PIC 16, Winter 2018 – Assignment 6M (alternate)

Assigned 2/12/2018. Code (a single `.py` file) due by the end of class 2/16/2018 on CCLE. Hand in a printout of this document with the self-assessment portion completed by the end of class on 2/16/2018.

In this assignment, you will use SymPy to confirm a relatively complex calculation performed in a research paper.

Task

Download the accepted version of Tails in Biomimetic Design: Analysis, Simulation, and Experiment.[1] The ultimate goal is to derive Equation 18 from Equations 11 and Equations 12 using the assumptions presented in the paper. You are welcome to go about it in any way you wish; I outline a procedure if you get stuck because it can be *very* tricky. You will find that my solution does not contain any "weird" tricks and is quite readable, but there are subtleties in the code that require a lot of care. For instance, SymPy functions may return objects other than regular SymPy expressions; you'll need to isolate the part you need in order to move on. If you are careful, the hints below will save you a lot of time.

1. Enter Equation 11 in SymPy. I suggest that you write the equation as an expression *assumed to equal zero* rather than an equality. For example, if the equation were $a = b$, I would suggest you write it as $a - b$, and remember the assumption that this is assumed to be zero. A reasonable variable name for the resulting expression is `e11`; you may want to follow this convention to store intermediate results because you typically do not want to redefine existing symbols. (This will become more clear in the next step.) Note that $\dot{\omega}$ is shorthand notation for $\frac{d\omega}{dt}$. If you do not explicitly tell SymPy that ω is a function of t when you take the derivative, then it will assume it is independent of t and evaluate to zero. For example, if `f` and `x` have already been declared as SymPy symbols, then you can indicate that `f` is a function of `x` by typing `f(x)` rather than just `f`; i.e. (`diff(f(x),x)` rather than `diff(f,x)`).
2. Enter Equation 12 in SymPy. Again, remember that ω is a function of t when you enter the equation; you have to be explicit. Again, I suggest that you write the important part of the equation, the right hand side, as an expression assumed to be zero rather than an equality relation. Also, I again recommend that you store it as something like `e12` rather than `T` because the latter will redefine the symbol `T`, complicating the substitution of this result into Equation 11 (next). Assume these tips hold for the rest of the assignment.
3. Substitute 12 into 11. Verify that the result is equivalent to Equation 13.
4. Solve Equation 13 (it's a differential equation). However, your result will not be so close to Equation 14 because the constant of integration is represented differently. Note that the `dsolve` function returns an `Equality` object rather than a normal expression. To continue, you'll need to figure out how to get the right hand side of the `Equality` out. There's a post on stackoverflow that addresses this; I suggest you search for it on Google. If you get stuck, ask me.
5. Solve for the constant of integration given the initial condition that $\omega(t = 0) = 0$. Note that the `Solve` function returns a list even if there is only one solution, which could cause trouble if you were to try to substitute the result directly into the general solution of Equation 13….
6. Substitute the expression for the constant of integration into the solution of Equation 13. If you have trouble, read 5 carefully! Verify that your result is equivalent to Equation 14, given that $\omega(0) = 0$.

[1] Published version and citation information available at http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6386240

7. Substitute Equation 14 into Equation 12, then perform the integration indicated in the paper to generate Equation 15. Note that you will end up with a `Piecewise` object, but you're only interested in the case of positive T_0. There are several ways this can be resolved, explained here.
8. Maximize Equation 15 as indicated in the section of the paper labeled "E. Optimization". You may wish to simplify Equation 15 before and after finding the maximum. The result should be equivalent to Equation 18, although SymPy has a hard time verifying it.

Self-Assessment

Which of the following equations were you able to derive from Equation 11 and Equation 12? (20 pts each)

- ☐ Equation 13
- ☐ Equation 14
- ☐ Equation 15
- ☐ Equation 17
- ☐ Equation 18

Indicate your total score: