



Euler equations and money market interest rates: The role of monetary policy and risk premium shocks



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HIGHLIGHTS

- In theory, Euler equation and money market interest rates are equal.
- VAR evidence suggests that they are negatively correlated.
- Stochastic risk premium disturbances can account for this observation.
- Implementing collateral constraints tied to housing might be promising.

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ABSTRACT

We challenge the view that the negative correlation between the Federal Funds and the Euler equation interest rate is linked to monetary policy. Using Monte Carlo experiments, we show that the negative correlation can be explained by risk premium disturbances.

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1. Introduction

The limited performance of consumption Euler equations is well known. Recently, Canzoneri et al. (2007) present another failure of consumption Euler equations. Using a novel approach, they challenge the view that the money market interest rate targeted by the central bank is equal to the rate implied by a Euler equation, as is commonly assumed in standard New Keynesian (NK) models. Canzoneri et al. (2007) use US data and derive conditional moments of consumption and inflation from an estimated vector autoregression (VAR). These moments and actual observations are then used to compute interest rates implied by consumption Euler equations obtained from alternative specifications of preferences. By comparing implied with actual interest rates, two results stand out. First, the behavior of implied rates differs significantly from

the Federal Funds rate. In particular, real interest rates implied by Euler equations are strongly negatively correlated with the observed money market rate. Second, using standard regression analysis and impulse response functions, Canzoneri et al. (2007) report that the Federal Funds rate and the Euler equation rate move in opposite directions following a monetary policy tightening.

The purpose of this paper is to explore the link between the correlation between implied and actual interest rates and the stance of monetary policy. As explained by Canzoneri et al. (2007), the fact that the two rates do not coincide is intuitive if the representative household has standard, additively separable CRRA preferences. Empirical studies show that consumption responds in a hump-shaped fashion to a monetary contraction (see Christiano et al., 2005). That is, in the quarters following a monetary contraction, interest rates and consumption growth are negatively correlated. Standard preferences, however, imply that consumption growth and interest rates are positively correlated. Consequently, using a standard Euler equation to compute implied interest rates

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results in a negative correlation between actual and implied interest rates. With this intuition in mind, adding habit persistence to household preferences seems to be a promising candidate to reconcile the dynamics of money market interest rates and rates implied by Euler equations.¹ Most prominently, [Fuhrer \(2000\)](#), [Christiano et al. \(2005\)](#), and [Smets and Wouters \(2007\)](#) rely on habit persistence to explain the observed dynamics of output and consumption in response to a monetary policy shock. From this perspective, the finding in [Canzoneri et al. \(2007\)](#) that the implied Euler equation rate and the Federal Funds rate do not coincide across a large number of preference specifications that explicitly allow for habit formation is quite surprising.²

To investigate the sources of the negative correlation between implied and actual interest rates, we make use of a Monte Carlo experiment. We assume that the model economy is defined by a full-fledged NK dynamic stochastic general equilibrium (DSGE) model. We use this model as a data-generating process and compute replications of simulated data. We then use the simulated data to construct implied Euler equation rates following the methodology set forth by [Canzoneri et al. \(2007\)](#). Based on this setup, counterfactual simulations allow us to explore the sources of the spread between implied and actual interest rates in a direct way. We choose to use the estimated model in [Smets and Wouters \(2007\)](#) (henceforth, SW) as our data-generating process. We do so because of several reasons. First, the SW model features complex dynamics with a rich set of structural shocks that aims to describe a fairly complete quantitative description of the US economy. Second, the consumption Euler equation in the SW model deviates from a standard Euler equation along two dimensions. On one hand, it features habit formation. On the other hand, it allows for nonseparability between consumption and labor effort. This is relevant because [Collard and Dellas \(2012\)](#) find that this feature limits the failure of consumption Euler equations identified by [Canzoneri et al. \(2007\)](#). Third, we choose to use the SW model for our Monte Carlo experiment, because the model features a wedge between the money market interest rate and the interest rate implied by the consumption Euler equation. A shock to this wedge (risk premium shock) distorts the equality between the two rates and causes a change in the consumption pattern of households. Hence, given that in the data-generating process implied by the SW model the spread between Euler equation and actual interest rates is simply a statistical noise, we are able to disentangle the impact of monetary policy on the correlation between the two rates from the effect that arises from the assumption of risk premium disturbances.

