对称值问题的计算方法实验报告

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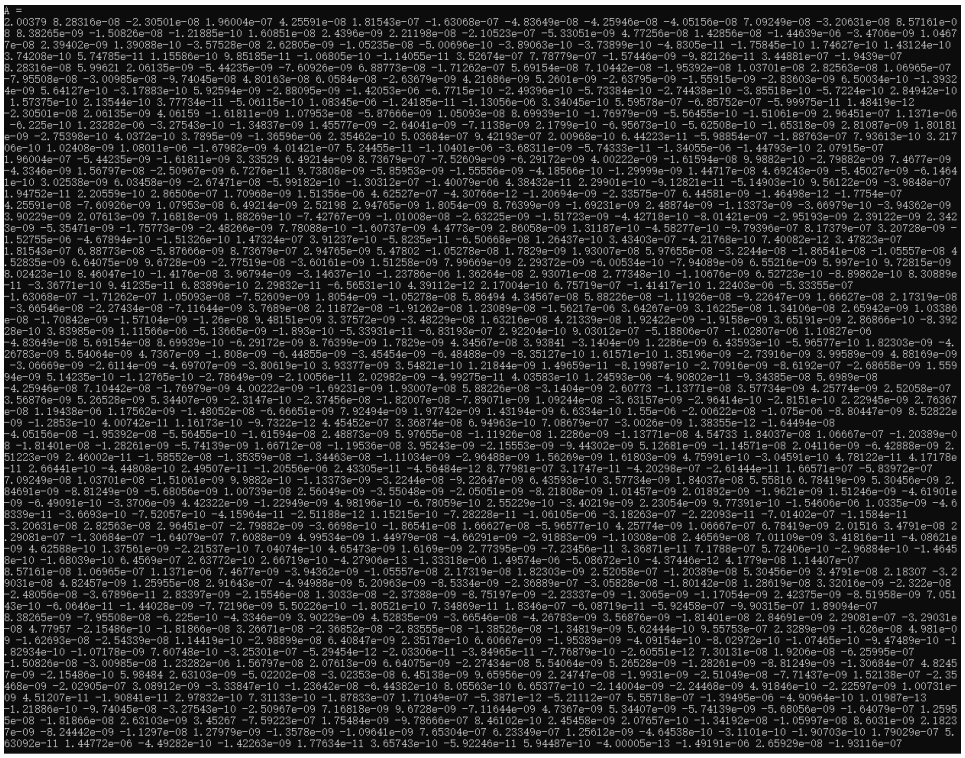
1 问题描述

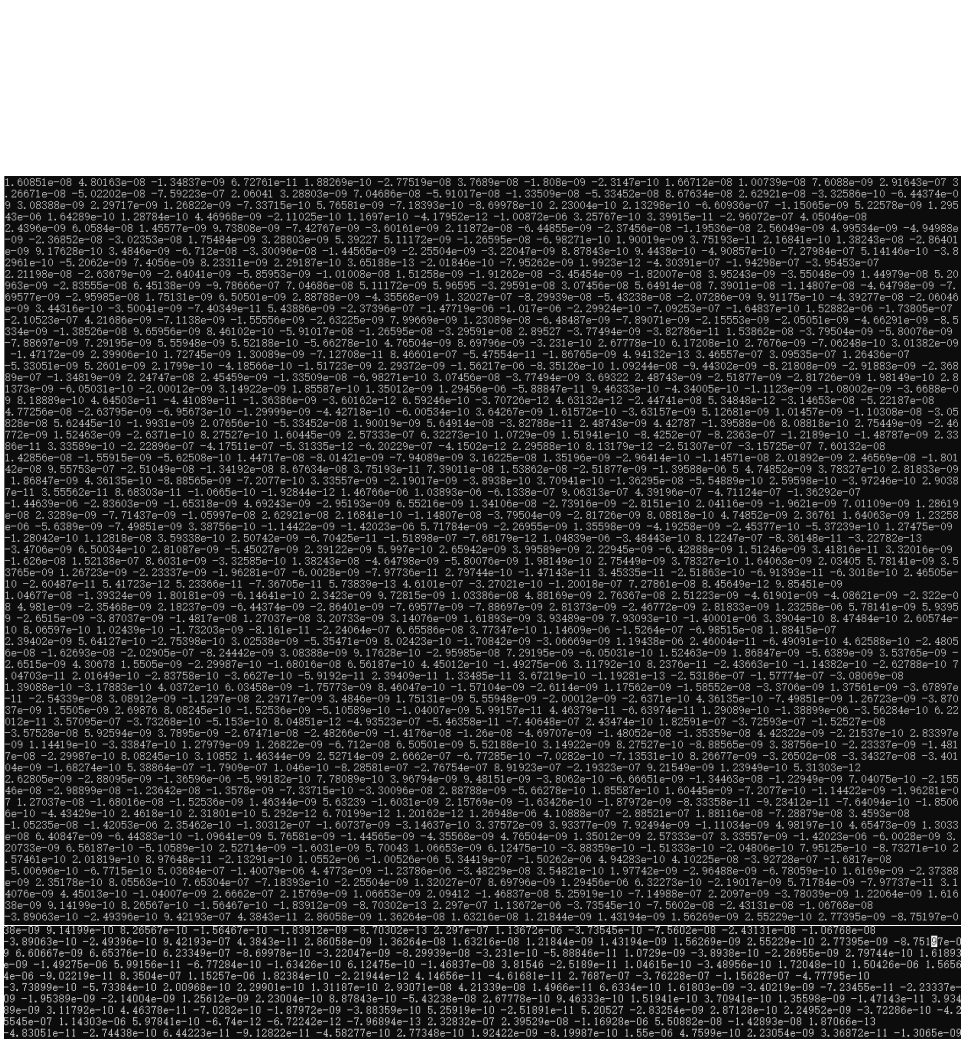
1. 利用过关 Jacobi 方法求实对称三对角阵全部特征值和特征向量
2. 先利用二分法求实对称三对角阵指定特征值，再利用反幂法求对应特征向量

