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Which decision theory?

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HIGHLIGHTS

- A new laboratory study to identify the best descriptive decision theories.
- We use a representative sample of binary choice problems.
- We use a lottery set with a small number of outcomes and probabilities.
- We find that a simple heuristic, rank-dependent utility and expected utility theory provide the best goodness of fit.

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ABSTRACT

A new laboratory experiment is designed to identify the best theories for describing decisions under risk. The experimental design has two noteworthy features: a representative sample of binary choice problems (for fair comparison across theories) and a lottery set with a small number of outcomes and probabilities (for ease of non-parametric estimation). We find that a simple heuristic, rank-dependent utility and expected utility theory provide the best goodness of fit.

1. Introduction

The aim of this paper is to identify descriptive decision theories that provide the best goodness of fit to experimental data. This experimental study has two noteworthy features. First, we use a representative sample of binary choice problems (*i.e.*, experimental questions are not selected by an experimenter). A design where an experimenter selects choice problems might not be optimal for comparing different decision theories. A decision theory may fit better for certain types of choice problems and over- or under-representation of these problems leads to over- or under-estimation of the theory's goodness of fit. Second, we use a set of lotteries with a small number of outcomes and probabilities. This allows us to estimate all decision theories without any parametric assumptions.

The paper is organized as follows. Section 2 describes the design and implementation of our experiment. Section 3 summarizes ten decision theories considered in this paper. Section 4 presents our econometric model of discrete choice based on a latent dependent variable. Section 5 outlines our estimation procedure. Section 6 summarizes the results.

2. Experiment

We designed our experiment to facilitate non-parametric estimation of various theories. Specifically, all risky alternatives used in the experiment have a small number of outcomes and probabilities. We restrict risky alternatives to have no more than four possible outcomes. These four outcomes are €5, €20, €25 and €40. Using only probability values 0, 0.25, 0.5, 0.75 and 1, it is possible to construct 23 distinct probability distributions over these outcomes. Using only these 23 lotteries, it is possible to construct a total of 140 binary choice problems where none of the alternatives stochastically dominates the other.[1]

The experiment was conducted as a paper-and-pencil classroom experiment. Subjects received a booklet with 140 decision

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[1] In fact, a power test shows that for all model comparisons considered in this paper it is sufficient to use only 86 binary choice problems for the false negative rate (probability of a Type II error) 0.2.

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Fig. 1. An example of a decision problem as displayed in the experiment.

problems. Each problem was printed on a separate page. For each subject, pages with 140 problems were rearranged in a random order. Probability information was explained through the distribution of standard playing cards. Fig. 1 shows an example of one decision problem as it was displayed in the experiment.

The experiment was conducted in the University of Innsbruck. Altogether, 38 undergraduate students took part in two experimental sessions, which were conducted on the same afternoon. Twenty out of 38 subjects (52.6%) were female. The average age of experimental participants was 21.5 years (minimum age was 18, maximum age was 34). Fourteen out of 38 subjects (36.8%) were economics majors. All subjects had no previous experience with economic experiments.

Subjects were allowed to go through experimental questions at their own pace with no time restriction. After answering all 140 questions, each subject was asked to spin a roulette wheel. The number of sectors on the roulette wheel corresponded to the total number of questions asked in the experiment. The question randomly selected on the roulette wheel was played out for real money.

Subjects who opted for a sure monetary payoff in the selected question simply received this amount in cash. Subjects who opted for a lottery were shown the corresponding composition of playing cards. The cards were subsequently reshuffled and subjects had to draw one card. Depending on the suit of their drawn card, they received the corresponding payoff. Upon observing the suit of their drawn card, subjects inspected all remaining cards to verify that card composition did not change after reshuffling.

Each experimental session lasted about 1.5 h. About one third of this time was spent on using physical randomization devices at the end of the experiment. On average, subjects earned €25. Two subjects earned €5, 19 subjects earned €20, 8 subjects earned €25 and 9 subjects earned €40.