In the next section, we use US data to compute interest rates implied by consumption Euler equations for two sets of preferences and compare these rates to the Federal Funds rate. In Section 3, we use a Monte Carlo experiment to explore the relationship between implied and actual interest rates. Section 4 concludes the paper.

2. Comparing Euler equation and money market interest rates

Here, we follow the approach in [Canzoneri et al. \(2007\)](#) and compute nominal and real interest rates implied by consumption Euler equations. We compute implied interest rates for the specification of preferences as in [Smets and Wouters \(2007\)](#) and for standard, additively separable CRRA preferences.³ As in [Smets](#)

and [Wouters \(2007\)](#), the consumer's objective function is assumed to be

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1 - \sigma_c} (C_t - H_t)^{1 - \sigma_c} \right) \exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_t^{1 + \sigma_l} \right), \quad (1)$$

where E_0 denotes the expectation operator at period $t = 0$, C_t denotes consumption relative to a habit stock, H_t , and L_t is hours worked. The parameter σ_c is the coefficient of relative risk aversion, and σ_l is the inverse elasticity of labor supply. The habit stock is external and is defined by $H_t = \lambda C_{t-1}$, where λ governs the degree of habit formation. Smets and Wouters' specification of consumer preferences nests the standard CRRA utility function with separability between consumption and hours worked and no habit formation. If σ_c approaches 1 and $h = 0$, the period utility function implied by (1) approaches a standard log utility function, so that lifetime utility reads

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t). \quad (2)$$

The corresponding Euler equations to conditions (1) and (2) are

$$\frac{\exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_t^{1 + \sigma_l} \right)}{(C_t - \lambda C_{t-1})^{\sigma_c}} = \beta E_t \left(\frac{\exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_{t+1}^{1 + \sigma_l} \right)}{(C_{t+1} - \lambda C_t)^{\sigma_c}} \frac{R_t \epsilon_t^b}{\Pi_{t+1}} \right) \quad (3)$$

$$\text{and } \frac{1}{C_t} = \beta E_t \left(\frac{1}{C_{t+1}} \frac{R_t \epsilon_t^b}{\Pi_{t+1}} \right), \quad (4)$$

where R_t is the gross nominal interest rate controlled by the central bank, Π_t is the gross inflation rate, and ϵ_t^b is a risk premium shock that represents a wedge between R_t and the return on bonds held by households. The shock is assumed to follow an AR(1) process in logs. Following the analysis in [Canzoneri et al. \(2007\)](#), we abstract from the shock term when we compute implied Euler equation interest rates.

Log-linearizing (3) around the steady state balanced growth path of the model yields the following dynamics of nominal, respectively, real interest rates⁴

$$r_t = (1/c_3)(c_1 c_{t-1} - c_t + (1 - c_1) E_t c_{t+1} + c_2(l_t - E_t l_{t+1})) + E_t \pi_{t+1} \quad (5)$$

and

$$rr_t = (1/c_3)(c_1 c_{t-1} - c_t + (1 - c_1) E_t c_{t+1} + c_2(l_t - E_t l_{t+1})), \quad (6)$$

where $c_1 = \frac{\lambda/\gamma}{1 + \lambda/\gamma}$, $c_2 = \frac{(\sigma_c - 1)(W_{\#}^h L_{\#}/C_{\#})}{\sigma_c(1 + \lambda/\gamma)}$, $c_3 = \frac{1 - \lambda/\gamma}{\sigma_c(1 + \lambda/\gamma)}$, and γ is the steady state growth rate. The log-linear dynamics of nominal and real interest rates implied by (4) are given by