2 程序运行结果

2.1 过关 Jocobi 方法

$n = 50 \quad Q_k, A_k$





不同阶数矩阵特征值

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exercise7_1
n=50, 迭代次数为：4979
用时:607.05ms

特征值从小到大的排列为：
2.00379 2.01516 2.03405 2.06041 2.09412 2.13506 2.18307 2.23798 2.29957 2.36761 2.44184 2.52198 2.60773 2.69876 2.79473 2.89527 3 3.10852 3.22043 3.33529 3.45
267 3.57213 3.69522 3.81546 3.93341 4.06159 4.18454 4.30678 4.42787 4.54733 4.66471 4.77957 4.89143 5 5.10473 5.20527 5.30124 5.39227 5.47802 5.55816 5.63239
5.70043 5.76202 5.81693 5.86494 5.90588 5.93959 5.96595 5.98484 5.99621


n=60, 迭代次数为：7416
用时:1137.14ms

特征值从小到大的排列为：
2.00265 2.0106 2.02382 2.04229 2.06594 2.09473 2.12857 2.16737 2.21103 2.25943 2.31245 2.36994 2.43176 2.49774 2.5677 2.64145 2.71881 2.79957 2.88351 2.97041
3.06005 3.15217 3.24654 3.34292 3.44103 3.54062 3.64144 3.7432 3.84565 3.9485 4.0515 4.15435 4.2568 4.35856 4.45938 4.55897 4.65708 4.75346 4.84783 4.93995 5.
02959 5.11649 5.20043 5.28119 5.35855 5.4323 5.50226 5.56824 5.63006 5.68755 5.74057 5.78897 5.83263 5.87143 5.90527 5.93406 5.95771 5.97618 5.9894 5.99735


n=70, 迭代次数为：10163
用时:1779.38ms

特征值从小到大的排列为：
2.00196 2.00783 2.01759 2.03124 2.04875 2.07007 2.09517 2.124 2.1565 2.19261 2.23226 2.27537 2.32186 2.37163 2.42458 2.48063 2.53964 2.60152 2.66613 2.73335 2
.80306 2.8751 2.94935 3.02565 3.10387 3.18383 3.2654 3.3484 3.43268 3.51806 3.6044 3.6915 3.77921 3.86735 3.95576 4.04424 4.13265 4.22079 4.3085 4.3956 4.4819
4 4.56732 4.6516 4.7346 4.81617 4.89613 4.97435 5.05065 5.1249 5.19694 5.26665 5.33387 5.39848 5.46036 5.51937 5.57542 5.62837 5.67814 5.72463 5.76774 5.80739
5.8435 5.876 5.90483 5.92993 5.95125 5.96876 5.98241 5.99217 5.99804


n=80, 迭代次数为：13931
用时:3207.49ms

特征值从小到大的排列为：
2.0015 2.00601 2.01352 2.02402 2.03749 2.05391 2.07326 2.0955 2.12061 2.14855 2.17927 2.21273 2.24888 2.28767 2.32902 2.3729 2.41921 2.46791 2.51891 2.57214 2
.62752 2.68496 2.74438 2.80568 2.86879 2.93359 3 3.06791 3.13723 3.20784 3.27964 3.35253 3.42639 3.50112 3.57659 3.6527 3.72934 3.80638 3.88371 3.96122 4.0387
3 4.11629 4.19362 4.27066 4.3473 4.42341 4.49888 4.57361 4.64747 4.72036 4.79216 4.86277 4.93209 5 5.06641 5.13121 5.19432 5.25562 5.31504 5.37248 5.42786 5.4
8109 5.53209 5.58079 5.6271 5.67098 5.71233 5.75112 5.78727 5.82073 5.85145 5.87939 5.9045 5.92674 5.94609 5.96251 5.97598 5.98648 5.99399 5.9985


