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A note on cointegration of international stock market indices

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ABSTRACT

Cointegration is frequently used to assess the degree of interdependence of financial markets. We show that if a stock's price follows a stock specific random walk, market indices cannot be cointegrated. Indices are a mere combination of n different random walks which itself is non-stationary by construction. We substantiate the theoretical propositions using a sample of 28 stock indices as well as a simulation study. In the latter we simulate stock prices, construct indices and test whether these indices are cointegrated. We show that while heteroscedasticity misleads cointegration tests, it is not sufficient to explain the high correlation between stock market index returns. A common random walk component and correlated price innovations are necessary to reproduce this feature.

1. Introduction

Building on the time series properties of stock market index data, a vast literature has emerged which studies the dependence and the degree of integration of international financial markets by means of cointegration analysis. Due to the relatively strong comovement of financial markets, the assumption of a shared common trend seems plausible at first sight. This is the reason why cointegration analysis has been a major tool in the study of interrelations between financial markets. However, there are two major issues that have to be taken into account.

First, the cointegration relationship seems to be a very fragile one. Different studies using the same indices do not necessarily find an identical number of cointegrating vectors. Most of the studies are conducted in the spirit of Kasa (1992) who can identify one common stochastic trend for the stock markets of the USA, Japan, United Kingdom, Germany, and Canada. He uses monthly and quarterly data over a period of almost 16 years which suits the notion that cointegration is a long term concept while short run deviations from the common trend are possible. As opposed to these findings, Pascual (2003) finds no cointegration relationship between the French, German, and UK stock market using quarterly data for an even longer sample from 1960 to 1999. Statistically, if the time series are not found to be cointegrated in the larger sample, they should not be found to be cointegrated on any subsample like the one employed by Kasa (1992). There are numerous further examples in the literature where a slight alteration of the approach leads to different results. For example, Aggarwal and Kyaw (2005) and Phengpis and Swanson (2006) both investigate the NAFTA countries. While Aggarwal and Kyaw (2005) find evidence for cointegration in the post-NAFTA era, Phengpis and Swanson (2006) do not. Detecting cointegration, thus, seems to critically depend upon the time span under consideration and the precise specification of the statistical model.

The second issue is that Johansen's (1988) test for cointegration—the major tool in empirical work—is prone to misjudgement. Financial data are marked by heteroscedasticity which is known to bias the test (cp. Lee & Tse, 1996). Also, in particular in early studies like Kasa (1992), a small sample size has been a major issue. Even though accounting for heteroscedasticity (e.g. Cavaliere, Rahbek, & Taylor, 2010) and small sample size is possible (cp. Barkoulas & Baum, 1997; Johansen, 2002), it is hardly ever done. Recent studies, however, use in general daily data so that at least the latter issue can be regarded as overcome.

The fact that cointegration among stock market indices is a delicate issue has first been addressed by Richards (1995). He relies on the Capital Asset Pricing Model (CAPM) for stock prices and shows that indices, constructed as weighted averages of stock prices in a country, cannot be cointegrated. We contribute to the literature by translating his argument to international stock markets. In addition, we abandon the CAPM assumption in favour of the more flexible random walk model for stock prices, pursuing a twofold aim: Building on theoretical and statistical arguments, we demonstrate that stock market indices cannot be cointegrated. Furthermore, we foster an intuition for the heterogeneous results found in the literature.

The argumentation is based on three pillars. First, we use the random walk model for stock prices to derive that statistically cointegration

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between two markets is impossible. In our setting, stock market indices are a weighted average of random walk asset prices. These individual stochastic trends never cancel out in a cointegration regression and consequently prevent the indices from being cointegrated. Second, we perform an empirical exercise to show that using standard methodology cointegration is very unstable and that the results are at odds with the notion of long term comovement. In particular, we employ subsamples and show that detection of cointegration among pairs of stock market indices is basically random. And third, we simulate the theoretical model to gain an idea of the components of stock market indices. The aim of the simulation study is to reproduce data series which exhibit similar properties as the observed stock market indices in a cointegration framework.

We document that in a cointegration analysis of international stock market indices, every desired outcome can be produced by suitably restricting the sample period and adjusting the model. Standard cointegration tests will reject the null hypothesis of no cointegration far too often if the properties of the data (in particular heteroscedasticity) are disrespected. Our simulation study suggests that most likely stock market indices share an additional common stochastic trend and that returns are correlated. However, cointegration analysis is not a suitable methodology to investigate the comovement of stock markets if individual stocks are random walks themselves.

The paper proceeds as follows. Section 2 reviews the related literature and points out the critical issues raised in the Introduction. Section 3 outlines the random walk model of stock prices and highlights the implications for stock indices and cointegration. Section 4 presents the results of a cointegration analysis of 28 stock market indices in order to highlight inconsistencies when applying the cointegration methodology to empirical data. Section 5 holds a simulation study of the theoretical model and Section 6 concludes.

