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News impact curve for stochastic volatility models

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HIGHLIGHTS

- We propose two news impact curves for stochastic volatility models.
- We use a rejection sampling method to compute the conditional density of the (log) volatility.
- We show a scatter plot of the return and the volatility using the Markov chain Monte Carlo scheme.

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ABSTRACT

This paper proposes a couple of new methods to compute the news impact curve for stochastic volatility (SV) models. The new methods incorporate the joint movement of return and volatility, which has been ignored by the extant literature. The first method employs the Bayesian Markov chain Monte Carlo scheme and the other one employs the rejection sampling. The both methods are simple, versatile, and applicable to various SV models. Contrary to the monotonic news impact functions in the extant literature, the both methods give the U-shaped news impact curves comparable to the GARCH models. They also capture the volatility asymmetry for the asymmetric SV models.

1. Introduction

Modeling and forecasting financial asset volatility have attracted many researchers and practitioners since the seminal work of Engle (1982) that proposes the autoregressive conditional heteroskedasticity (ARCH) model. Bollerslev (1986) proposes the generalized ARCH (GARCH) model and a number of extensions, including asymmetric GARCH models such as the exponential GARCH (EGARCH) model of Nelson (1991) and the GJR model of Glosten et al. (1993), have followed. Extensive literature such as Taylor (1986), Ghysels et al. (1996) and Shephard (1996) has studied another class of volatility models called the stochastic volatility (SV) models. These models have various specifications on volatility dynamics which imply different impact of past return shocks, or information, on the return volatility.

Engle and Ng (1993) define the news impact curve which measures how the new information affects the return volatility in the context of GARCH models. In GARCH models, today's volatility is a function of observations up to yesterday and therefore today's news shock is a change of today's return not explained by the estimated today's volatility. With today's volatility fixed, typically at the unconditional volatility, a plot of tomorrow's volatility against today's news shock shows the well known U-shaped news impact curve. The news impact curve also reflects the volatility asymmetry or leverage effect (a negative shock yields higher volatility than a positive shock) for asymmetric GARCH models such as the EGARCH and GJR models.

The news impact curve for SV models has been defined similarly in the extant literature. The news impact function is

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typically defined as the expectation of tomorrow's volatility conditional on today's return with today's volatility fixed at the unconditional volatility. Contrary to the U-shaped news impact curve for GARCH models, this news impact function is a flat line for symmetric SV models and downward-sloping curve for asymmetric SV (ASV) models. The monotonic news impact curve, instead of the U-shaped curve, for SV models is due to the different specification of volatility process. Contrary to GARCH models, SV models treat today's volatility as a latent variable and thus a change of today's return can be due to either a change of volatility or news shock, or both. Therefore, it is problematic to define the news impact function with today's volatility fixed as in GARCH models.

Considering the joint move of today's volatility and news shock, this paper proposes a couple of new methods to compute the news impact curve for SV models. The first method employs the Markov chain Monte Carlo (MCMC) scheme and the other one employs the rejection sampling. The both methods are versatile and applicable to various SV extensions such as the realized SV models proposed by Takahashi et al. (2009) and Koopman and Scharth (2013). An empirical example with Spyder, the S&P 500 exchange-traded fund, shows that the both methods give a U-shaped news impact curve comparable to the GARCH models and also capture the asymmetry for the ASV models.

The rest of this paper is organized as follows. Next section illustrates the problem in the traditional method to compute the news impact curve for SV models. Section 3 proposes the new methods. Then, we demonstrate the news impact curve with actual daily returns of Spyder in Section 4. The final section concludes.

2. News impact curve

To illustrate a news impact curve, consider an asset return,

$$r_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0, 1), \tag{1}$$

where r_t is the asset return and we call σ_t^2 volatility in this paper.

Engle and Ng (1993) define the news impact function as a relation between r_t and σ_{t+1}^2, implied by a volatility specification, with all lagged conditional variances evaluated at the level of the unconditional variance of the asset return, σ^2. GARCH models specify σ_{t+1}^2 as a function of the information up to t. Since σ_t^2 is known at $t-1$, a change of r_t is solely due to a change of ϵ_t. This feature of GARCH models justifies the news impact function with lagged conditional variances fixed at σ^2. For example, GARCH(1, 1) model specifies the volatility as follows.

