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A variable addition test for exogeneity in structural threshold models

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HIGHLIGHTS

- We propose a variable addition test for exogeneity in structural threshold models.
- The proposed test is implemented as a functional form test.
- The good finite sample properties of the test are shown by Monte Carlo analysis.

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ABSTRACT

This paper proposes a variable addition test for exogeneity in threshold regression models with potentially endogenous right-hand-side variables. An accurate Monte Carlo study is undertaken and the results show the good finite sample properties of the suggested test.

1. Introduction

Tong's (1983) original threshold regression model embodies the maintained assumption of exogenous right-hand-side variables. Caner and Hansen (2004) extend Tong's (1983) model to allow for potentially endogenous explanatory variables while keeping the threshold variable exogenous. Kourtellos et al. (2011) generalise Caner and Hansen's (2004) specification by allowing for potentially endogenous right-hand-side variables including the threshold variable. Kapetanios (2010) suggests Hausman-type exogeneity tests in Caner and Hansen's (2004) model where the threshold variable is *a priori* known to be exogenous. To the very best of our knowledge, testing for exogeneity under Kourtellos et al.'s (2011) framework where all right-hand-side variables are potentially endogenous is still an open issue. This paper aims at filling this gap and suggests a variable addition test for exogeneity in threshold models with potentially endogenous right-hand-side variables.

The paper is organised as follows: Section 2 introduces the model; the exogeneity test is described in Section 3; a Monte Carlo analysis is performed in Section 4; concluding remarks are given in Section 5. Concerning notation, $\mathbf{I}(\cdot)$ denotes the indicator function; $\xrightarrow{d}$ denotes convergence in distribution; $\mathbf{\Phi}(\cdot)$ denotes the cumulative distribution function of the standard normal distribution.

2. Model

We consider the structural form

$$y_t = \mathbf{z}_{1t}'\boldsymbol{\beta} + \mathbf{z}_{1t}'\boldsymbol{\delta}\mathbf{I}(z_{2t} > \alpha) + u_t, \quad t = 1, \ldots, T, \tag{1}$$

where $y_t \in \mathcal{Y} \subseteq \Re$ is the dependent variable; $\mathbf{z}_{1t} \in \mathcal{Z}_1 \subseteq \Re^k$ is a $k \times 1$ vector of potentially endogenous variables; $z_{2t} \in \mathcal{Z}_2 \subseteq \Re$ is the potentially endogenous threshold variable and α is the threshold value; $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ are $k \times 1$ vectors of slope coefficients. The reduced forms for $\mathbf{z}_{1t}$ and z_{2t} are

$$\mathbf{z}_{1t} = \mathbf{\Pi}_1\mathbf{x}_{1t} + \boldsymbol{\varepsilon}_{1t}, \qquad z_{2t} = \mathbf{x}_{2t}'\boldsymbol{\pi}_2 + \varepsilon_{2t}, \quad t = 1, \ldots, T, \tag{2}$$

respectively, where $\mathbf{x}_{1t} \in \mathcal{X}_1 \subseteq \Re^{m_1}$ and $\mathbf{x}_{2t} \in \mathcal{X}_2 \subseteq \Re^{m_2}$ are $m_1 \times 1$ and $m_2 \times 1$ vectors of instruments, respectively, with $m_1 \geq k$;

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$\mathbf{\Pi}_1$ is a $k\times m_1$ matrix of parameters; $\boldsymbol{\pi}_2$ is a $m_2\times 1$ vector of parameters. For all $k\times 1$ vectors $\mathbf{w}\in\Re^k$ we assume $0<\mathrm{P}\left(z_{2t}\neq\mathbf{z}_{1t}'\mathbf{w}\right)\leq 1$: should this condition fail to hold then (9) below would have to be suitably modified to avoid multicollinearity issues. Finally, the $(k+2)\times 1$ vector of error terms $\left(u_t,\boldsymbol{\varepsilon}_{1t}',\varepsilon_{2t}\right)'$ satisfies the conditional moment constraints $\mathrm{E}\left[\left(u_t,\boldsymbol{\varepsilon}_{1t}',\varepsilon_{2t}\right)'|\mathbf{x}_{1t},\mathbf{x}_{2t}\right]=\mathbf{0}$.