3. Decision theories

Let $X = \{€5, €20, €25, €40\}$ denote the set of possible outcomes and let $Q = \{0, 0.25, 0.5, 0.75, 1\}$ denote the set of probability values. Let $L : X \to Q$ denote a typical lottery used in the experiment, *i.e.*, $L(x) \in Q$ for all $x \in X$ and $\Sigma_{x\in X} L(x) = 1$. For any lottery L, the cumulative distribution function $F_L(x)$ is defined as $F_L(x) = \Sigma_{y\in X, x\geq y} L(y)$, for all $x \in X$. Similarly, the decumulative distribution function $G_L(x)$ of lottery L is defined as $G_L(x) = \Sigma_{y\in X, y\geq x} L(y)$, for all $x \in X$.

$$U(L) = \begin{cases} \dfrac{[L(€20)\cdot u(€20) + L(€25)\cdot u(€25)]\cdot(1+\beta) + L(€40)}{1+\beta\cdot[L(€5)+L(€20)+L(€25)]}, \\ \quad \text{if } 0 \leq U(L) \leq u(€20) \\ \dfrac{L(€20)\cdot u(€20)\cdot(1+\beta) + L(€25)\cdot u(€25) + L(€40)}{1+\beta\cdot[L(€5)+L(€20)]}, \\ \quad \text{if } u(€20) \leq U(L) \leq u(€25) \\ \dfrac{L(€20)\cdot u(€20) + L(€25)\cdot u(€25) + L(€40)}{1+\beta\cdot L(€5)}, \\ \quad \text{if } u(€25) \leq U(L) \leq 1. \end{cases} \tag{1}$$

For each subject we estimated 9 decision theories that are presented in Table 1. We also consider the possibility of decisions to be driven by some simple heuristic. At least two observations point in this direction. First, despite a large number of questions, subjects cope with the experiment very quickly. Typically, they need about 30 s for each decision. Only fast and frugal heuristics can result in such speedy decision making.

Second, the best fitting parameters of EUT and RDU reveal that quite a few subjects behave as if maximizing an extremely risk averse utility function $u(€5) = 0$ and $u(€20) = u(€25) = u(€40) = 1$. These subjects apparently minimize the probability of the lowest outcome. This is the second step in the priority heuristic (Brandstätter et al., 2006). Yet, the priority heuristic itself cannot be estimated on our dataset (the priority heuristic is inconclusive in a decision problem depicted on Fig. 1). In the context of our experiment, it is very easy (*i.e.*, with little cognitive effort) to apply the following simple rule of thumb (abbreviated as H):

(a) pick a lottery with a smaller probability of the lowest outcome €5;
(b) if two lotteries yield the lowest outcome €5 with the same probability, then pick a lottery with the highest probability of the greatest outcome €40.

Note that there is no concept of utility value in H (as it is typical in the psychological literature on heuristics). There are no subjective parameters to be estimated in H. H is not nested in any other decision theory.

4. Econometric model of discrete choice

Each of 140 decision problems used in the experiment is a binary choice between two lotteries L and R. Existing literature

Table 1
Decision theories that are estimated on experimental data.