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha r_t^2, \tag{2}$$

where it is assumed that $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$ to assure that the volatility σ_t^2 is always positive and that $|\alpha + \beta| < 1$ to guarantee that the volatility is stationary. The news impact function is then

$$\sigma_{t+1}^2 = \omega + \beta\sigma^2 + \alpha r_t^2. \tag{3}$$

This implies the well known U-shaped news impact curve.

Under SV models, however, σ_t^2 is a latent variable and hence it is unknown at $t-1$. For example, consider the following standard SV model.

$$\begin{aligned} h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2), \\ \mathrm{cov}[\epsilon_t, \eta_t] &= \rho\sigma_\eta^2, \end{aligned} \tag{4}$$

where $h_t = \log \sigma_t^2$, $|\phi| < 1$ for stationarity, and ρ captures the volatility asymmetry. If $\rho = 0$, this model becomes the symmetric SV model. If $\rho < 0$, it is consistent with the volatility asymmetry or leverage effect observed in stock markets.

Following Yu (2005), we define the news impact function for SV models as a relation between r_t and h_{t+1} in this section while we also consider the relation between r_t and σ_{t+1}^2 in the next section. Contrary to GARCH models, a change of r_t is due to a change of either ϵ_t or h_t, or both. This implies a stochastic relation between r_t and h_{t+1} instead of the deterministic relation in GARCH models. This relation can be expressed as a conditional expectation of h_{t+1},

$$\begin{aligned} E[h_{t+1}|r_t] &= \mu + \phi(E[h_t|r_t] - \mu) + E[\eta_t|r_t] \\ &= \mu + \phi(E[h_t|r_t] - \mu) + \rho\sigma_\eta r_t E[\exp(-h_t/2)|r_t]. \end{aligned} \tag{5}$$

Replacing the conditional expectations with the unconditional expectations yields the following news impact function,[1]

$$E[h_{t+1}|r_t] \approx \mu + \rho\sigma_\eta \exp\left\{-\frac{\mu}{2} + \frac{\sigma_\eta^2}{8(1-\phi^2)}\right\} r_t. \tag{6}$$

If $\rho = 0$, this is a flat line. If $\rho < 0$, this is a downward sloping line.

Such a monotonic news impact line is due to ignoring the dependence between r_t and h_t and replacing the conditional expectations with the unconditional ones. If $E[h_t|r_t]$ is increasing in the absolute return $|r_t|$, the conditional expectation in (5) implies a non-monotonic news impact curve. Thus, incorporating the joint distribution of r_t and h_t may give the U-shaped news impact curve.

In the next section, we propose a couple of new methods which incorporate the joint movement to compute the news impact curve. Instead of directly computing the conditional expectations, $E[h_t|r_t]$ and $E[\exp(-h_t/2)|r_t]$, we take a simulation based approach in two different ways. The first method employs the Bayesian MCMC scheme, which does not require possibly complicated conditional distributions but only a stationary distribution of h. The other method estimates the conditional density by a simple rejection sampling. The both methods are versatile to various SV specifications.

3. New method

We illustrate our new methods to compute a news impact curve for the standard SV model given by Eqs. (1) and (4). We assume that $|\phi| < 1$ for a stationary log-volatility process, $h_0 = \mu$, and

$$\eta_0 \sim N\left(0, \frac{\sigma_\eta^2}{1-\phi^2}\right). \tag{7}$$

3.1. New method via Bayesian MCMC scheme

We incorporate the joint movement of h_t and r_t (or ϵ_t) by generating h_t, r_t, and h_{t+1} from the joint distribution via the Bayesian MCMC scheme. Specifically, we implement the following procedure.

(i) Set parameters $(\phi, \mu, \rho, \sigma_\eta)$.
  (a) Generate h from its stationary distribution,
  $$h \sim N\left(\mu, \frac{\sigma_\eta^2}{1-\phi^2}\right), \tag{8}$$
  and ϵ from the standard normal distribution, $\epsilon \sim N(0, 1)$.
  (b) Compute daily return,
  $$r = \epsilon \exp(h/2), \tag{9}$$
  and one day ahead volatility forecast,
  $$\hat{h} = \mu + \phi(h - \mu) + \rho\sigma_\eta\epsilon. \tag{10}$$
(ii) Repeat step (i) for K times.
(iii) Estimate the news impact curve by fitting curves in the generated K pairs of r and $\hat{h}$.