The model in (1) and (2) was formally introduced in Kourtellos et al. (2011); it generalises Caner and Hansen's (2004) model in which $\mathbf{z}_{1t}$ is potentially endogenous and the threshold variable z_{2t} is *a priori* known to be exogenous. Kapetanios (2010) suggests Hausman-type tests for the null hypothesis of exogeneity in Caner and Hansen's (2004) model. To the very best of our knowledge, no test for exogeneity has been proposed so far for the model in (1) and (2), in which both $\mathbf{z}_{1t}$ and z_{2t} are potentially endogenous: this paper aims at filling this gap and suggests a variable addition test for exogeneity.

In defining the null and the alternative hypotheses of interest we follow Blundell and Horowitz (2007): given (1) and the maintained constraint $\mathrm{E}(u_t|\mathbf{x}_{1t},\mathbf{x}_{2t})=0$, we test the null hypothesis of exogeneity

$$\begin{aligned}\mathbb{H}_0:\ &\mathrm{E}\left[y_t-\mathrm{E}\left(y_t|\mathbf{z}_{1t},z_{2t}\right)|\mathbf{x}_{1t},\mathbf{x}_{2t}\right]\\ &=0\Leftrightarrow\mathrm{E}\left(y_t|\mathbf{z}_{1t},z_{2t}\right)=\mathbf{z}_{1t}'\boldsymbol{\beta}+\mathbf{z}_{1t}'\boldsymbol{\delta}\mathbf{I}\left(z_{2t}>\alpha\right),\end{aligned}\tag{3}$$

against the alternative of endogeneity

$$\begin{aligned}\mathbb{H}_1:\ &\mathrm{E}\left[y_t-\mathrm{E}\left(y_t|\mathbf{z}_{1t},z_{2t}\right)|\mathbf{x}_{1t},\mathbf{x}_{2t}\right]\\ &\neq 0\Leftrightarrow\mathrm{E}\left(y_t|\mathbf{z}_{1t},z_{2t}\right)\neq\mathbf{z}_{1t}'\boldsymbol{\beta}+\mathbf{z}_{1t}'\boldsymbol{\delta}\mathbf{I}\left(z_{2t}>\alpha\right).\end{aligned}\tag{4}$$

The null hypothesis in (3) is equivalent to $\mathrm{E}(u_t|\mathbf{z}_{1t},z_{2t})=0$; the alternative in (4) is equivalent to $\mathrm{E}(u_t|\mathbf{z}_{1t},z_{2t})\neq 0$. Under the null hypothesis $\mathbf{z}_{1t}$ and z_{2t} are *both* exogenous and the parameters in (1) are consistently estimated by least squares. Chan (1993) shows that the least squares estimator for the threshold value α is super-consistent (i.e., T consistent) and asymptotically independent of that for $\left(\boldsymbol{\beta}',\boldsymbol{\delta}'\right)'$, the latter being $T^{1/2}$ asymptotically normally distributed. We assume the validity of the least squares estimator under $\mathbb{H}_0$ in (3): see Chan (1993) and Hansen (2000) for a detailed discussion about the underlying assumptions. Notice that we are not interested in running inference on α: unlike Hansen (2000), Caner and Hansen (2004), Kapetanios (2010) and Kourtellos et al. (2011), we do not require the "small threshold assumption", namely $\boldsymbol{\delta}\to\mathbf{0}$ at an appropriate rate as $T\to\infty$.

3. Exogeneity test

3.1. Testing procedure

For expositional purposes, to motivate the exogeneity test suggested below we first consider the case $\boldsymbol{\delta}=\mathbf{0}$; the structural form in (1) becomes

$$y_t=\mathbf{z}_{1t}'\boldsymbol{\beta}+u_t,\quad t=1,\ldots,T,\tag{5}$$

with reduced form for $\mathbf{z}_{1t}$ given in (2). Following Wooldrige (2012), the linear projection of u_t on $\boldsymbol{\varepsilon}_{1t}\equiv\mathbf{z}_{1t}-\mathbf{\Pi}_1\mathbf{x}_{1t}$ is written in error form as