2. Related literature

Based on the assumption that stock markets in different countries share common stochastic trends, numerous studies have tried to detect those. One of the first was Kasa (1992) who can identify one common stochastic trend for the stock markets of the U.S., Japan, England, Germany, and Canada. He used monthly and quarterly data over a period of almost 16 years. More recent contributions include Choudhry, Lu, and Peng (2007), Lagoarde-Segot and Lucey (2007) and Valadkhani and Chancharat (2008). These studies have in common that they all identify exactly one common stochastic trend. However, there is no economic or financial theory predicting the number of common stochastic trends. Empirically, Click and Plummer (2005), for example, investigate the relationship between five ASEAN stock markets on a daily basis for four years and find that these markets are cointegrated. However, the authors can identify only one cointegrating vector. This implies four stochastic trends which influence the cointegration relationship. The authors conclude that in this case the integration of these financial markets is far from being perfect. Empirical work, thus, cannot unambiguously deduce the number of stochastic trends shared by financial markets. The number of detected cointegrating vectors critically depends on the number of markets analysed, the sample time span, data frequency and the properties of financial data like fat tails or heteroscedasticity.

Empirical evidence is not only mixed with respect to the number of common trends. The question whether financial markets share a common stochastic trend at all is also not answered unambiguously. The studies cited above find evidence for the existence of a cointegration relationship. In contrast, Chan, Gup, and Pan (1997) who analyse 18 stock market indices, find that these markets are not cointegrated. The analysis is conducted using monthly data from 1961 to 1992. Pascual (2003) studies whether the degree of integration between the French, German, and UK stock markets increases. He does not find a cointegration relationship using quarterly observations from 1960 to 1999 either. The results of Narayan and Smyth (2005) who investigate the relationships between the stock markets of New Zealand, Australia and the G7 countries, are mixed, depending on which test they use to detect cointegration. Their analysis is based on real monthly observations from 1967 to 2003.

With respect to financial theory, the existence of cointegration relationships would contradict the Efficient Market Hypothesis (EMH) which requires that returns—and with them future prices—are not predictable in the long run.[1] A common model, frequently used in the literature, which captures this behaviour of stock or index returns at high frequencies, is the random walk model for stock prices. It dates back to work by Fama (1965) and Malkiel (1973) and has ever since frequently been applied (see, inter alia, Black, 1986; Godfrey, Granger, & Morgenstern, 2007) and tested, albeit with mixed results (see, inter alia, De Bondt & Thaler, 1985; Fama, 1995; Worthington & Higgs, 2009). Cointegration, by contrast, would allow for some kind of predictability in the long run, even though short run predictions are not possible. This argument is not limited to stock markets. Granger (1986) shows that gold and silver prices are not cointegrated once these prices are generated on an efficient market. The very same is true for stock prices. That cointegration based analysis of market efficiency is unreliable has then been pointed out by Barkoulas and Baum (1997) in the context of foreign exchange markets.

This paper suggests that under the assumption that stock prices are generated according to the random walk model, international financial markets are not cointegrated in the Engle and Granger (1987) sense. For the most part, we follow arguments that have been put forward by Richards (1995) who claims that stock return indices in one stock market cannot be cointegrated if one assumes that excess returns are generated according to the CAPM. He argues that in order to be cointegrated, the company specific shocks of one company need to offset the shocks of the other company. However, both of these shocks would have to be completely unexpected, but identical in size and direction. He states that this would rule out the possibility that any management decision permanently affected a company's stock price. He summarises that these company specific shocks "will not translate into a cointegrating relationship between the actual return indices for the two (or more) assets." It seems that this result has been neglected in some of the literature on cointegration of financial markets since then. This paper will therefore reinforce the argumentation that company specific shocks eventually inhibit the existence of cointegration relations (as defined by Engle & Granger, 1987) between international stock market indices. In contrast to Richards (1995) who seeks to explain the results of Kasa (1992) obtained on low frequencies, our line of argumentation will keep features of high frequency data in mind. Our model will therefore be different from Richards (1995) in that we will not rely on the CAPM, but the more general random walk model for stock prices. It is widely accepted that on high frequencies stock prices are modelled best by a random walk. Further, Richards (1995) attributed some of the results in the literature specifically to a small sample bias in the Johansen (1988) cointegration testing framework. This issue can be regarded as overcome since high frequency data (in particular daily data) are nowadays widely (and even freely) available. However, daily data are marked by other features (e.g. heteroscedasticity) which have to be taken into account when testing international financial markets for cointegration.

3. Stock prices, indices, and cointegration

The basic model for stock prices which is widely used in the literature, assumes that log-prices *individually* follow a random walk. The model can be written as

$$p_{i,t} = p_{i,t-1} + e_{i,t}, \tag{1}$$

[1] Short-run predictions may be possible due to market frictions and investor behaviour.

where $p_{i,t}$ is the price of stock i at time t (in logarithms). The innovation term $e_{i,t}$ is a white noise process with $\mathbb{E}\left[e_{i,t}\right] = 0$ and variance $\sigma^2_{i,t}$. Whether the variance is time dependent will not influence the theoretical result, so we suppress the time subscript in σ^2_i in the subsequent outline. The model may contain a drift term $\delta_{i,t}$. However, there is an ongoing debate on whether a drift term is compatible with informational efficiency of markets (Edwards & Magee, 2001; Malkiel, 1973). Our results hold irrespective of the inclusion of a drift term which is therefore neglected in the following.

