will be the QML estimator rather than the ML estimator.

4. A simulation study

We assess the performance of the EF, ML and QML methods through a simulation study. This study is conducted using the BCACD(1, 1) and EXACD(1, 1) models with various popular error distributions, namely the standardized Exponential distribution, standardized Weibull distribution and standardized Generalized Gamma distribution.

Tables 1 and 2 show the simulation results for BCACD(1, 1) and EXACD(1, 1) models with various error distributions for sample size of $n = 500$ and $n = 2000$.

For a sample size of $n = 500$, the BCACD(1, 1) models give smaller bias for all the estimates than the EXACD(1, 1) models. We also observe that all the estimates in these two models with different error distributions are fairly close to the target value. The standard errors (SE) and mean squared errors (MSE) of these two models are also reported. When the error distribution is exponentially distributed, as expected, the estimated standard errors for all estimates based on ML method are smaller than those of the EF method. As the error distributions follow the Weibull and Generalized Gamma distributions, the QML method gives larger estimated standard errors for all parameters than the ML method. The same phenomenon also happens to the EF method.

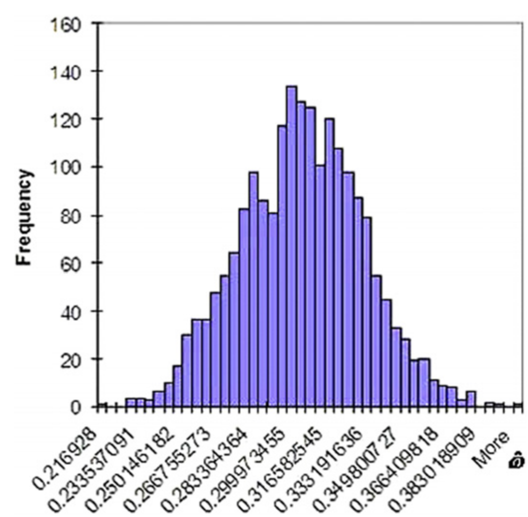

Fig. 1. The histogram for the $\hat{\omega}$ in the BCACD(1, 1) model.

From the Tables 1 and 2, it is clear that the estimated standard error and mean squared error have been reduced with the increase in the sample size to $n = 2000$. It can be seen from the values of the estimated bias that the estimates are close to their true values in the EF method.

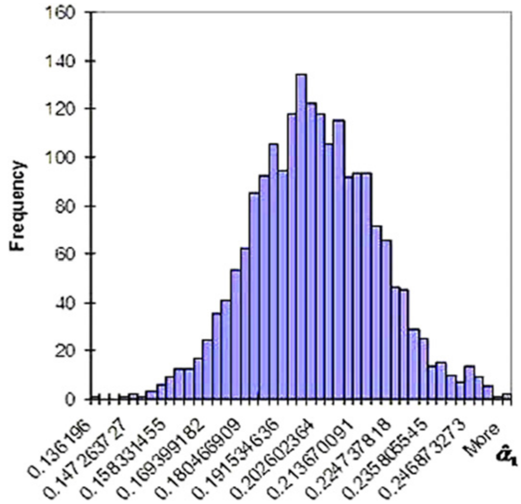

Fig. 2. The histogram for the $\hat{\alpha}_1$ in the BCACD(1, 1) model.

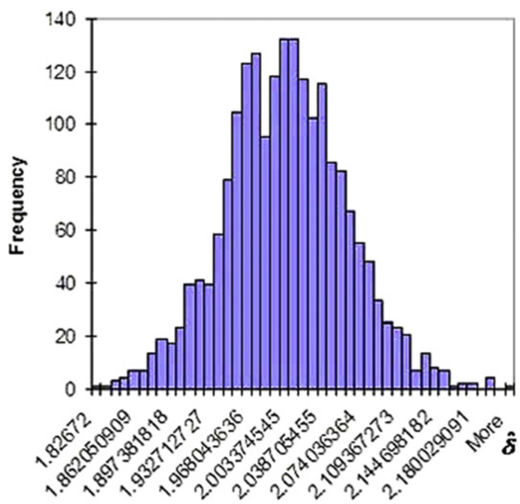

Fig. 3. The histogram for the $\hat{\delta}$ in the BCACD(1, 1) model.

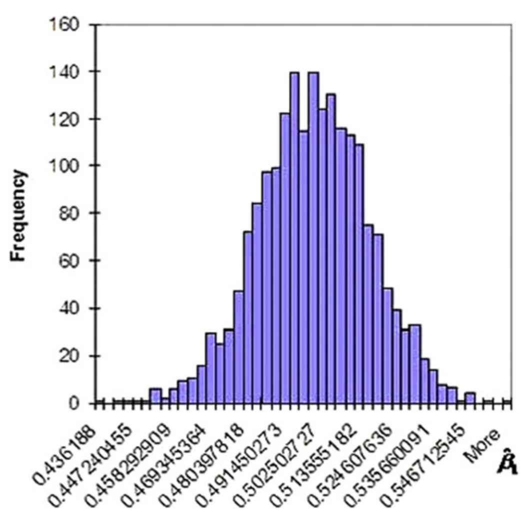

Fig. 4. The histogram for the $\hat{\beta}_1$ in the BCACD(1, 1) model.

In addition to the above simulation study, we give the finite sample results of standard errors for parameter estimates based on the EF method for BCACD(1, 1) and EXACD(1, 1) models when $n = 2000$. To find the finite sample standard errors of the EF estimate $\hat{\boldsymbol{\theta}}$, we use a similar technique to Allen et al. (2012).