Theory	Utility of lottery L	Normalized parameters	Subjective parameters to be estimated	Nested models (number of parameter restrictions)
Expected value (EV)	$U(L) = \Sigma_{x\in X} L(x) \cdot x$	No	No	No
Expected utility theory (EUT)	$U(L) = \Sigma_{x\in X} L(x) \cdot u(x)$	$u(€5) = 0, u(€40) = 1$	$u(€20), u(€25)$	EV (2)
Yaari (1987) dual model (Y)	$U(L) = \Sigma_{x\in X}[w(G_L(x)) - w(1 - F_L(x))] \cdot x$	$w(0) = 0, w(1) = 1$	$w(0.25), w(0.5), w(0.75)$	EV (2)
Quiggin (1981) rank-dependent utility (RDU)	$U(L) = \Sigma_{x\in X}[w(G_L(x)) - w(1 - F_L(x))] \cdot u(x)$	$w(0) = 0, w(1) = 1$, $u(€5) = 0, u(€40) = 1$	$w(0.25), w(0.5), w(0.75)$, $u(€20), u(€25)$	EV (5); EUT (3); Y (2)
Blavatskyy (2010) mean-variance approach (MV)	$U(L) = \Sigma_{x\in X} L(x) \cdot u(x) - 0.5\rho \cdot \Sigma_{y\in X} L(y) \cdot \lvert \Sigma_{x\in X} L(x) \cdot u(x) - u(y)\rvert$	$u(€5) = 0, u(€40) = 1$	$u(€20), u(€25)$ and $\rho \in [-1, 1]$	EV (3); EUT (1)
Chew (1983) weighted utility (WU) = regret theory	$U(L) = \dfrac{\Sigma_{x\in X} L(x) \cdot u(x) \cdot v(x)}{\Sigma_{x\in X} L(x) \cdot v(x)}$	$u(€5) = 0, u(€40) = 1$, $v(€5) = 1, v(€40) = 1$	$u(€20), u(€25), v(€20), v(€25)$	EV (4); EUT (2)
Chew et al. (1991) quadratic utility (QU)	$U(L) = \Sigma_{x\in X} \Sigma_{y\in X} L(x) \cdot L(y) \cdot \varphi(x, y)$	$\varphi(€5, €5) = 0$, $\varphi(€40, €40) = 1$	$\varphi(€5, €20), \varphi(€5, €25)$, $\varphi(€5, €40), \varphi(€20, €20)$, $\varphi(€20, €25), \varphi(€20, €40)$, $\varphi(€25, €25), \varphi(€25, €40)$,	EV (8); EUT (6)
Gul (1991) disappointment aversion (DA)	Formula (1)	$u(€5) = 0, u(€40) = 1$	$u(€20), u(€25), \beta > -1$	EV (3); EUT (1)
Viscusi (1989) prospective reference (PR)	$U(L) = \lambda \cdot \Sigma_{x\in X} L(x) \cdot u(x) + (1 - \lambda) \cdot \Sigma_{x\in X} \mathrm{sign}(L(x)) \cdot u(x) / \Sigma_{x\in X} \mathrm{sign}(L(x))$	$u(€5) = 0, u(€40) = 1$, $\mathrm{sign}(q) = 1$ if $q > 0$, $\mathrm{sign}(q) = 0$ if $q = 0$	$u(€20), u(€25), \lambda \in [0, 1]$	EV (3); EUT (1)

(*e.g.*, Hey and Orme, 1994; Hey, 2001) typically employs the Fechner model of random errors (strong utility). A decision maker chooses L over R if

$$U(L) - U(R) \geq \xi, \tag{2}$$

where ξ is a random variable (with zero mean) that is independently and identically distributed across all lottery pairs. Unfortunately, model (2) has at least three shortcomings:

(a) the distribution of ξ is affected by arbitrary affine transformation of utility function;
(b) the notion of risk aversion is not defined (see Wilcox, 2011; Blavatskyy, 2008, Axiom 5);
(c) first-order stochastic dominance is violated (see Blavatskyy, forthcoming).

Other existing models share some of these shortcomings. Problem (a) also appears in Luce's choice model (Luce, 1959; Holt and Laury, 2002) and the model of Blavatskyy (2009, 2011, 2012) with a non-homogeneous function $\varphi(\cdot)$. Problem (c) also appears in a tremble model (Harless and Camerer, 1994), heteroscedastic Fechner model (*e.g.*, Hey, 1995; Buschena and Zilberman, 2000; Blavatskyy, 2007) and a model of Wilcox (2008, 2011).

Another popular econometric model is a random preference approach (*e.g.*, Falmagne, 1985; Loomes and Sugden, 1995) including random utility (*e.g.*, Gul and Pesendorfer, 2006). Unfortunately, this approach allows for intransitive choice cycles (similar to Condorcet's paradox). Such cycles are rarely observed in the data (*e.g.*, Rieskamp et al., 2006, p. 648).