In the spirit of latent factor models in finance, we allow the innovation term in Eq. (1) to consist of different components, namely a global, a local and an idiosyncratic component (cp. Jung, Liesenfeld, & Richard, 2011). These components are collected in a new vector such that $e_{i,t}$ is a composite error term calculated as $e_{i,t} = \epsilon'_{i,t}\iota$. $\epsilon_{i,t}$ is a multivariate white noise process with $\mathbb{E}\left[\epsilon_{i,t}\right] = 0$. ι is a (3×1)-vector of ones. The individual components in $\epsilon_{i,t}$ are serially uncorrelated, but may be cross-sectionally correlated, such that $\mathbb{E}\left[\epsilon_{i,t}\epsilon'_{i,s}\right] = \Sigma$ for $t \neq s$ and zero for $t \neq s$.

A stock market index is usually calculated as a weighted and normalised sum of individual stock prices. Without loss of generality we assume that an index X_j is calculated as

$$X_{j,t} = \sum_{i=1}^{n} w_i \cdot p_{i,t}, \tag{2}$$

where w_i is the weight for asset i. As X_j is composed of n price series which are assumed to follow a random walk, the index will be a weighted sum of n random walks and, thus, also be non-stationary.

Given the price model in Eq. (2), the crucial question is whether any two stock market indices X_1 and X_2 of countries 1 and 2, respectively, are cointegrated in the sense of Engle and Granger (1987). This is the case if and only if $X_1 - \beta X_2$ is stationary.[2] This will happen if the linear combination of the indices successfully eliminates the stochastic trends which compose the individual stock prices. However, if stock i is an element of X_1 and at the same time not an element of X_2 (for all stocks i), X_1 and X_2 cannot be cointegrated. The reason is that the individual stochastic trends which are contained in the individual stocks, do not cancel out: the random walk contained in one specific stock is different from the random walk component contained in any other stock. As in the present framework two stock market indices are weighted averages of distinct $I(1)$-series, no linear combination exists which removes all stochastic trends. So for any β, $X_1 - \beta X_2$ is $I(1)$ and the stationarity requirement is violated. No cointegrating vector exists which would assure that $X_1 - \beta X_2 \sim I(0)$. Therefore, two (or more) such indices are not cointegrated in the sense of Engle and Granger (1987).

This result also holds for market indices in one country as long as their basis, i.e. the stocks used to calculate them, are not identical. The same is true when cross-listed stocks are used to calculate indices in two different countries. The cross-listed stocks themselves will most likely be cointegrated due to the law of one price (e.g. Grammig, Melvin, & Schlag, 2005; Hasbrouck, 1995) which allows only for temporary price deviations, but no fundamental ones (cp. the price model of Stock & Watson, 1988). The indices, however, could only be cointegrated if all stocks were the same, i.e. one index would have to be the exact reproduction of the other. Two such indices are, to the best of our knowledge, not calculated on any stock exchanges.

In order to illustrate the result that stock market indices of different countries are not cointegrated in the assumed context, we limit ourselves to two indices which are composed of only two stock prices each. Rewrite these four stock prices as

$$\begin{aligned}
p_{1,t} &= p_{1,t-1} + g_t + l_{1,t} + \varepsilon_{1,t} = \sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{1,s} + \sum_{s=1}^{t} \varepsilon_{1,s} \\
p_{2,t} &= p_{2,t-1} + g_t + l_{1,t} + \varepsilon_{2,t} = \sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{1,s} + \sum_{s=1}^{t} \varepsilon_{2,s} \\
p_{3,t} &= p_{3,t-1} + g_t + l_{2,t} + \varepsilon_{3,t} = \sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{2,s} + \sum_{s=1}^{t} \varepsilon_{3,s} \\
p_{4,t} &= p_{4,t-1} + g_t + l_{2,t} + \varepsilon_{4,t} = \sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{2,s} + \sum_{s=1}^{t} \varepsilon_{4,s},
\end{aligned} \tag{3}$$

where g_t is the global, $l_{j,t}$ the local and $\varepsilon_{i,t}$ the idiosyncratic innovation in $\epsilon_{i,t}$. We assume without loss of generality initial values $g_0 = l_{j,0} = \varepsilon_{i,0} = 0$. The indices are then constructed as

$$\begin{aligned}
X_{1,t} &= w_1 p_{1,t} + (1-w_1)p_{2,t} \\
X_{2,t} &= w_2 p_{3,t} + (1-w_2)p_{4,t}.
\end{aligned} \tag{4}$$