[1] See, e.g., Yu (2005) and Asai and McAleer (2009) for other approximation methods.

There are several ways to implement steps (i) and (ii). We can repeat step (i) by simply fixing the parameters at the estimated values such as posterior means of the parameter samples generated by the MCMC method. We can also implement step (i) with different parameter values each time. For example, we can generate r and $\hat{h}$ for each parameter samples generated in the MCMC estimation of the SV models. This method requires the MCMC estimation scheme but enables us to take account of the parameter uncertainty.

There are also several ways to estimate the news impact curve in step (iii). For example, we can estimate the news impact curve defined as a relation between r_t and h_{t+1} by the local linear Gaussian kernel regression. We can also estimate the news impact curve defined as a relation between r_t and σ^2_{t+1} simply by transforming the generated samples of the log-volatility forecast ($\hat{h}$) to those of the volatility forecast ($\exp(\hat{h})$).

This procedure generates r_t and h_{t+1} from the joint distribution and then estimate the news impact curve by fitting the curve in the samples. Step (i) in the procedure resembles the first step of a particle filter.[2] While the particle filter generates h_{t+1} from its conditional distribution given r_t, the above procedure estimates the relation between h_{t+1} and r_t using the samples generated from the joint distribution.

3.2. New method with rejection sampling

Given the parameter estimates, we can also compute the news impact curve by employing the rejection sampling in the following way.[3]

(I) Set $r_t^{(l)}$ in an arbitrary interval $[\underline{r}, \bar{r}]$. For example,

$$r_t^{(l)} = \underline{r} + \frac{\bar{r} - \underline{r}}{L} l, \quad l = 0, 1, 2, \ldots, L. \tag{11}$$

(II) Compute the mode of $\ln f(h_t|r_t^{(l)})$, denoted as $\hat{h}_t^{(l)}$, where

$$\ln f(h_t|r_t) = \text{const.} - \frac{1}{2}h_t - \frac{1}{2}r_t^2\exp(-h_t) - \frac{1-\phi^2}{2\sigma_\eta^2}(h_t - \mu)^2. \tag{12}$$

(III) Generate $h_t^{(m)}$, $m = 1, \ldots, M$, from $f(h_t|r_t^{(l)})$ by the rejection sampling using a proposal density of normal with mean $\tilde{\mu}$ and variance $\tilde{\sigma}^2$, where

$$\tilde{\sigma}^2 = \frac{\sigma_\eta^2}{1-\phi^2}, \qquad \tilde{\mu} = \mu + \frac{\tilde{\sigma}^2\{(r_t^{(l)})^2\exp(-h_t^*) - 1\}}{2}, \tag{13}$$

and h_t^* is an arbitrary value. We set $h_t^* = \hat{h}_t^{(l)}$ for higher acceptance rate.

(IV) Set $h_{t+1}^{(n)}$ in an arbitrary interval $[\underline{h}, \bar{h}]$. For example,

$$\underline{h} = \mu + \phi(\hat{h}_t^{\min} - \mu) - 1.96\sigma_\eta, \quad \bar{h} = \mu + \phi(\hat{h}_t^{\max} - \mu) + 1.96\sigma_\eta, \tag{14}$$

and

$$h_{t+1}^{(n)} = \underline{h} + \frac{\bar{h} - \underline{h}}{N} n, \quad n = 0, 1, 2, \ldots, N, \tag{15}$$

where $\hat{h}_t^{\min}$ and $\hat{h}_t^{\max}$ are the minimum and maximum values of $\{\hat{h}_t^{(l)}\}$, respectively.

(V) Compute the conditional density as

$$\hat{f}(h_{t+1}^{(n)}|r_t^{(l)}) = \frac{1}{M}\sum_{m=1}^{M} f(h_{t+1}^{(n)}|h_t^{(m)}, r_t^{(l)}), \tag{16}$$

where

$$f(h_{t+1}|h_t, r_t) = \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma_\eta^2}} \times \exp\left(-\frac{\{\bar{h}_{t+1} - \phi\bar{h}_t - \rho\sigma_\eta r_t\exp(-h_t/2)\}^2}{2(1-\rho^2)\sigma_\eta^2}\right), \tag{17}$$

and $\bar{h}_t = h_t - \mu$.

