MTH 131b: Analysis

Homework 2

Due Date: April 20, 2018

Instructions:

- All questions have equal weight.
- Answer all questions.
- Three randomly selected problems will be graded from all the problems below. Each problem carries a maximum of three points. One point will be awarded for completing the remaining problems.
- Maximum score in this homework set is 10 points.

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1. Let (X, d) be a metric space and let Y be a subset viewed as a metric space with the relative topology. Let $\{y_n\}$ be a sequence with $y_n \in Y$. Suppose that $\{y_n\}$ is a convergent sequence in X, show that $\{y_n\}$ is a Cauchy sequence in (Y, d_Y). Is it necessarily true that it is a convergent sequence in (Y, d_Y)?

2. Let X be a metric space and suppose that $\{x_n\}$ is a Cauchy sequence. Suppose that $\{x'_n\}$ is another sequence in X such that $d(x_n, x'_n) \to 0$ as $n \to \infty$. Show that $\{x'_n\}$ is also Cauchy.

3. Show that any set X with the discrete metric is complete.

4. If Y is a non-empty compact subset of $\mathbb{R}$, then there exits A and B such that $A \leq x \leq B$ for all $x \in Y$.

5. Let X be a metric space. Let K_i be a sequence of non-empty compact sets such that

$$K_1 \supset K_2 \supset K_3 \supset \cdots .$$

Show that the intersection $\cap_{i=1}^{\infty} K_i \neq \emptyset$.