Substituting the individual prices in Eq. (4) by the respective stock prices in Eq. (3) gives

$$\begin{aligned}
X_{1,t} &= w_1 p_{1,t-1} + (1-w_1)p_{2,t-1} + g_t + l_{1,t} + w_1\varepsilon_{1,t} + (1-w_1)\varepsilon_{2,t} \\
&= \sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{1,s} + w_1\sum_{s=1}^{t} \varepsilon_{1,s} + (1-w_1)\sum_{s=1}^{t} \varepsilon_{2,s}
\end{aligned} \tag{5}$$

$$\begin{aligned}
X_{2,t} &= w_2 p_{3,t-1} + (1-w_2)p_{4,t-1} + g_t + l_{2,t} + w_2\varepsilon_{3,t} + (1-w_2)\varepsilon_{4,t} \\
&= \sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{2,s} + w_2\sum_{s=1}^{t} \varepsilon_{3,s} + (1-w_2)\sum_{s=1}^{t} \varepsilon_{4,s}.
\end{aligned} \tag{6}$$

In order for X_1 and X_2 to be cointegrated, the linear combination $X_1 - \beta X_2$ would have to eliminate the global, the two local as well as the four stock specific stochastic trends. Denote by u_t the residuals of a cointegration regression:

$$\begin{aligned}
u_t &= X_{1,t} - \beta X_{2,t} \\
&= \sum_{s=1}^{t} g_s - \beta\sum_{s=1}^{t} g_s + \sum_{s=1}^{t} l_{1,s} - \beta\sum_{s=1}^{t} l_{2,s} + w_1\sum_{s=1}^{t} \varepsilon_{1,s} \\
&\quad + (1-w_1)\sum_{s=1}^{t} \varepsilon_{2,s} - \beta w_2\sum_{s=1}^{t} \varepsilon_{3,s} - \beta(1-w_2)\sum_{s=1}^{t} \varepsilon_{4,s}.
\end{aligned} \tag{7}$$

Cointegration would require u_t to be stationary. This is, however, not the case as it still contains random walk components. As can easily be seen, for $\beta = 1$ only the global stochastic trend is eliminated. The local and the stock specific stochastic trends, however, are still present. Thus, $X_1 - \beta X_2$ still contains a combination of stochastic trends and is not stationary. More precisely,

$$\begin{aligned}
u_t &= u_{t-1} + l_{1,t} - \beta l_{2,t} + w_1\varepsilon_{1,t} \\
&\quad + (1-w_1)\varepsilon_{2,t} - \beta w_2\varepsilon_{3,t} - \beta(1-w_2)\varepsilon_{4,t}
\end{aligned} \tag{8}$$

which is an $I(1)$ process. The last result holds for any possible $\beta \in \mathbb{R}$. The only difference in u_t will be that it also contains $(1 - \beta)g_t$, i.e. the global innovation.

A point of critique might be the assumption that the error term is composed of global, local and idiosyncratic components simultaneously. It may seem that the composite error assumption artificially and needlessly increases the number of stochastic trends in the price process. Allowing for one innovation term only, however, does not alter the key result. As long as there are stock specific innovations which are modelled as a random walk, the stock market index will always be a weighted average of stock specific stochastic trends. The only difference would be that local innovations $l_{1,t} - \beta l_{2,t}$ would not appear in the process u_t in Eq. (8). Global innovations g_t are either

[2] In a bivariate cointegration analysis one of the coefficients in the cointegrating vector is usually normalised to 1 as in $X_1 - \beta X_2$ where the cointegrating vector is $(1 - \beta)$.

not present in the first place or cancel out anyway. In economic terms, this means that there is always stock specific information which permanently affects the share price of one company only and does not affect the share price of another company. It may happen that a specific news event affects the distribution of the innovations of other companies, i.e. the innovations are correlated, but they still are not exactly identical. Identity and corresponding weights, however, are what is required for the indices to be cointegrated in the sense of Engle and Granger (1987).

An important feature of financial time series which has been often documented in empirical studies is that return models exhibit heteroscedastic innovations. In the theoretical derivation of the result that international financial markets are not cointegrated, the presence of heteroscedasticity does not matter. A time-varying variance does only influence the behaviour of the random walk in such a way that it would be more volatile. The important feature, the non-stationarity, is not affected.

The argumentation easily extends to the multivariate case where it will be impossible to find a cointegration vector β such that a linear combination $\beta' x$ with x an $(n \times 1)$ vector of stock market indices, will be stationary. The global trend again may cancel out, but the stock specific innovations do not.

A further property which has frequently been documented in the empirical literature, is the high correlation between stock market index returns (Longin & Solnik, 1995, e.g.). According to the model framework above, there are two possible sources which can induce correlation between the return series. First, there may be stochastic trends which are common to the individual prices. If there is a global stochastic trend, this very same trend will be contained in both stock indices. Thus, the indices would not be independent any longer and the returns would exhibit some degree of correlation, depending on the variance of the global trend relative to the variance of the idiosyncratic innovations. Second, index return correlation could also be induced by cross-sectionally correlated innovations which are not ruled out by the above setting. Again, it is important to note that high correlation between stock markets is not a necessary condition for markets to be integrated. On the contrary, Adler and Dumas (1983) show that completely integrated markets, i.e. markets where risk is priced in the same way, can be uncorrelated.