$$r_t = E_t c_{t+1} - c_t + E_t \pi_{t+1} \quad (7)$$

$$\text{and } rr_t = E_t c_{t+1} - c_t. \quad (8)$$

To compute implied interest rates from Eqs. (5)–(8), we use the posterior mean estimates for the model parameters as reported in [Smets and Wouters \(2007\)](#) and for the conditional forecasts we follow [Canzoneri et al. \(2007\)](#) and assume that the dynamics of consumption, employment, and inflation can be captured in a VAR defined as

$$Z_t = A_0 + A_1 Z_{t-1} + \dots + A_p Z_{t-p} + u_t, \quad (9)$$

¹ See [Schmitt-Grohé and Uribe \(2008\)](#) and [Dennis \(2009\)](#) for a review on the concept of habit formation in macroeconomic models.

² See [Canzoneri et al. \(2007\)](#) for a further discussion.

³ In each model it is assumed that the representative household is infinitely lived and chooses consumption, labor effort, and one-period nominal bonds to maximize lifetime utility subject to a budget constraint.

⁴ A lower case letter denotes the log-linear deviation of the corresponding upper case letter from the balanced growth path, and starred variables refer to steady state values (see [Smets and Wouters, 2007](#)). Note that [Canzoneri et al. \(2007\)](#) compute implied interest rates under the assumption of conditional lognormality. As they have already pointed out, the assumption of lognormality results in Euler equations that differ from those derived by log-linearization only by a constant.

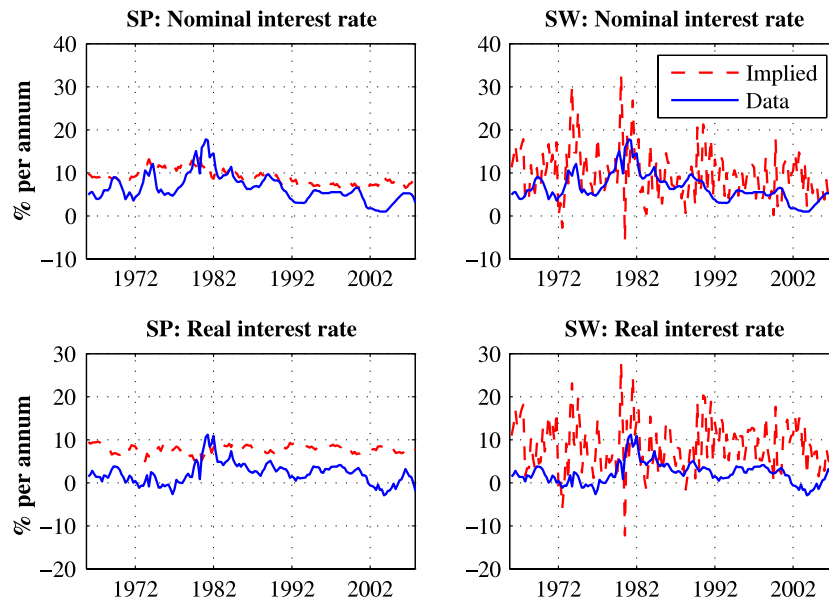


Fig. 1. Euler equation versus Federal Funds rate.

where u_t is a vector of IID-normal error terms. The variables in the VAR are the log of per capita real consumption expenditures on nondurable goods and services, the inflation rate, a measure of hours worked, the log CRB price index, the log of per capita real disposable income, the log of per capita real nonconsumption GDP and the Federal Funds rate.⁵

In Fig. 1 we present the results by comparing the time series of the Federal Funds rate and the rates implied by the two sets of consumption Euler equations. Table 1 summarizes the properties of actual and implied interest rates.

