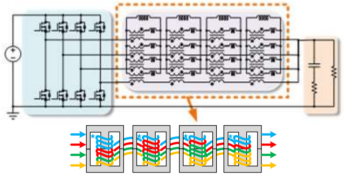
Discrete Symmetrical Coupled Inductor Structure and its Matrix-type Implementation for DC-DC Converter

Xiang Zhang, Shangzhi Pan, Praveen Jain

Queen's University, Kingston, ON, Canada

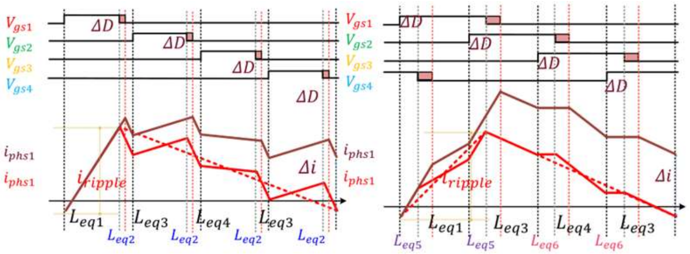
ECCE 2023 IEEE ENERGY CONVERSION CONGRESS & EXPO Nashville, TN | OCT.29-Nov.2



Proposed $\mathbf{Z_m} = \begin{bmatrix} L_{s1} & \pm M_{12} & \pm M_{13} & \cdots & \pm M_{1m} \\ \pm M_{12} & L_{s2} & \pm M_{23} & \cdots & \pm M_{2m} \\ \pm M_{13} & M_{23} & L_{s3} & \cdots & \pm M_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \pm M_{1m} & \pm M_{2m} & \pm M_{3m} & \cdots & L_{sm} \end{bmatrix}$

Conventional $\mathbf{Z_m} = \begin{bmatrix} L_s & M & M & \cdots & M \\ M & L_s & M & \cdots & M \\ M & M & L_s & \cdots & M \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M & M & M & \cdots & L_s \end{bmatrix}$

We proposed a new symmetrical coupled inductor structure for dc-dc converters, the main difference is the mutual inductance between phases can be either positive or negative

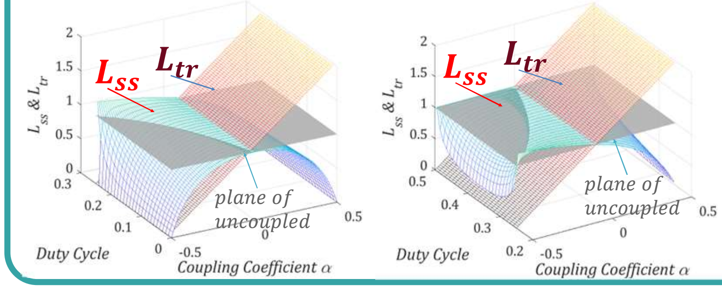


Phase current can analyzed under time intervals, with its equivalent interval inductance, related to the duty cycle and coupling coefficient

EXPRESSION OF EQUIVALENT INTERVAL INDUCTANCE IN ALL DUTY CYCLE RANGES

Duty cycle range	Related time interval	Symbol	Expression
$0 < D < 0.25$	Interval I	L_{eq1}	$\frac{(1-D)(4\alpha^2-1)}{2\alpha^2-2D\alpha+(D-1)}L_s$
	Interval II, IV, VI, VIII	L_{eq2}	$\frac{(4\alpha^2-1)}{2\alpha-1}L_s$
	Interval III, VII	L_{eq3}	$\frac{(-D)(4\alpha^2-1)}{(1-2D)\alpha+D}L_s$
	Interval V	L_{eq4}	$\frac{(-D)(4\alpha^2-1)}{-2\alpha^2-2D\alpha+D}L_s$
$0.75 < D < 1$	Interval I	L_{eq1*}	$\frac{(-D)(4\alpha^2-1)}{-2\alpha^2+(2-2D)\alpha+D}L_s$
	Interval III, VII	L_{eq3*}	$\frac{(1-D)(4\alpha^2-1)}{(1-2D)\alpha+(D-1)}L_s$
	Interval V	L_{eq4*}	$\frac{(1-D)(4\alpha^2-1)}{\alpha^2+(2-2D)\alpha+(D-1)}L_s$
$0.25 < D < 0.5$	Interval I, III	L_{eq5}	$\frac{(1-D)(4\alpha^2-1)}{2\alpha^2+(1-2D)\alpha+(D-1)}L_s$
$0.5 < D < 0.75$	Interval V, VII	L_{eq6}	$\frac{D(4\alpha^2-1)}{2\alpha^2+(2D-1)\alpha-D}L_s$

Similar to conventional coupled multiphase buck converter, it is possible to have larger steady-state inductor $\boldsymbol{L_{SS}}$ and smaller transient inductor $\boldsymbol{L_{tr}}$



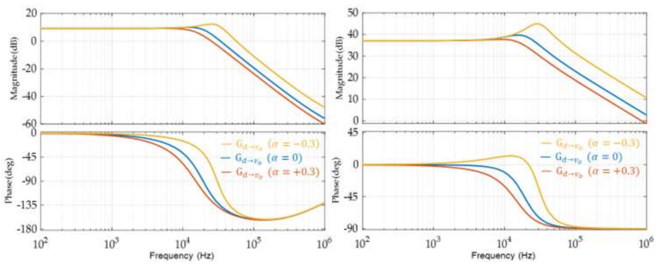
- The proposed coupled inductor dynamic performance in dc-dc converter can be analyzed through state space model, by inverse the inductance matrix $\mathbf{Z_m}$ to reluctance matrix $\mathbf{B}$.
- The duty cycle to output current and output voltage dynamics ($G_{d\to v_o}$ and $G_{d\to\Sigma i_k}$) can be calculated using the derived model. The output capacitor esr and winding resistors are considered.