4. An empirical example

In order to evaluate the theoretical result in Section 3, we first analyse a dataset of 28 stock indices. According to the theoretical model, we expect the cointegration tests between any two stock market indices to reject the cointegration hypothesis. The setting is a quasi-experiment so that overall we expect wrong implications of the cointegration test in accordance to the chosen significance level. More precisely, when testing on the 5% significance level, we expect to find approximately 5% of all combinations to be cointegrated. We restrict the study to bivariate cointegration for reasons of clearness. However, any test involving more than two market indices should lead to the same conclusion. With respect to correlation between the indices, we expect either a low correlation if only the stocks' innovations are correlated. If there are common stochastic trends together with correlated innovations, correlation between the indices should be comparatively higher.

The dataset covers daily close values between 1st March 2001 and 28th February 2009, i.e. we have 2084 observations. The data are taken from finance.yahoo.com (see columns one and two of Table 1 for an overview). In case of a national holiday in one country, the previous closing value has been substituted by the previous value as is standard in the literature. A necessary condition for any pair of two indices to be cointegrated is that each single index is integrated of order 1. We therefore perform augmented Dickey–Fuller tests with individual lag-length selection using the Schwarz information criterion. The log-level indices are all found to be non-stationary while log-returns are $I(0)$. Correlation of the latter is found to be rather volatile (with a standard deviation of 0.2052). It is lowest between ATS and KLSE returns (0.0366) and highest for DJIA and S&P 500 returns (0.9780).

Table 1
Index overview.

Index	Country	Cointegrated with
Europe		
ATX	Austria	AEX, CAC40, MIBTEL
AEX	Netherlands	ATX
CAC40	France	ATX, NIKKEI225, MIBTEL
DAX	Germany	MIBTEL, NIKKEI225, OMXC, OMXS
FTSE	United Kingdom	S&P500
MIBTEL	Italy	AORD, ATX, CAC40, DAX, KLSE, NIKKEI225, SMI
OMXC	Denmark	AORD, DAX, OMXS, S&P TSX
OMXS	Sweden	AORD, DAX, HSI, KLSE, KSE, OMXC, SGX, TA100
OSEAX	Norway	BSE, JKSE
SMI	Switzerland	MIBTEL, NIKKEI225
TA100	Israel	KSE, OMXS
Asia/Pacific		
AORD	Australia	KLSE, MIBTEL, NIKKEI225, OMXC, OMXS, SGX, S&P400, S&P TSX
BSE	India	BVSP, IPC, JKSE, MERV, OSEAX
HSI	Hong Kong	OMXS, SGX
JKSE	Indonesia	BSE, BVSP, IPC, MERV, OSEAX
KLSE	Malaysia	AORD, MIBTEL, NASDAQ, OMXS, S&P400
NIKKEI225	Japan	AORD, CAC40, DAX, MIBTEL, SMI
SGX	Singapore	AORD, HSI, KSE, OMXS
TSEC	Taiwan	NASDAQ
KSE	South Korea	OMXS, SGX, TA100
Americas		
DJIA	USA	
NASDAQ	USA	KLSE, TSEC
S&P500	USA	FTSE
S&P400	USA	AORD, KLSE, S&P TSX
S&P TSX	Canada	AORD, OMXC, S&P400
BVSP	Brazil	BSE, IPC, JKSE
IPC	Mexico	BSE, BVSP, JKSE
MERV	Argentina	BSE, JKSE

The table presents in columns one and two a list of the indices used for the cointegration study. The results of one of the bivariate cointegration studies (no intercept, no trend, $\alpha = 0.05$) are summarized in column three. It presents the indices for which cointegration is found together with the column one index.

As financial time series are usually found to exhibit heteroscedastic errors, we use the Ljung–Box test for autocorrelation on the squared levels and the squared log-returns of each index, the squared variables being a crude measure of the variance of the respective time series. The tests are conducted using ten lags. We find that both the stock market indices as well as the respective return time series exhibit time dependence in the variance.[3]

We perform bivariate cointegration tests among all possible combinations of the indices. As we have 28 indices, there are 378 possible index pairs. To perform cointegration tests we rely on the Johansen (1988) methodology. We use the Johansen (1988) test instead of the Engle and Granger (1987) two-step method in order to keep the study comparable to Kasa (1992) and Richards (1995). We restrict the analysis to the trace test as the maximum eigenvalue test leads in general to the same conclusions. The model used for testing is a simple VAR without intercept (as the time series do not exhibit a deterministic trend) and one lagged term (as suggested by the Schwarz information criterion). As regards the cointegration relationship, we implement both the specification with and without intercept. P-values are calculated via the response surface tables of MacKinnon, Haug, and Michelis (1999).