Apparently, implied Euler equation rates behave significantly different compared to actual interest rates. While habit formation and nonseparability lead to a positive correlation between implied and actual real interest rates, we find that the implied Euler equation rates are extremely volatile. On one hand, this is in line with Canzoneri et al. (2007) who find that the excess volatility arises across a number of preference specifications that include habit formation. On the other hand, the finding of excess volatility of implied interest rates is in contrast to Collard and Dellas (2012). They find that nonseparability between consumption and labor can in principle solve this issue. Note that our finding of excess volatility do not challenge their result, because we can report that choosing $h = 0.5$ leads to a perfect match between the volatilities of implied and actual real interest rates, while the corresponding correlation remains at about 0.10. Finally, the excess volatility of interest rates derived under habit formation and nonseparability is reflected in the persistence of interest rates. While standard consumption Euler equation interest rates can account for the high AR(1) coefficient of the Federal Funds rate, implied interest rates using the SW preference specification generates too less persistence.

3. Monte Carlo experiment

In this section, we challenge the findings in the previous section by making use of a Monte Carlo experiment. We take the estimated model in Smets and Wouters (2007), assuming that it is

⁵ The inflation rate is measured as the log change in the deflator for expenditures on nondurable goods and services. The data on hours worked is constructed as in Smets and Wouters (2007). The VAR is estimated over the sample from 1966:1 to 2008:II. We use two lags.

Table 1

Statistics for interest rates (% p.a.)

	FFr	Euler equation	
		SP	SW
<i>Nominal interest rates</i>			
St. dev	3.28	1.69	5.86
AR(1)	0.94	0.91	0.24
Corr (FFr, implied)	–	0.53	0.19
<i>Real interest rates</i>			
St. dev.	2.53	0.97	5.86
AR(1)	0.89	0.78	0.19
Corr (FFr, implied)	–	–0.07	0.10

Table 2

Correlation of model and implied interest rates.

	SP		SW	
	Median	95% CI	Median	95% CI
<i>Nominal interest rates</i>				
Corr (model, implied)	0.11	[–0.40; 0.55]	0.08	[–0.05; 0.22]
<i>Real interest rates</i>				
Corr (model, implied)	0.08	[–0.29; 0.39]	0.04	[–0.12; 0.21]

the true data-generating process, and we compute replications of simulated data. For each replication we then compute implied interest rates as outlined in Section 2. Finally, counterfactual model simulations allow us to explore the relationship between implied and actual interest rates.

3.1. Baseline results

In Table 2 we show the distribution characteristics of the correlation between implied and actual (model generated) interest rates based on 1000 replications of simulated time series of consumption, inflation, hours worked, and interest rates of the same length as the data that is used in Section 2.⁶

It is apparent from the table that the standard Euler equation as well as the SW Euler equation fails to mimic the dynamics

⁶ We simulate data from the model evaluated at its posterior mean. Hence, we rule out parameter uncertainty when we construct implied interest rates.

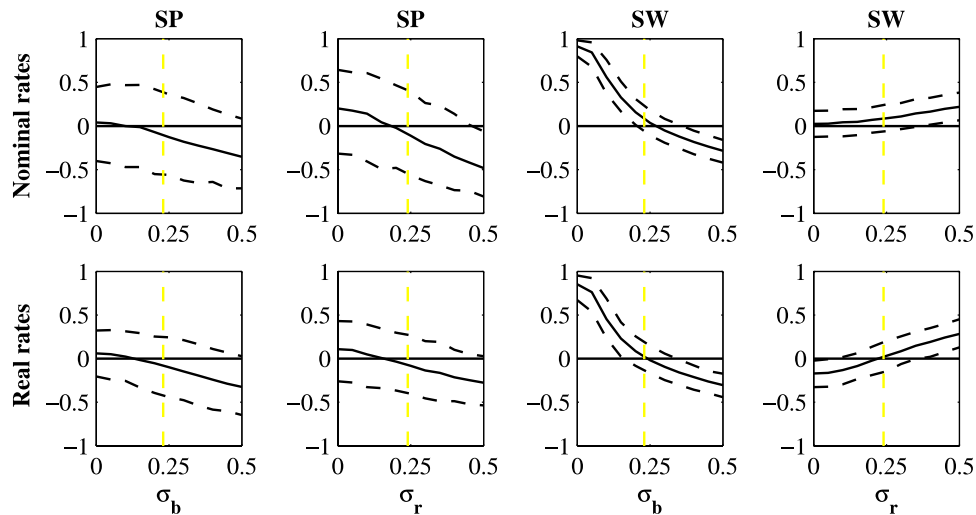


Fig. 2. Distribution of correlation between actual and implied interest rates. Note: Median and 95% interval. The dashed vertical line represents the posterior mean for σ_b , respectively, σ_r , as estimated in Smets and Wouters (2007).