In this paper we use a modification of model (2) which avoids problems (a)–(c). For any two lotteries L and R, let $L \vee R$ denote a lottery that yields outcome $x \in X$ with a probability

$$\min\{F_L(x), F_R(x)\} + \max\{G_L(x), G_R(x)\} - 1. \tag{3}$$

Similarly, let $L \wedge R$ denote a lottery that yields outcome $x \in X$ with a probability

$$\max\{F_L(x), F_R(x)\} + \min\{G_L(x), G_R(x)\} - 1. \tag{4}$$

First, consider the case when lottery L stochastically dominates lottery R. In this case, $U(L) - U(R) = U(L \vee R) - U(L \wedge R)$. Thus, to avoid violations of stochastic dominance, we need to make sure that the realization of a random variable ξ is never greater than the difference $U(L \vee R) - U(L \wedge R)$. In other words, inequality (2) must be always satisfied if L stochastically dominates R. Thus, stochastic dominance imposes an upper bound on possible errors:

$$\xi \leq U(L \vee R) - U(L \wedge R). \tag{5}$$

Second, consider the case when R stochastically dominates L. In this case, $U(L) - U(R) = U(L \wedge R) - U(L \vee R)$. To avoid violations of stochastic dominance, we need to make sure that the realization of a random variable ξ is never less than the difference $U(L \wedge R) - U(L \vee R)$. In other words, inequality (2) must always hold with a reversed sign if R stochastically dominates L. Thus, stochastic dominance also imposes a lower bound on possible errors:

$$\xi \geq U(L \wedge R) - U(L \vee R). \tag{6}$$

Inequalities (5) and (6) imply that random variable ξ must be distributed on a bounded interval. In general, this interval varies across lottery pairs. Thus, random variable ξ cannot be independently and identically distributed across all lottery pairs. Yet, it is possible to write random error ξ as $\varepsilon \cdot [U(L \vee R) - U(L \wedge R)]$. Inequalities (5) and (6) are then both satisfied if random variable ε is independently and identically distributed on the interval $[-1, 1]$.

Hence, we use the following econometric model of discrete choice: L is chosen over R if

$$U(L) - U(R) \geq \varepsilon \cdot [U(L \vee R) - U(L \wedge R)], \tag{7}$$

where ε is a random variable symmetrically distributed around zero on the interval $[-1, 1]$. Note that we also solve shortcoming (a) of model (2). Multiplying utility function $U(\cdot)$ by an arbitrary positive constant does not affect the distribution of random error ε in model (7). Blavatskyy (forthcoming, Section 6) demonstrates that problem (b) is also resolved for model (7).

Let $\Phi : [-1, 1] \to [0, 1]$ be the cumulative distribution function of random error ε. A decision maker then chooses L over R with probability (8).

$$P(L, R) = \Phi\left(\frac{U(L) - U(R)}{U(L \vee R) - U(L \wedge R)}\right). \tag{8}$$

We assume that $\Phi(v) = I_{\eta,\eta}(0.5 + 0.5v)$ for all $v \in [-1, 1]$, where $I_{\eta,\eta}(\cdot)$ is the cumulative distribution function (the regularized incomplete beta function) of a symmetric beta distribution with parameters η and η. Beta distribution is quite flexible and includes the uniform distribution ($\eta = 1$), unimodal distribution ($\eta > 1$) and bimodal (U-shaped) distribution ($\eta < 1$) as special cases. Subjective parameter $\eta \in R_+$ can be interpreted as a measure of noise. If $\eta \to +\infty$ then model (8) converges to a deterministic decision theory.

We can use model (8) for estimating all decision theories considered in Section 3 except for a simple heuristic H which lacks utility function $U(\cdot)$. Since H already specifies a deterministic choice rule, we can easily extend it into a model of probabilistic choice as follows. A decision maker chooses lottery L over lottery R with probability (9).

$$P(L, R) = \begin{cases} \eta, & \text{if } L(€5) < R(€5) \text{ or } L(€5) = R(€5) \\ & \text{and } L(€40) > R(€40) \\ 1-\eta, & \text{if } L(€5) > R(€5) \text{ or } L(€5) = R(€5) \\ & \text{and } L(€40) < R(€40). \end{cases} \quad (9)$$

Again, subjective parameter $\eta \in [0.5, 1]$ can be interpreted as a measure of noise. If $\eta = 1$ then a decision maker literary applies heuristic H in every decision problem. If $\eta = 0.5$ then a decision maker chooses at random.