In the first case without intercept in the cointegration relationship (model 1), the trace test indicates that 46 out of the 378 combinations (i.e. 12.17%) are cointegrated when performed on a 5% significance

[3] To conserve space, the results of the $I(1)$ and heteroscedasticity tests are not printed.

level. The pairs of stock market indices found to be cointegrated are presented in Table 1. The results seem rather coincidental: first, geographical proximity is not a determinant for cointegration as, for example, not even the US indices DJIA, NASDAQ and S&P500 are cointegrated. Second, there also seems to be no link between cointegration and economic power (G7 countries versus rest of the world). When adding an intercept in the cointegrating equation (model 2), we find that (based on the trace test) 36 out of 378 (i.e. 9.52%) stock index combinations are cointegrated. As we have 2084 observations, we repeat the test on a more conservative significance level of 1%. We now find that 17 out of 378 combinations (4.50%) or 9 out of 378 (2.38%) combinations, respectively, are cointegrated. Hence, we find a tendency to reject the null hypothesis of no cointegration about twice as often as the significance level would allow given the assumption that indices are not cointegrated is true. This over-rejection, however, may very well be due to a characteristic feature of the return data: heteroscedasticity. Lee and Tse (1996), for example, showed that the presence of heteroscedasticity leads to over-rejection of the null hypothesis of no cointegration between two time series.

Whether the null hypothesis of no cointegration is rejected or not also depends on the time period which is analysed. Intuitively, this should not be the case: if a cointegration relationship existed and was identified between t and $t + 1000$ days, then it should also be identifiable between $t + 5$ and $t + 1005$ days. In order to illustrate that this assumption does not hold in the context of international stock market indices, we perform the cointegration test on a window of 1042 observations[4] moving through the full dataset in weekly steps (i.e. we have 208 times 378 tests). The result is graphically illustrated in Fig. 1. The solid line gives the rejection rates of the null hypothesis of no cointegration based on model 1 while the dashed line represents the rejection rates based on model 2. The first observation depicted corresponds to the sample window starting 1st March 2001 and ending 25th February 2005 while the last corresponds to 24th February 2005 to 25th February 2009. Across all windows we find on average 20% of the stock market indices to be cointegrated, both based on model 1 and model 2. As the graph in Fig. 1 stresses, the rejection rates are quite volatile when moving through the sample, ranging between 6 and 46%. This range is similar for both models used. Still, for the same period, the two models generally identify a different number of stock market indices as cointegrated. This is particularly true for the subsamples starting between December 2001 (depicted as 40 in Fig. 1) and August 2003 (130). Further, a pair identified by the model without intercept is not necessarily identified by the model with intercept in the same subsample period. In the subsequent simulation study we will show that a possible explanation for this behaviour lies in the presence of common stochastic trends, i.e. the global and local shocks, and heteroscedasticity in the error term in Eq. (1).

5. A simulation experiment

The unanswered question remaining is whether stock market indices do share a common stochastic trend, i.e. whether globally important information exists which is permanently absorbed into prices. As we have ruled out a standard cointegration analysis to answer this question, we conduct a simulation experiment and check whether the theoretical considerations in Section 3 are in line with the empirical findings in Section 4. We simulate prices according to the model

$$p_{i,t} = p_{i,t-1} + \iota' \epsilon_t, \tag{9}$$

[4] 1042 observations correspond to approximately 4 years of data and half of the observations in the full dataset. Results are qualitatively robust to shorter or longer periods.

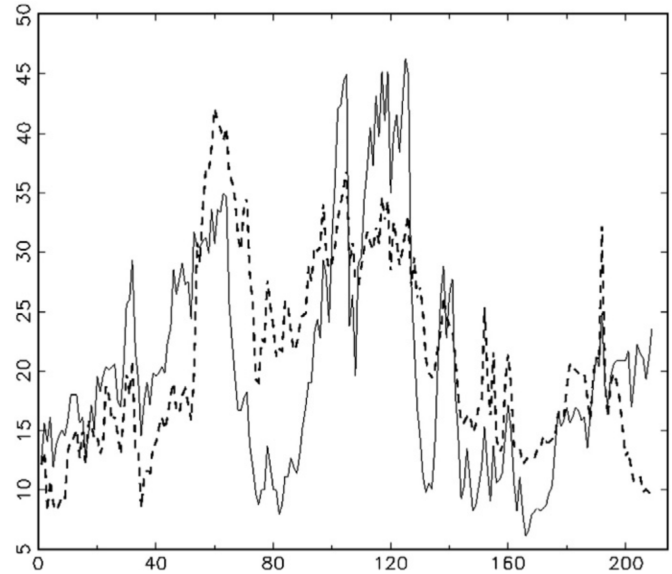

Fig. 1. Rolling cointegration test — rejection rates of $H_0 : r = 0$. The graphic depicts the rejection rates of the null hypothesis that the rank of the cointegration matrix is zero resulting from the rolling cointegration test. The solid line presents rates based on the model without intercept in the cointegration relationship while the dashed line presents the rates based on the model with intercept in the cointegration relationship. The abscissa represents the 208 subsamples. Tests were conducted on a 5% significance level.