of actual interest rates. For both Euler equations the correlation between implied and actual interest rates is centered around zero. While for standard preferences the correlation is highly volatile, the correlation is tight for the specification of preferences as in the SW model. The latter finding is due to the excess volatility of interest rates implied by consumption habits that mechanically ties the correlation to zero. Noteworthy, for both Euler equations the correlation between the Federal Funds rate and the Euler equation interest rate as computed in Section 2 lies well within the corresponding distribution of the correlation between actual and implied interest rates based on the Monte Carlo experiment.

3.2. The role of monetary policy and risk premium shocks

In our experiment we can identify three sources that obviously account for the spread between actual and implied interest rates. A first source is model misspecification. Clearly, this is the case for standard preferences. A second source stems from the fact that in order to compute implied interest rates, information on households' forecasts has to be drawn. In our experiment we can easily address this issue by using the relevant information from the simulated model. By doing so, we find that the baseline results remain virtually unchanged, so that we can conclude that this source of divergence is of little relevance. A third source arises due to the omission of risk premium shocks as implied by Eqs. (5)–(8). Hence, if actual consumption dynamics are influenced by risk premium disturbances, implied Euler rates and actual interest rates diverge.

To explore whether the spread is systematically linked to monetary policy (see Canzoneri et al., 2007) and to investigate the role of risk premium disturbances, we make use of counterfactual model simulations. Fig. 2 shows the distribution of the correlation between actual and implied interest rates based on replications of artificial data as a function of the variance of risk premium (σ_b) and monetary policy shocks (σ_r). With this setup at hand, we are able to disentangle the impact of monetary policy shocks on the correlation between the two rates from the effect that arises from the assumption of risk premium disturbances.⁷ The

results are clear cut. In the case of standard preferences, a higher variance of both monetary and risk premium shocks results in a negative correlation. This is not the case when implied interest rates are computed using the SW Euler equation. While a higher variance of monetary policy shocks induces a positive correlation, a higher variance of risk premium shocks leads to a negative correlation. In that sense, an increasing importance of monetary policy disturbances stabilizes the correlation between implied and actual interest rates. In sum, the analysis reveals that only risk premium shocks have the capability to drive a wedge between actual and implied interest rates, so that the observed correlation between the two rates is negative.

In Fig. 3 we gain further insights into the link between implied and actual interest rates by computing impulse response functions of implied interest rates for a negative risk premium as well as positive monetary policy shock. The spread between implied and actual interest rates is related to monetary policy in the case of standard preferences. This is not the case when preferences exhibit habits and nonseparability between consumption and labor effort. Interest rate dynamics implied by the SW Euler equation are equal to actual dynamics, namely, a rise in interest rates. In the case of the SW Euler equation, the spread between implied and actual interest rates is linked exclusively to risk premium shocks. Clearly, a negative shock to the wedge between the money market interest rate targeted by the central bank and the return on bonds held by households causes consumption, labor, inflation, and interest rates to rise. Interest rates implied by the consumption Euler equation, however, drop initially. Hence, in the wake of risk premium disturbances the observed correlation between implied and actual interest rates is negative.