$$u_t=\boldsymbol{\varepsilon}_{1t}'\boldsymbol{\lambda}+\xi_{1t},\quad t=1,\ldots,T,\tag{6}$$

where $\boldsymbol{\lambda}$ is a $k\times 1$ vector of parameters and the error term ξ_{1t} satisfies $\mathrm{E}(\xi_{1t}|\mathbf{z}_{1t},\mathbf{x}_{1t})=\mathbf{0}$. If $\boldsymbol{\varepsilon}_{1t}$ were observable (i.e., if $\mathbf{\Pi}_1$ were known) then $\boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ could be estimated by least squares from

$$y_t=\mathbf{z}_{1t}'\boldsymbol{\beta}+\boldsymbol{\varepsilon}_{1t}'\boldsymbol{\lambda}+\xi_{1t},\quad t=1,\ldots,T,\tag{7}$$

which is obtained by plugging (6) into (5); $\boldsymbol{\varepsilon}_{1t}$ is not observable and can be consistently estimated as $\hat{\boldsymbol{\varepsilon}}_{1t}\equiv\mathbf{z}_{1t}-\hat{\mathbf{\Pi}}_1\mathbf{x}_{1t}$ for $t=1,\ldots,T$, where $\hat{\mathbf{\Pi}}_1$ is the least squares estimator for $\mathbf{\Pi}_1$ in (2). The parameters vectors $\boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ can then be consistently estimated by least squares from

$$\begin{aligned}y_t&=\mathbf{z}_{1t}'\boldsymbol{\beta}+\hat{\boldsymbol{\varepsilon}}_{1t}'\boldsymbol{\lambda}+\nu_{1t},\\ \nu_{1t}&\equiv\xi_{1t}+\left(\boldsymbol{\varepsilon}_{1t}-\hat{\boldsymbol{\varepsilon}}_{1t}\right)'\boldsymbol{\lambda},\quad t=1,\ldots,T,\end{aligned}\tag{8}$$

where $\hat{\boldsymbol{\varepsilon}}_{1t}$ controls for the potential endogeneity of $\mathbf{z}_{1t}$ in (5). Testing the null hypothesis that $\mathbf{z}_{1t}$ is exogenous in (5) is equivalent to testing for $\boldsymbol{\lambda}=\mathbf{0}$ in (8): this is a variable addition test. Given $\mathrm{E}(y_t|\mathbf{z}_{1t},\mathbf{x}_{1t})=\mathbf{z}_{1t}'\boldsymbol{\beta}+\boldsymbol{\varepsilon}_{1t}'\boldsymbol{\lambda}$ in (7), testing $\boldsymbol{\lambda}=\mathbf{0}$ is equivalent to testing the null hypothesis $\mathrm{E}(y_t|\mathbf{z}_{1t})=\mathbf{z}_{1t}'\boldsymbol{\beta}$ against the alternative $\mathrm{E}(y_t|\mathbf{z}_{1t})\neq\mathbf{z}_{1t}'\boldsymbol{\beta}$: a variable addition test for exogeneity is then a functional form test for $\mathrm{E}(y_t|\mathbf{z}_{1t})$.