where ϵ_t is a (3×1)-vector which collects the possibly correlated global, local, and stock specific innovations. ι is a (3×1)-vector of ones. The individual elements of the vector ϵ exhibit heteroscedasticity and are modelled as GARCH(1,1) processes according to

$$\begin{aligned} \epsilon_{i,t} &= \sqrt{h_{i,t}}\, \nu_{i,t} \\ h_{i,t} &= 0.01 + \gamma\epsilon_{i,t-1}^2 + \omega h_{i,t-1} \\ \nu_{i,t} &\sim N(0,1). \end{aligned} \tag{10}$$

The parameters of the GARCH-model vary: ω follows a uniform distribution (between 0.90 and 0.98 and $\gamma = 1 - \omega - 0.01$). The model, thus, exhibits the commonly documented pattern of high volatility persistence of financial data (e.g. Akgiray, 1989; Enders, 2010) while the variance process itself is ensured to be stationary. Initial values $p_{i,0}$ are set to zero. The first 200 observations of the simulated price series are discarded.

Indices are then calculated as a weighted sum of the individual price series. In order to follow the simple model in Section 4, we use two price series to construct an index. As stock market indices are never composed of two stocks only, we also construct two indices using 30 price series for each index (as in the DAX or the DJIA, for example). In both cases the weights are $w_i = \frac{1}{n}$, where n is the number of price series used to calculate the index. The study is conducted for sample sizes of $T = 500$ and $T = 1000$ observations. The simulations are run with Gauss using the KM random number generator and 10,000 replications. The Johansen (1988) test is conducted using only a model without drift and critical values are obtained using the response surface tables of MacKinnon et al. (1999).

In the basic setting we simulate the stock price model free of common components (elements $\epsilon_{2,t} = \epsilon_{3,t} = 0$ in ϵ_t in Eq. (9)) and let the structural innovations be independent, but heteroscedastic according to Eq. (10). We find that correlation of the returns is on average close to zero. At the same time the test for cointegration becomes less reliable. The trace test rejects the null hypothesis that the time series are not cointegrated about two to three times as often as suggested by the chosen significance level (cp. Case 1 in Table 2). So the presence of heteroscedasticity in the level price series seems to mislead the Johansen test. This is mainly due to the high volatility persistence.

Table 2
Simulation results.

Number of observations	α (%)	Number of prices in X_i	$H_0: r = 0$ (%)	$H_0: r = 1$ (%)	Return correlation
Case 1: no common components and uncorrelated innovations					
500	5	2	11.0000	5.7079	−0.0027
500	5	30	11.1800	6.7327	0.0025
1000	1	2	2.7600	0.6787	0.0099
1000	1	30	2.5200	0.5950	−0.0097
Case 2: common components and uncorrelated innovations					
500	5	2	12.2000	5.9909	0.3552
500	5	30	12.4400	5.8931	0.4440
1000	1	2	2.8400	0.7616	0.3489
1000	1	30	2.9600	0.6801	0.4465
Case 3: common components and correlated innovations					
500	5	2	11.9100	6.3571	0.8081
500	5	30	11.4900	5.4570	0.5827
1000	1	2	3.2000	0.7128	0.8233
1000	1	30	3.3500	0.8795	0.7294

The table presents simulation results for the model in Eq. (9) where the innovation term consists of a global, a local, and an idiosyncratic component. Each component follows an individual GARCH(1,1) process with normally distributed innovations. They may be uncorrelated (cases 1 and 2) or correlated (case 3). α is the significance level on which tests are conducted. $H_0: r = 0$ and $H_0: r = 1$ are the null hypotheses that the rank of the cointegration matrix is zero or one, respectively, in the Johansen trace test; the table reports rejection rates of these hypotheses; the second test is only performed once the first null hypothesis has been rejected.

However, if we lower ω such that it varies between 0.55 and 0.65, the cointegration test performs well within the expected limits.

In the second setting we add the common global and local components, specifying $\epsilon_{2,t}$ and $\epsilon_{3,t}$ as in Eq. (10). The structural innovations $\nu_{i,t}$ are still uncorrelated. Again we find the tendency of the cointegration test to slightly over-reject the null hypothesis of no cointegration (cp. Case 2 in Table 2). This level is also similar to what we find in the empirical example. The correlation between the returns of the simulated indices increases due to the common random walk component in both index series. It varies around 0.4, which is higher than in the benchmark case, but still lower than in the empirical example which suggests that there may be a further source of correlation in the stock market return series.

In the third setting we allow the individual innovations to be correlated.[5] We assume that the global and local components are moderately positively correlated and that individual innovations within one area are correlated with each other as well as with the local component. In the latter case, correlation can be negative. The correlation between the individual innovations of different areas is still zero.