(VI) Compute the mean of h_{t+1} conditional on $r_t^{(l)}$ as

$$\hat{E}[h_{t+1}|r_t^{(l)}] = \sum_{n=1}^{N} h_{t+1}^{(n)}\hat{f}(h_{t+1}^{(n)}|r_t^{(l)})\Delta h^{(n)}, \tag{18}$$

where $\Delta h^{(n)} = h_{t+1}^{(n)} - h_{t+1}^{(n-1)}$, $n = 1, \ldots, N$.

Plotting $\hat{E}[h_{t+1}|r_t^{(l)}]$ against each value of $r_t^{(l)}$ shows the news impact curve. We can also estimate the news impact curve defined as a relation between r_t and σ^2_{t+1} simply by replacing Eq. (18) with

$$\hat{E}[\sigma_{t+1}^2|r_t^{(l)}] = \sum_{n=1}^{N} \exp(h_{t+1}^{(n)})\hat{f}(h_{t+1}^{(n)}|r_t^{(l)})\Delta h^{(n)}. \tag{19}$$

4. Empirical illustration

This section gives an empirical example of the news impact curve proposed in the previous section. We use 1,758 samples of daily returns for Spyder, the S&P 500 exchange-traded fund, from February 1, 2001 to January 31, 2007.

We estimate the symmetric and asymmetric SV models using the Bayesian MCMC estimation method with the following prior distributions,

$$\mu \sim N(-0.1, 1), \qquad \frac{\phi + 1}{2} \sim \text{Beta}(20, 1.5),$$

$$\sigma_\eta^{-2} \sim \text{Gamma}(2.5, 0.025), \qquad \frac{\rho + 1}{2} \sim \text{Beta}(1, 1).$$

To sample h efficiently, we employ the estimation procedure with the block sampler of Watanabe and Omori (2004) for the SV model and Omori and Watanabe (2008) for the ASV model.[4] We discard the first 5,000 samples as the burn-in and use the subsequent 20,000 samples for the estimation. Table 1 shows the MCMC estimation results which are consistent with the extant literature. For example, the posterior mean of ϕ is close to one, which implies the high persistence of volatility. Additionally, the posterior mean

Table 1
MCMC estimation results of symmetric and asymmetric SV (SV and ASV) models. 95%L and 95%U are the lower and upper quantiles of 95% credible interval, respectively. The last two columns are the *p*-value of the convergence diagnostic test by Geweke (1992) and the inefficiency factor.

Model	RM	Mean	Stdev.	95%L	Median	95%U	CD	Inef.
SV	ϕ	0.9727	0.0107	0.9444	0.9744	0.9878	0.399	219.52
	σ	0.2239	0.0531	0.1701	0.2087	0.3992	0.348	394.20
	μ	−0.1453	0.2247	−0.5780	−0.1501	0.3154	0.528	23.73
ASV	ϕ	0.9705	0.0064	0.9570	0.9708	0.9821	0.787	28.53
	σ	0.2085	0.0175	0.1783	0.2070	0.2455	0.586	88.98
	ρ	−0.5517	0.0580	−0.6545	−0.5552	−0.4295	0.047	79.21
	μ	−0.1697	0.1553	−0.4618	−0.1751	0.1510	0.290	7.48

[2] We thank the anonymous referee for pointing out this analogy. For the details of the particle filter in this context, see, e.g., Kim et al. (1998), Pitt and Shephard (1999) and Omori et al. (2007).

[3] We thank the anonymous referee for suggesting the alternative simulation strategy.

[4] See, e.g., Ghysels et al. (1996) and Shephard (1996) for other estimation methods.