We now suggest a variable addition test for the null hypothesis that $\mathbf{z}_{1t}$ and z_{2t} are both exogenous in the general structural form in (1): see DeBenedictis and Giles (1998) for a comprehensive description of variable addition tests. Compared to a Hausman-type test, a variable addition test is appealing as it does not require estimation of (1) under the alternative hypothesis of endogeneity: see Kourtellos et al. (2011) for further details on this issue, including the additional distributional assumptions. Unlike the linear model in (5), a closed-form expression for $\mathrm{E}(y_t|\mathbf{z}_{1t},z_{2t},\mathbf{x}_{1t},\mathbf{x}_{2t})$ is not available for the nonlinear specification in (1): for testing purposes, following the same approach that underlies Ramsey's (1969) RESET test, we *approximate* it by using polynomials in $\hat{\boldsymbol{\varepsilon}}_{1t}$ and $\hat{\varepsilon}_{2t}$, where $\hat{\varepsilon}_{2t}\equiv z_{2t}-\mathbf{x}_{2t}'\hat{\boldsymbol{\pi}}_2$ for $t=1,\ldots,T$, and $\hat{\boldsymbol{\pi}}_2$ is the least squares estimator for $\boldsymbol{\pi}_2$ in (2). Under the null hypothesis in (3), the conditional moment $\mathrm{E}(y_t|\mathbf{z}_{1t},z_{2t})$ is made of the linear part $\mathbf{z}_{1t}'\boldsymbol{\beta}$ and of the nonlinear component $\mathbf{z}_{1t}'\boldsymbol{\delta}\mathbf{I}(z_{2t}>\alpha)$: as in the linear model in (5), misspecification in $\mathrm{E}(y_t|\mathbf{z}_{1t},z_{2t})$ due to endogeneity of $\mathbf{z}_{1t}$ can be parsimoniously captured by adding $\hat{\boldsymbol{\varepsilon}}_{1t}$; by polynomial approximation of nonlinear functions, misspecification in $\mathrm{E}(y_t|\mathbf{z}_{1t},z_{2t})$ due to endogeneity of z_{2t} through $\mathbf{I}(z_{2t}>\alpha)$ can be parsimoniously captured by adding $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{2t}^2$. This leads to the augmented model

$$\begin{aligned}y_t&=\mathbf{z}_{1t}'\boldsymbol{\beta}+\mathbf{z}_{1t}'\boldsymbol{\delta}\mathbf{I}\left(z_{2t}>\alpha\right)+\hat{\boldsymbol{\varepsilon}}_{1t}'\boldsymbol{\kappa}+\hat{\varepsilon}_{2t}\phi_1+\hat{\varepsilon}_{2t}^2\phi_2+\varsigma_t,\\ &t=1,\ldots,T,\end{aligned}\tag{9}$$

where $\boldsymbol{\kappa}$ is a $k\times 1$ parameters vector, and ϕ_1 and ϕ_2 are scalar parameters: a variable addition test for the null hypothesis in (3) against the alternative in (4) is obtained by estimating (9) by least squares and testing the null hypothesis

$$\mathbb{H}_0:(\boldsymbol{\kappa}=\mathbf{0})\cap(\phi_1=0)\cap(\phi_2=0)\tag{10}$$

against the alternative

$$\mathbb{H}_1:(\boldsymbol{\kappa}\neq\mathbf{0})\cup(\phi_1\neq 0)\cup(\phi_2\neq 0).\tag{11}$$

Let SSR_0 and SSR_1 denote the sum of squared residuals obtained from least squares estimation of (9) under $\mathbb{H}_0$ and $\mathbb{H}_1$ in (10) and (11), respectively. Since the least squares estimator for α is super-consistent (i.e., T consistent), the test statistic

$$\mathcal{F}_T=T\frac{SSR_0-SSR_1}{SSR_0}\tag{12}$$

is such that as $T\to\infty$: (a) $\mathcal{F}_T\xrightarrow{d}\chi^2(k+2)$ under $\mathbb{H}_0$ in (10); and (b) $\mathrm{P}(\mathcal{F}_T>C)=1$ under $\mathbb{H}_1$ in (11) for any $C\in\Re$.

Comments on the augmented model in (9). The augmented model in (9) has been motivated by an argument of parsimony; however, other augmented models leading to valid tests for exogeneity of $\mathbf{z}_{1t}$ and z_{2t} in (1) can be constructed. We discuss two possible and not mutually exclusive extensions. First, potential endogeneity of $\mathbf{z}_{1t}$ can be captured by accounting for the interaction between $\mathbf{z}_{1t}$ and z_{2t}: the cross products $\hat{\boldsymbol{\varepsilon}}_{1t}\hat{\varepsilon}_{2t}$ and $\hat{\boldsymbol{\varepsilon}}_{1t}\hat{\varepsilon}_{2t}^2$ could then be added to (9). Further, potential endogeneity of z_{2t} can be captured by higher order polynomial approximations in $\hat{\varepsilon}_{2t}$ to better approximate the shape of $\mathbf{I}(z_{2t}>\alpha)$. These extensions produce augmented models that are less parsimonious than (9): the net effect on the power of the resulting exogeneity tests would have to be assessed by Monte Carlo methods. Notice that exogeneity tests when either $\mathbf{z}_{1t}$ or z_{2t} are *a priori* known to be exogenous can also be constructed from (9). When $\mathbf{z}_{1t}$ is exogenous the augmented model becomes