4. Conclusion

At first glance, the message of our analysis for the problem posed by the failure of consumption Euler equations identified by Canzoneri et al. (2007) is straightforward. Given that the model economy of our Monte Carlo experiment is true, we can conclude that risk premium shocks have the capability to drive a wedge between money market interest rates and interest rates implied by consumption Euler equations, so that the observed correlation between the two rates is negative. However, the analysis in our paper is not without controversy, because whether the model put forward by Smets and Wouters (2007) represents the true data-generating process is subject of debate. Chari et al. (2009)

⁷ Note that the counterfactual exercise of varying one volatility can be misleading, as the overall empirical fit of the model given the data deteriorates. As a robustness check we conducted a similar exercise where we re-estimated the model, while imposing different values for the volatilities. All results are quantitatively similar and are available upon request.

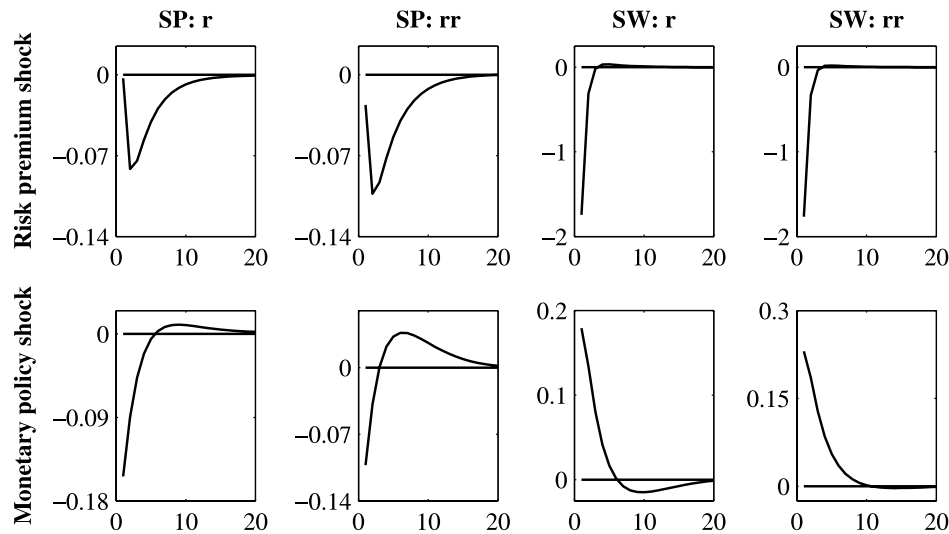


Fig. 3. Implied interest rate responses for risk premium and monetary policy shock.

make the point that the model lacks a structural modeling of risk premium disturbances and argue that the estimated variance of the risk premium shock seems to be implausibly large compared to the variance of the Federal Funds rate. To our opinion, there is no necessary conflict between their concerns and our result that risk premium shocks may resolve the evidence on implied Euler equation rates. In fact, both are simply two sides of the same coin. Nevertheless, we should note that our analysis does not rely on whether shocks to the wedge between the money market interest rate and the Euler equation rate are truly risk premium disturbances. In principle, any disturbance term that alters households' intertemporal optimality condition for consumption has the capability to induce a negative correlation between observed and implied Euler equation interest rates. With respect to this, the message of our paper is that more has to be done to fully reconcile observed consumption dynamics and the structural underpinning implied by the consumption Euler equation. One promising way is to implement financial factors in the form of collateral constraints along the lines of [Kiyotaki and Moore \(1997\)](#). In a highly influential paper, [Iacoviello \(2005\)](#) extends the standard NK framework to account for borrowing constraints tied to housing values. Using structural estimation, he finds that collateral effects are crucial to explain US consumption dynamics in response to fluctuations in house prices. In a related work, [Iacoviello and Neri \(2010\)](#) provide evidence that housing collateral account for 12% of the total variance of US consumption growth in the period from 1989:Q4 to 2006:Q4. Given this quantitatively large effect of borrowing constraints on US consumption dynamics, this friction

might limit the dependence of US consumption dynamics on risk premium disturbances implied by the [Smets and Wouters \(2007\)](#) framework and thereby might be a candidate to resolve the failure of consumption Euler equations implied by the analysis in [Canzoneri et al. \(2007\)](#).

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