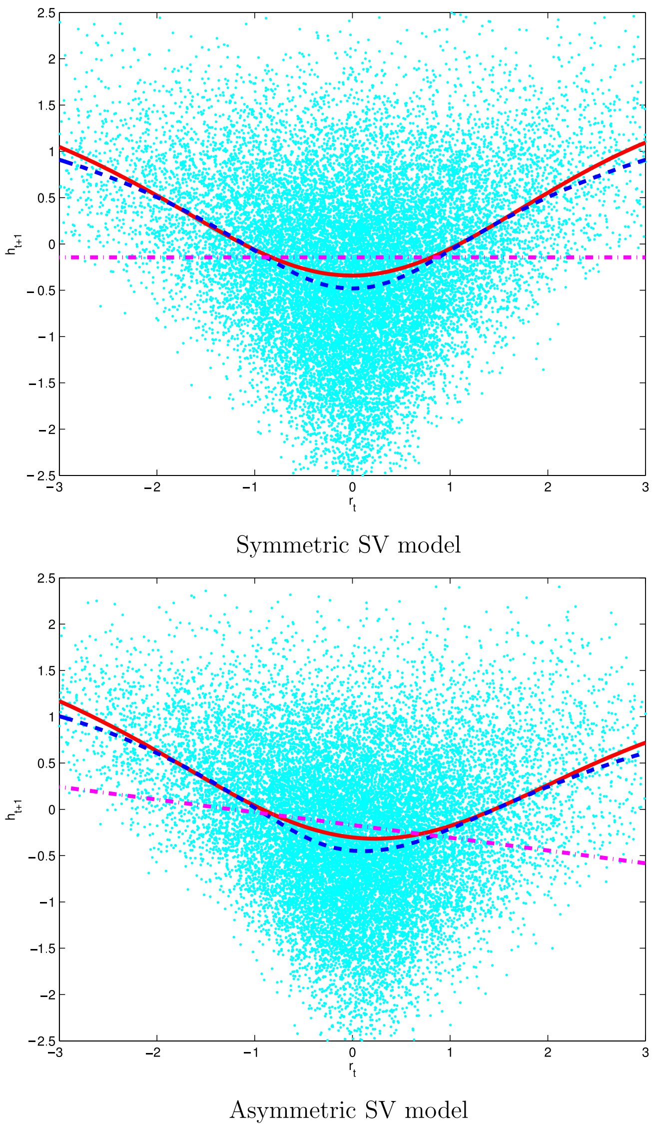

Fig. 1. News impact curves for symmetric (top) and asymmetric SV (bottom) models by the conventional method (dash–dot line) using Eq. (6), the new method via Bayesian MCMC scheme (solid line), and the method with the rejection sampling (dashed line). Horizontal and vertical axes represent today's daily return (r_t) and tomorrow's log-volatility (h_{t+1}), respectively. In the both methods, we use the posterior means in Table 1 as the parameter values. We implement the new method via the MCMC scheme with the local linear Gaussian kernel regression, where we set the bandwidth of 1 and the number of grid points of 100. The scatter plots illustrate the generated samples of r_t and h_{t+1}. We also implement the new method with the rejection sampling, where we set $[\underline{r}, \bar{r}] = [-3, 3]$, $r_t^{(l)}$ as in Eq. (11) with $L = 100$, $M = 200{,}000$, $[\underline{h}, \bar{h}]$ as in Eq. (14), and $h_{t+1}^{(n)}$ as in Eq. (15) with $N = 100$.