$$y_t = \mathbf{z}'_{1t}\boldsymbol{\beta} + \mathbf{z}'_{1t}\boldsymbol{\delta}\mathbf{I}(z_{2t} > \alpha) + \hat{\varepsilon}_{2t}\phi_1 + \hat{\varepsilon}^2_{2t}\phi_2 + \varsigma_t, \quad t = 1, \ldots, T,$$

and the null hypothesis is $\mathbb{H}_0 : (\phi_1 = 0) \cap (\phi_2 = 0)$. When z_{2t} is exogenous (9) simplifies to

$$y_t = \mathbf{z}'_{1t}\boldsymbol{\beta} + \mathbf{z}'_{1t}\boldsymbol{\delta}\mathbf{I}(z_{2t} > \alpha) + \hat{\boldsymbol{\varepsilon}}'_{1t}\boldsymbol{\kappa} + \varsigma_t, \quad t = 1, \ldots, T,$$

and the null hypothesis is $\mathbb{H}_0 : \boldsymbol{\kappa} = \mathbf{0}$: in this case it would be interesting to compare the power of the proposed variable addition test with that of Kapetanios's (2010) Hausman-type tests.

3.2. Wild bootstrap

As proved in Chan (1993), least squares estimation of threshold models leads to a super-consistent estimator for the threshold value which is asymptotically independent of the estimator for the slope coefficients. However, as argued in Kapetanios (2000), this asymptotic independence property is unlikely to hold in finite samples. We then follow Kapetanios (2010) and propose a bootstrap-based version of the suggested exogeneity test: as discussed in Horowitz (2001), the bootstrap is expected to produce consistent confidence intervals since the test statistic $\mathcal{F}_T$ in (12) is constructed from the asymptotically normally distributed estimators for the slope coefficients in (9). We focus on the wild bootstrap. Let b and B denote the b-th bootstrap replication and the total number of replications, respectively; for $b = 1, \ldots, B$, the wild bootstrap involves the following steps:

1. Construct the test statistic $\mathcal{F}_T$ in (12).
2. Construct the sequence of residuals

   $$\tilde{\varsigma}_t \equiv y_t - \mathbf{z}'_{1t}\tilde{\boldsymbol{\beta}}_T - \mathbf{z}'_{1t}\tilde{\boldsymbol{\delta}}_T\mathbf{I}(z_{2t} > \tilde{\alpha}_T), \quad t = 1, \ldots, T,$$

   where $\tilde{\boldsymbol{\beta}}_T$, $\tilde{\boldsymbol{\delta}}_T$ and $\tilde{\alpha}_T$ are the restricted least squares estimators for $\boldsymbol{\beta}$, $\boldsymbol{\delta}$ and α in (9), respectively, under the null hypothesis in (10).
3. Generate the bootstrap sample under the null hypothesis in (10) as

   $$y^b_t = \mathbf{z}'_{1t}\tilde{\boldsymbol{\beta}}_T + \mathbf{z}'_{1t}\tilde{\boldsymbol{\delta}}_T\mathbf{I}(z_{2t} > \tilde{\alpha}_T) + v^b_t\tilde{\varsigma}_t,$$
   $$v^b_t \sim IID\mathcal{N}(0, 1), \quad t = 1, \ldots, T, \ b = 1, \ldots, B,$$

   and compute the test statistic

   $$\mathcal{F}^b_T = T\frac{SSR^b_0 - SSR^b_1}{SSR^b_0}, \quad b = 1, \ldots, B,$$

   where SSR^b_0 and SSR^b_1 are the sum of squared residuals under $\mathbb{H}_0$ and $\mathbb{H}_1$ in (10) and (11), respectively, from the b-th replication.
4. Estimate the p-value as

   $$\hat{p}_T = \frac{1}{B}\sum_{b=1}^{B}\mathbf{I}\left(\mathcal{F}_T < \mathcal{F}^b_T\right).$$