5. Estimation procedure

Estimation is done separately for each subject. Subjective parameters of ten decision theories plus noise parameter η are estimated by maximizing total log-likelihood (formulas (8) and (9) show the likelihood of one decision). Non-linear optimization is solved in the Matlab 7.2 package (based on the Nelder–Mead simplex algorithm).

We begin by estimating two theories with no subjective parameters: EV and H. We then compare EV and H in terms of their goodness of fit to the revealed choice pattern of each subject. We use the Vuong likelihood ratio test for strictly non-nested models (see Vuong, 1989; Appendix A.2 in Loomes et al., 2002). If one of the theories provides a significantly better fit (at the 5% significance level) than the other, it is tentatively labeled as the best descriptor for the corresponding subject. If two theories do not significantly differ in terms of their goodness of fit, both are tentatively labeled as the best descriptors.

Next, we consider EUT and Y and compare their goodness of fit with that of the best descriptor(s). We use a standard likelihood ratio test for nested models and the Vuong likelihood ratio test for strictly non-nested models. In the latter case, the Akaike information criterion is used to penalize EUT or Y for a greater number of parameters. If EUT (or Y) significantly outperforms the best descriptor(s), it tentatively becomes the best descriptor. If both EUT and Y significantly outperform the best descriptor(s), then EUT and Y are compared with each other using the Vuong likelihood ratio test for overlapping models.

Finally, we consider all remaining non-expected utility theories from Section 2 and repeat the same routine as for EUT and Y. At the end of this exercise, for each subject we identify one or several decision theories such that none of the remaining theories provides a significantly better goodness of fit to the subject's revealed choice pattern.

6. Results

Fig. 2 summarizes estimation results. There are three best-fitting decision theories: EUT, RDU and H. Each of these theories can account for about a quarter of individual choice patterns. Most of choice patterns best rationalized by EUT can be equally well described by Y. Some of choice patterns best rationalized by RDU can be equally well described by MV.

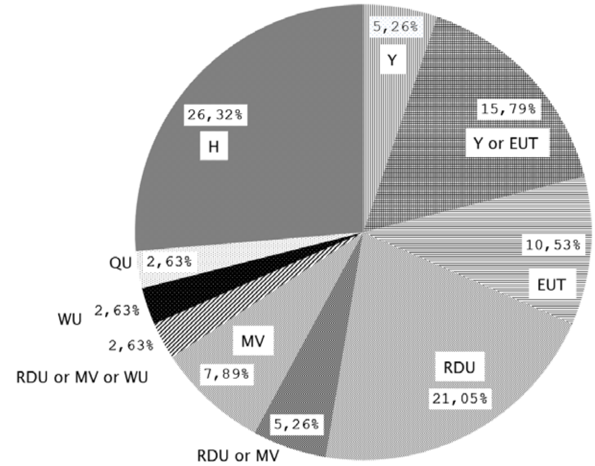

Fig. 2. Best-fitting decision theories (in percentage of all subjects).

At the same time, there are three decision theories (EV, DA and PR) that always (i.e., for every subject) provide a significantly worse goodness of fit than some other theory. Two more theories (QU and WU) provide the best description only for one or two subjects. Thus, we can confidently delete EV, DA, PR, QU and WU from the list of promising descriptive decision theories.

For most of the subjects for whom EUT fits best the best-fitting utility function is concave, i.e. subjects reveal risk averse behavior. Only one such subject behaved as if maximizing a convex utility function. For many subjects for whom RDU fits best the best-fitting utility function is concave but we observe a lot of heterogeneity. Three subjects behave as if maximizing an extremely risk averse utility function $u(€5) = 0$ and $u(€20) = u(€25) = u(€40) = 1$. They apparently minimized the probability of the lowest outcome but used some other heuristic than H. For many subjects the best-fitting probability weighting function turns out to be a concave function. Only one subject (#20) revealed a convex probability weighting function. A textbook inverse S-shaped probability weighting function is found only for one subject (#10). Two subjects (#12 and #34) behave as if they have an S-shaped probability weighting function.

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