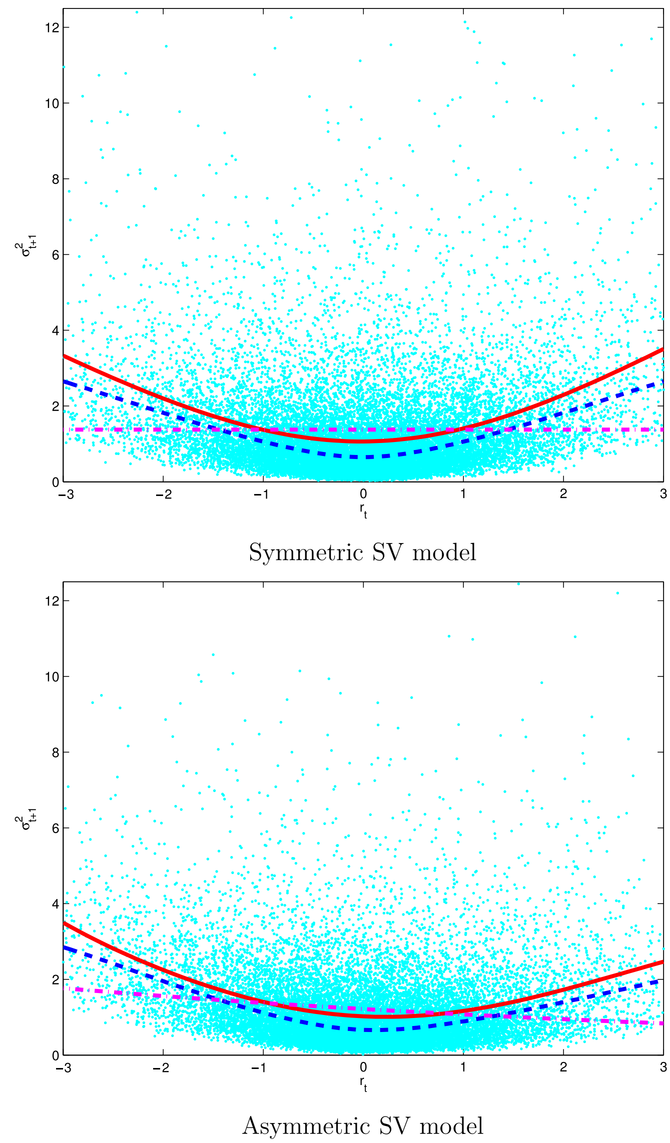

Fig. 2. News impact curves for symmetric (top) and asymmetric SV (bottom) models by the conventional method (dash–dot line) using Eq. (3.4) in Asai and McAleer (2009), the new method via Bayesian MCMC scheme (solid line), and the method with the rejection sampling (dashed line). Horizontal and vertical axes represent today's daily return (r_t) and tomorrow's volatility ($\sigma_{t+1}^2 = \exp(h_{t+1})$), respectively. In the both methods, we use the posterior means in Table 1 as the parameter values. We implement the new method via the MCMC scheme with the local linear Gaussian kernel regression, where we set the bandwidth of 1 and the number of grid points of 100. The scatter plots illustrate the generated samples of r_t and σ_{t+1}^2. We also implement the new method with the rejection sampling, where we set $[\underline{r}, \bar{r}] = [-3, 3]$, $r_t^{(l)}$ as in Eq. (11) with $L = 100$, $M = 200{,}000$, $[\underline{h}, \bar{h}]$ as in Eq. (14), and $h_{t+1}^{(n)}$ as in Eq. (15) with $N = 100$.

of ρ is negative and its 95% interval does not contain zero, which indicates the well known volatility asymmetry.

Using the posterior means in Table 1 as the parameter values, we compute the news impact curves by the two methods proposed in the previous section. We implement the new method via the MCMC scheme with the local linear Gaussian kernel regression, where we set $K = 20{,}000$, the bandwidth of 1, and the number of grid points of 100.[5] We also implement the new method with the rejection sampling, where we set $[\underline{r}, \bar{r}] = [-3, 3]$, $r_t^{(l)}$ as in Eq. (11) with $L = 100$, $M = 200{,}000$, $[\underline{h}, \bar{h}]$ as in Eq. (14), and $h_{t+1}^{(n)}$ as in Eq. (15) with $N = 100$.

Fig. 1 shows the news impact curves defined as a relation between r_t and h_{t+1} by the conventional and new methods with the scatter plots of the generated samples by the MCMC method. Fig. 2 shows the news impact curves defined as a relation between r_t and σ_{t+1}^2 with the scatter plots. In the both figures, the new methods give the familiar U-shaped news impact curves, comparable to GARCH models, while the conventional method gives the flat or downward-sloping lines. The both figures also show that the both methods capture the volatility asymmetry for the ASV models.

5 We also compute the news impact curve using each parameter samples generated in the MCMC estimation and obtain the similar news impact curves in Figs. 1 and 2.

5. Conclusion

This paper proposes a couple of new methods to compute the news impact curve for SV models. The new methods incorporate the joint movement of return and volatility, which has been ignored by the extant literature. The first method employs the Bayesian MCMC scheme and the other method employs the rejection sampling. Empirical results with Spyder, the S&P 500 exchange-traded fund, show that both new methods give the familiar U-shaped news impact curves and capture the volatility asymmetry.

Although we illustrate the new methods with a simple SV model, the both methods are versatile and easily applicable to various SV models. For example, it is straightforward to compute a news impact curve for the SV model of Nakajima and Omori (2012) where a more general distribution is assumed for ϵ_t and the realized SV model of Takahashi et al. (2009) which specifies daily returns and realized volatility measures jointly.

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