4. Monte Carlo analysis

4.1. Design

The data generating process (DGP) is

$$y_t = \left(\beta_1 + z^r_{1t}\beta_2\right) + \left(\delta_1 + z^r_{1t}\delta_2\right)\mathbf{I}\left(z^r_{2t} > \alpha\right) + u^r_t, \quad t = 1, \ldots, T, \ r = 1, \ldots, R,$$

where the superscript r refers to the replication and R is the total number of replications: the threshold effects on the intercept and on the slope coefficient associated to z^r_{1t} are measured by δ_1 and δ_2, respectively. We set $\alpha = 2$ and $\beta_1 = \beta_2 = \beta = 0.25$, α and β fixed in repeated samples. The variables z^r_{1t} and z^r_{2t} are generated as

$$z^r_{it} = \pi_1 + x^r_{it}\pi_2 + \left(1 - \pi^2_2\right)^{1/2}\varepsilon^r_{it}, \quad i = 1, 2, \ t = 1, \ldots, T, \ r = 1, \ldots, R.$$

Table 1
Size based on asymptotic distribution, 5% level.

T	$\pi_2 = 0.5$			$\pi_2 = 0.8$		
	$\delta = 0.25$	$\delta = 1.00$	$\delta = 1.75$	$\delta = 0.25$	$\delta = 1.00$	$\delta = 1.75$
Panel (a): $\delta_1 = \delta$, $\delta_2 = 0$						
50	0.0930	0.1180	0.0790	0.0930	0.1290	0.0790
100	0.0970	0.0750	0.0590	0.0880	0.0710	0.0610
200	0.0760	0.0470	0.0420	0.0600	0.0500	0.0480
400	0.0770	0.0510	0.0500	0.0510	0.0510	0.0500
Panel (b): $\delta_1 = \delta_2 = \delta$						
50	0.1060	0.0540	0.0490	0.1220	0.0570	0.0520
100	0.0910	0.0600	0.0590	0.0770	0.0600	0.0580
200	0.0550	0.0420	0.0420	0.0540	0.0480	0.0480
400	0.0500	0.0480	0.0480	0.0510	0.0500	0.0460

The experiment is run in Ox 6.21. The sample sizes T, the total number of Monte Carlo replications R, and the total number of bootstrap replications B are set equal to $T = 50, 100, 200, 400$, $R = 1000$ and $B = 300$, respectively. The seed of the random number generator is set equal to -1. We control for: (i) the magnitude of the threshold effect; (ii) the degree of endogeneity through the correlation between ε^r_{1t} and ε^r_{2t} on the one hand and u^r_t on the other hand; and (iii) the strength of the instruments x^r_{1t} and x^r_{2t} through π_2.

The exogenous variables x^r_{1t} and x^r_{2t} in the reduced forms for z^r_{1t} are z^r_{2t} are generated as

$$x^r_{it} = \mu(1-\rho) + \rho x^r_{i,t-1} + \left(1-\rho^2\right)^{1/2}\epsilon^r_{it}, \quad i = 1, 2, \ t = -49, \ldots, T, \ r = 1, \ldots, R, \ x_{i,-50} = 0,$$

where $\mu \sim \mathcal{N}(1, 1)$, $\rho \sim \mathcal{U}(0.05, 0.95)$, μ and ρ fixed in repeated samples. The shocks ϵ^r_{1t} and ϵ^r_{2t} are generated as

$$\epsilon^r_{it} = \left(f^r_{\epsilon t} + \eta^r_{\epsilon_i t}\right)/\sqrt{2}, \quad i = 1, 2, \ t = -49, \ldots, T, \ r = 1, \ldots, R,$$

where $f^r_{\epsilon t} \sim IID\mathcal{N}(0, 1)$, $\eta^r_{\epsilon_1 t} \sim IID\mathcal{N}(0, 1)$, and $\eta^r_{\epsilon_2 t} \sim IID\mathcal{N}(0, 1)$: in this way $\mathrm{Corr}\left(x^r_{1t}, x^r_{2t}\right) = \mathrm{Corr}\left(\epsilon^r_{1t}, \epsilon^r_{2t}\right) = 0.5$. The first 50 observations in the DGP for x^r_{1t} and x^r_{2t} are discarded to reduce the effect induced by the initial values $x^r_{1,-50} = x^r_{2,-50} = 0$.