In case of two constituents of an index, the covariance matrix Σ of all innovations ν_i—collected in a vector $(g, l_1, l_2, \varepsilon_{1,1}, \varepsilon_{1,2}, \varepsilon_{2,1}, \varepsilon_{2,2})$ with g the global, l the local and ε the idiosyncratic innovations—is specified as

	g	l_1	l_2	$\varepsilon_{1,1}$	$\varepsilon_{1,2}$	$\varepsilon_{2,1}$	$\varepsilon_{2,2}$
g	1.00						
l_1	0.60	1.00					
l_2	0.45	0.20	1.00				
$\varepsilon_{1,1}$	0.00	0.20	0.00	1.00			
$\varepsilon_{1,2}$	0.00	−0.30	0.00	−0.40	1.00		
$\varepsilon_{2,1}$	0.00	0.00	0.30	0.00	0.00	1.00	
$\varepsilon_{2,2}$	0.00	0.00	0.10	0.00	0.00	0.20	1.00

We assume that global and local components are dependent. The idiosyncratic shocks are independent from the global innovations (last four entries in the first column), but covary with the local component (second and third columns). In country 1 one stock is negatively correlated with the local component and with the other stock. Individual innovations are independent across countries (e.g. columns four and five contain zeros in rows six and seven). Although the choice of the entries will influence the outcome in this subsection, the general conclusions are not affected. For the thirty-stock index the structure of the covariance matrix is preserved (i.e. global and local components are still correlated while country 1 innovations and country 2 innovations are not). The entries, however, are random and can be positive or negative.

We now find the simulated data to fully reflect the properties found in the empirical analysis: the correlation is high (between 0.7 and 0.85) and the rejection rate of the null hypothesis that the indices are not cointegrated is approximately two to three times higher than the significance level would allow for, in particular if 1000 observations are simulated (cp. the lower part of Table 2). Wrong rejections seem to happen more frequently on the longer time series even though this is against the intuition that more data should deliver more reliable results. The rise of the return correlation is due to the additional source of dependence among the innovations which is induced through the structure of the covariance matrix. It is now in line with values as high as those found for DJIA and S&P 500 returns.

Variation of the parameters in the GARCH-model shows that as ω increases, the rejection rates of the null of no cointegration of the Johansen trace test increase. At the same time, the correlation measure diminishes slightly. So the more persistence there is in the variance equation, the less reliable the Johansen methodology seems to be.

In all simulation settings, we find that the number of stocks included in construction of the indices does not alter the results. As can be seen from Table 2, the rejection rates for the indices composed of two stocks and the indices composed of thirty stocks are virtually identical for each case. A possible explanation is that the number of common trends is the same: there is one global trend which is common to X_1 and X_2, irrespective of whether 2 or 30 stocks are used to construct the index. In other words, the addition of further unrelated stochastic trends—further stocks—does not influence the performance of the Johansen test.

From the above simulations we conclude that individual stock prices are best described by a model which contains a common global stochastic trend. This, together with correlated innovations and time-varying variance, translates into highly correlated, but not cointegrated stock market indices. In terms of information this means that there is global news which impacts on stock prices permanently while stock specific information also have a permanent effect and not only a transitory one. The latter, however, would be the necessary condition for cointegration being possible.

6. Summary and conclusion

The paper shows that under the assumption that stock prices follow the common random walk model, stock market indices of international financial markets cannot be cointegrated in the sense of Engle and Granger (1987). Cointegration is eventually inhibited by company specific innovations which are permanently absorbed into stock prices. These individual random walk components do not cancel out in a cointegration regression. Therefore, the resulting regression residuals are always non-stationary. The price model most likely includes a common global or local trend (as is usually the case in factor models) as well as correlated price innovations which is suitable to explain the observed high correlation between market indices, but not sufficient for cointegration. If the stock market indices were indeed cointegrated, only this one (!) global factor would be responsible for the price movement of all stocks worldwide which is rather unlikely.

In an empirical analysis using 28 stock market indices, we document that the detection of cointegration is rather coincidental. This is particularly highlighted by the fact that the time period which is analysed has a huge impact on the outcome of cointegration tests.

[5] The correlation matrices are constructed such that the fully specified model reflects the features found in Section 4 while still being compatible with other empirical findings (e.g. Harvey, 1991).

Even though structural breaks might be an issue for the whole sample (from 2001 to 2009), conducting the analysis on subperiods of four years should not alter the results as dramatically as was found in our analysis. A further feature which is known to influence cointegration tests (but largely ignored in studies dedicated to the analysis of international financial markets) is heteroscedasticity. It leads to an over-rejection of the null hypothesis of no cointegration and, thus, to faulty detection of cointegration between financial markets.

Based on the theoretical considerations in Section 3 and the empirical evidence in Section 4, we conclude that cointegration is not a suitable framework to analyse the interdependence or the integration of international financial markets. Even though a common stochastic trend may exist, it cannot be identified by means of cointegration analysis. Studying the interrelatedness or the degree of integration of financial markets via cointegration properties of stock market indices is therefore not fruitful and even inappropriate.

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