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{di_3}{dt} \\ \frac{di_4}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} A_s & A_1 & A_2 & A_1 & \frac{-(B_s+2B_1+B_2)R_{load}}{(R_{load}+R_{esr})} \\ A_1 & A_s & A_1 & A_2 & \frac{-(B_s+2B_1+B_2)R_{load}}{(R_{load}+R_{esr})} \\ A_2 & A_1 & A_s & A_1 & \frac{-(B_s+2B_1+B_2)R_{load}}{(R_{load}+R_{esr})} \\ A_1 & A_2 & A_1 & A_s & \frac{-(B_s+2B_1+B_2)R_{load}}{(R_{load}+R_{esr})} \\ X_c & X_c & X_c & X_c & -\frac{1}{(R_{load}+R_{esr})C} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ v_c \end{bmatrix} + V_{in} \underbrace{\begin{bmatrix} B_s & B_1 & B_2 & B_1 \\ B_1 & B_s & B_1 & B_2 \\ B_2 & B_1 & B_s & B_1 \\ B_1 & B_2 & B_1 & B_s \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$$

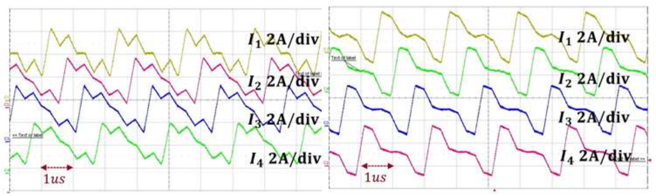
$$\boxed{\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u \\ y = \mathbf{E}x \end{cases}}$$

$$\begin{cases} v_o = \frac{R_{load}}{R_{load}+R_{esr}} v_c + (R_{load}//R_{esr})\Sigma i_k \\ \frac{dv_c}{dt} = \frac{R_{load}}{(R_{load}+R_{esr})C}\Sigma i_k - \frac{v_c}{(R_{load}+R_{esr})C} \end{cases}$$

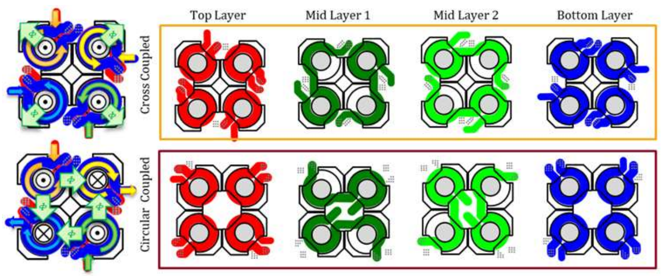
$$\begin{cases} G_{d\to v_o} = \frac{\widehat{v_o}}{\hat{d}} = \mathbf{E}_{\text{row2}}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \\ G_{d\to\Sigma i_k} = \frac{\widehat{\Sigma i_k}}{\hat{d}} = \mathbf{E}_{\text{row1}}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \end{cases}$$



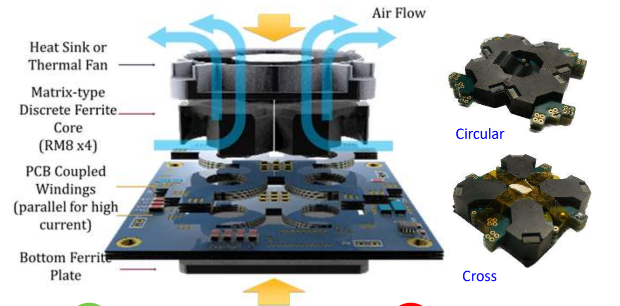
- The 4-phase example shows adjacent negative coupling can broaden the bandwidth, while positive coupling can have the opposite effect.
- Compared with traditional coupling structure, non-adjacent phase has little impact on the system dynamics.

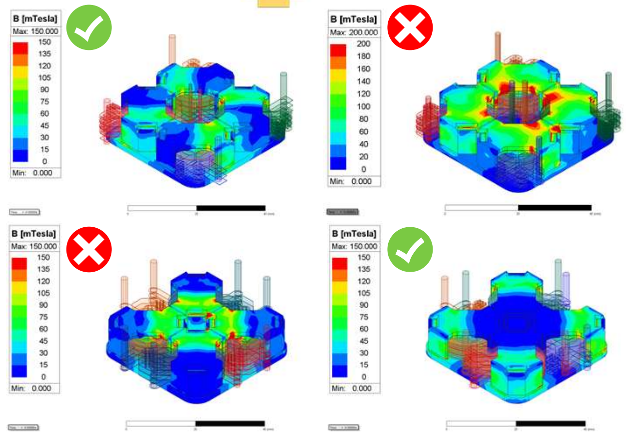


The waveform of 4-phase coupled buck converter shows the coupling impacts on phase current, suggesting that it is unnecessary to strictly control geometry symmetry between the phases to obtain a better current sharing and dynamic performance.



The proposed coupling structure can be implemented with matrix-type integration of RM cores, both cross and circular shape coupled.





According FEA simulation, when windings are cross negative coupled, the coupling will be strengthened, otherwise the coupling will be lower. From flux distribution, it is recommended to use cross shape for negative coupled and circular shape for positive coupled.