The shocks u^r_t, ε^r_{1t}, and ε^r_{2t} are generated as

$$u^r_t = \frac{\gamma_{\varepsilon u}f^r_{\varepsilon u,t} + \eta^r_{ut}}{\left(\gamma^2_{\varepsilon u} + 1\right)^{1/2}},$$

$$\varepsilon^r_{it} = \frac{\gamma_{\varepsilon u}f^r_{\varepsilon u,t} + \eta^r_{\varepsilon_i t}}{\left(\gamma^2_{\varepsilon u} + 1\right)^{1/2}}, \quad i = 1, 2, \ t = 1, \ldots, T, \ r = 1, \ldots, R,$$

where $f^r_{\varepsilon u,t} \sim IID\mathcal{N}(0, 1)$, $\eta^r_{ut} \sim IID\mathcal{N}(0, 1)$, $\eta^r_{\varepsilon_1 t} \sim IID\mathcal{N}(0, 1)$, and $\eta^r_{\varepsilon_2 t} \sim IID\mathcal{N}(0, 1)$: in this way $\mathrm{Corr}(u^r_t, \varepsilon^r_{1t}) = \mathrm{Corr}\left(u^r_t, \varepsilon^r_{2t}\right) = \mathrm{Corr}\left(\varepsilon^r_{1t}, \varepsilon^r_{2t}\right) = \gamma^2_{\varepsilon u}/\left(\gamma^2_{\varepsilon u} + 1\right)$.

Let $p_{z_2} \equiv \mathrm{P}\left(z^r_{2t} \le \alpha\right)$. Since $\mathrm{E}\left(z^r_{2t}\right) = \pi_1 + \mu\pi_2$ and $\mathrm{Var}\left(z^r_{2t}\right) = 1$ then $p_{z_2} = \boldsymbol{\Phi}(\alpha - \pi_1 - \mu\pi_2)$: we set $\pi_1 = \alpha - \mu\pi_2 - \boldsymbol{\Phi}^{-1}\left(p_{z_2}\right)$ for $p_{z_2} = 0.50$.

The magnitude of the threshold effect is controlled through $\boldsymbol{\delta} \equiv (\delta_1, \delta_2)'$ and two cases are considered: (a) $\delta_1 = \delta = 0.25, 1.00, 1.75$ and $\delta_2 = 0$; and (b) $\delta_1 = \delta_2 = \delta = 0.25, 1.00, 1.75$. The degree of endogeneity is controlled through $\gamma_{\varepsilon u}$: we set $\gamma_{\varepsilon u} = 0, 1, 3$, which correspond to $\gamma^2_{\varepsilon u}/\left(\gamma^2_{\varepsilon u} + 1\right) = 0, 0.5, 0.9$, respectively, where $\gamma_{\varepsilon u} = 0$ provides the size of the test, whereas $\gamma_{\varepsilon u} = 1, 3$ are informative about the power. Finally, $\mathrm{Corr}\left(z^r_{1t}, x^r_{1t}\right) = \mathrm{Corr}\left(z^r_{2t}, x^r_{2t}\right) = \pi_2$ and the strength of the instruments is controlled through π_2 by setting $\pi_2 = 0.5, 0.8$.

4.2. Results

Results for a nominal size equal to 5% are shown in Tables 1–3. When the empirical size is computed from the critical values of the asymptotic distribution of $\mathcal{F}_T$ (see Table 1) the test is slightly

Table 2
Size and power based on wild bootstrap, 5% level.