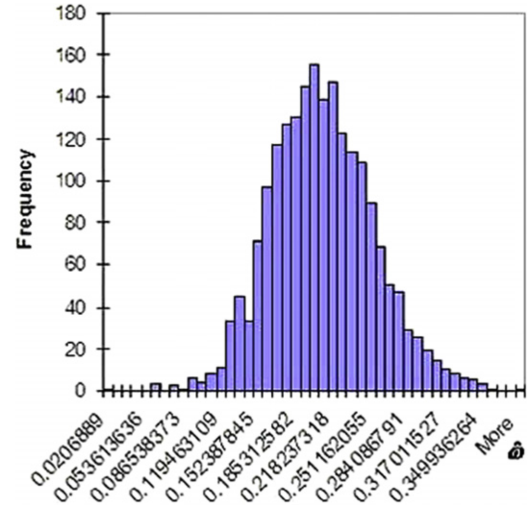

Fig. 5. The histogram for the $\hat{\omega}$ in the EXACD(1, 1) model.

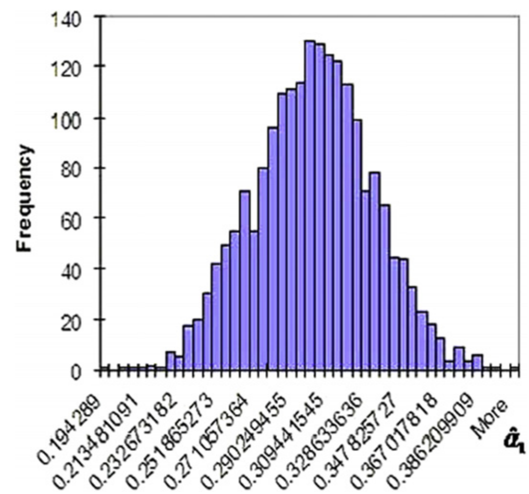

Fig. 6. The histogram for the $\hat{\alpha}_1$ in the EXACD(1, 1) model.

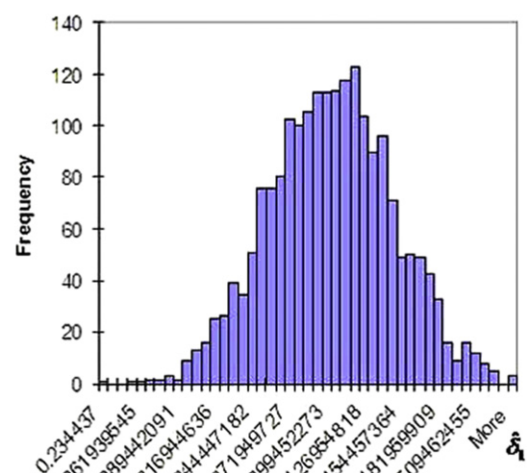

Fig. 7. The histogram for the $\hat{\delta}_1$ in the EXACD(1, 1) model.

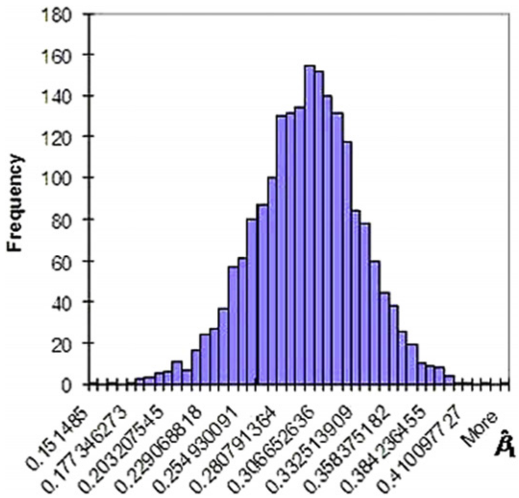

Fig. 8. The histogram for the $\hat{\beta}_1$ in the EXACD(1, 1) model.

For the BCACD(1, 1) model when the error distribution follows an Exponential distribution, the finite sample standard errors for ω, α_1, δ and β_1 are respectively 0.0371, 0.0210, 0.0652 and 0.0216. For the EXACD(1, 1) model with the error distribution following an Exponential distribution, the finite sample standard errors for $\omega, \alpha_1, \delta_1$ and β_1 are respectively 0.0621, 0.0302, 0.0451, 0.0352. These standard errors are close to the estimated standard errors as reported in Tables 1 and 2. Figs. 1–4 and Figs. 5–8 show the finite sample distribution of $\hat{\boldsymbol{\theta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\delta}, \hat{\beta}_1)$ of BCACD(1, 1) model and $\hat{\boldsymbol{\theta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\delta}_1, \hat{\beta}_1)$ of EXACD(1, 1) model with sample size of $n = 2000$ as reported in Tables 1 and 2. These histograms show that $\hat{\boldsymbol{\theta}}$ for both models respectively follow approximate Gaussian distributions with mean $\boldsymbol{\theta}$ and variance $\mathrm{Var}(\hat{\boldsymbol{\theta}})$. Using a similar technique, this can be extended to the general BCACD(p, q) and EXACD(p, q) models. We also note that the EF method's computation time is faster compared to those of the ML and QML methods.

5. Conclusion

This paper considered the estimation of BCACD and EXACD models based on estimating functions. Our simulation results suggest that the estimation of the parameters based on the EF method is not affected by the specification and the distribution of ε_i. Therefore, the EF method can serve as a semi-parametric method in parameter estimation. On the other hand the computation demands of this method are significantly lower than the ML and QML methods. This method could be useful in modelling duration data in financial economics.

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