T	$\gamma_{\varepsilon u}$	$\pi_2 = 0.5$			$\pi_2 = 0.8$		
		$\delta = 0.25$	$\delta = 1.00$	$\delta = 1.75$	$\delta = 0.25$	$\delta = 1.00$	$\delta = 1.75$
Panel (a): $\delta_1 = \delta, \delta_2 = 0$							
	0	0.0180	0.0310	0.0330	0.0130	0.0300	0.0290
50	1	0.0940	0.1490	0.1500	0.1010	0.1690	0.1590
	3	0.5770	0.6180	0.6380	0.6240	0.7200	0.7260
	0	0.0270	0.0390	0.0390	0.0480	0.0330	0.0360
100	1	0.6280	0.6890	0.6960	0.9510	0.9570	0.9630
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0540	0.0520	0.0490	0.0640	0.0490	0.0530
200	1	0.9880	0.9920	0.9920	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0560	0.0540	0.0560	0.0500	0.0430	0.0440
400	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Panel (b): $\delta_1 = \delta_2 = \delta$							
	0	0.0290	0.0310	0.0310	0.0310	0.0240	0.0260
50	1	0.1410	0.1540	0.1540	0.1570	0.1560	0.1590
	3	0.6210	0.6390	0.6390	0.7120	0.7290	0.7290
	0	0.0360	0.0340	0.0340	0.0370	0.0380	0.0390
100	1	0.6780	0.6990	0.7000	0.9530	0.9640	0.9660
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0500	0.0510	0.0510	0.0500	0.0570	0.0560
200	1	0.9920	0.9920	0.9920	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0490	0.0560	0.0560	0.0460	0.0440	0.0440
400	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 3
Size and power based on asymptotic distribution for infeasible benchmark, 5% level.

T	$\gamma_{\varepsilon u}$	$\pi_2 = 0.5$			$\pi_2 = 0.8$		
		$\delta = 0.25$	$\delta = 1.00$	$\delta = 1.75$	$\delta = 0.25$	$\delta = 1.00$	$\delta = 1.75$
Panel (a): $\delta_1 = \delta, \delta_2 = 0$							
	0	0.0510	0.0510	0.0510	0.0520	0.0520	0.0520
50	1	0.2300	0.2300	0.2300	0.2770	0.2770	0.2770
	3	0.7850	0.7850	0.7850	0.8680	0.8680	0.8680
	0	0.0580	0.0580	0.0580	0.0570	0.0570	0.0570
100	1	0.7720	0.7720	0.7720	0.9740	0.9740	0.9740
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0420	0.0420	0.0420	0.0500	0.0500	0.0500
200	1	0.9910	0.9910	0.9910	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0450	0.0450	0.0450	0.0460	0.0460	0.0460
400	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Panel (b): $\delta_1 = \delta_2 = \delta$							
	0	0.0510	0.0510	0.0510	0.0520	0.0520	0.0520
50	1	0.2300	0.2300	0.2300	0.2770	0.2770	0.2770
	3	0.7850	0.7850	0.7850	0.8680	0.8680	0.8680
	0	0.0580	0.0580	0.0580	0.0570	0.0570	0.0570
100	1	0.7720	0.7720	0.7720	0.9740	0.9740	0.9740
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0420	0.0420	0.0420	0.0500	0.0500	0.0500
200	1	0.9910	0.9910	0.9910	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0	0.0450	0.0450	0.0450	0.0460	0.0460	0.0460
400	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

oversized for combinations of low values of δ, π_2 and T; the problem is more pronounced when $\delta_2 = 0$, that is when the threshold effect is induced only on the intercept. This is consistent with the results in Kapetanios (2000): the asymptotic independence property deriving from the super-consistent estimator for the threshold value may not hold in finite samples. The test is correctly sized in the remaining cases.

When size and power are computed by wild bootstrap (see Table 2) the test is correctly sized; the power of the test monotonically increases in T, $\gamma_{\varepsilon u}$, π_2 and, to a lesser extent, in δ.

Finally, size and power are computed from the asymptotic distribution of $\mathcal{F}_T$ under an "infeasible benchmark" (see Table 3): this is obtained by replacing the estimated augmenting variables and threshold value by their true population values in the augmented model. This allows us to assess the effect of nuisance parameters on the test, as the benchmark produces a nuisance parameters free test. The results show that the test has the desired finite sample properties under the benchmark.

5. Concluding remarks

This paper has proposed a simple variable addition test for exogeneity in threshold regression models with potentially endogenous right-hand-side variables. A Monte Carlo study shows the good finite sample properties of the suggested test. Future work should investigate the applicability of the test we suggest to similar models with potentially endogenous right-hand-side variables, such as Areosa et al.'s (2011) smooth transition model and Massacci's (2012) exponential smooth transition model.

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