n=90, 迭代次数为：17573
用时:4581.57ms

特征值从小到大的排列为：
2.00119 2.00477 2.01072 2.01904 2.02972 2.04275 2.05812 2.07579 2.09576 2.118 2.14249 2.16918 2.19806 2.22909 2.26222 2.29743 2.33467 2.37389 2.41505 2.45809
2.50298 2.54965 2.59804 2.64811 2.69979 2.75302 2.80774 2.86387 2.92136 2.98013 3.04013 3.10126 3.16346 3.22667 3.29079 3.35576 3.4215 3.48792 3.55496 3.62252
3.69054 3.75893 3.8276 3.89648 3.96548 4.03452 4.10352 4.1724 4.24107 4.30946 4.37748 4.44504 4.51208 4.5785 4.64424 4.70921 4.77333 4.83654 4.89874 4.95987
5.01987 5.07864 5.13613 5.19226 5.24698 5.30021 5.35189 5.40196 5.45035 5.49702 5.54191 5.58495 5.62611 5.66533 5.70257 5.73778 5.77091 5.80194 5.83082 5.8575
1 5.882 5.90424 5.92421 5.94188 5.95725 5.97028 5.98096 5.98928 5.99523 5.99881


n=100, 迭代次数为：21955
用时:6723.79ms

特征值从小到大的排列为：
2.00097 2.00387 2.0087 2.01546 2.02414 2.03473 2.04722 2.0616 2.07786 2.09597 2.11593 2.13771 2.16129 2.18665 2.21377 2.24261 2.27316 2.30537 2.33922 2.37468
2.41172 2.45029 2.49035 2.53188 2.57483 2.61916 2.66482 2.71178 2.75998 2.80938 2.85994 2.91159 2.9643 3.01801 3.07267 3.12823 3.18463 3.24182 3.29975 3.35835
3.41757 3.47736 3.53765 3.59839 3.65951 3.72097 3.7827 3.84463 3.90672 3.9689 4.0311 4.09328 4.15537 4.2173 4.27903 4.34049 4.40161 4.46235 4.52264 4.58243 4
.64165 4.70025 4.75818 4.81537 4.87177 4.92733 4.98199 5.0357 5.08841 5.14006 5.19062 5.24002 5.28822 5.33518 5.38084 5.42517 5.46812 5.50965 5.54971 5.58828
5.62532 5.66078 5.69463 5.72684 5.75739 5.78623 5.81335 5.83871 5.86229 5.88407 5.90403 5.92214 5.9384 5.95278 5.96527 5.97586 5.98454 5.9913 5.99613 5.99903
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2.2 二分法 & 反幂法

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exercise7_2
二分法迭代次数，27用时:0.7229ms

最小特征值：0.000967443
最小特征向量，
0.0311036 0.0621771 0.0931905 0.124114 0.154917 0.18557 0.216044 0.246309 0.276335 0.306094 0.335557 0.364696 0.393481 0.421886 0.449883 0.477444 0.504544 0.5
31155 0.557253 0.582811 0.607806 0.632213 0.656008 0.679168 0.701672 0.723496 0.744621 0.765025 0.784689 0.803594 0.821722 0.839054 0.855575 0.871268 0.886118
0.900111 0.913234 0.925472 0.936816 0.947253 0.956773 0.965368 0.97303 0.979749 0.985521 0.99034 0.9942 0.997099 0.999033 1 1 0.999033 0.997099 0.9942 0.9903
4 0.985521 0.979749 0.97303 0.965368 0.956773 0.947253 0.936816 0.925472 0.913234 0.900111 0.886118 0.871268 0.855575 0.839054 0.821722 0.803594 0.784689 0.76
5025 0.744621 0.723496 0.701672 0.679168 0.656008 0.632213 0.607806 0.582811 0.557253 0.531155 0.504544 0.477444 0.449883 0.421886 0.393481 0.364696 0.335557
0.306094 0.276335 0.246309 0.216044 0.18557 0.154917 0.124114 0.0931905 0.0621772 0.0311036


二分法迭代次数，27用时:0.6627ms

最大特征值：3.99903
最大特征向量，
-0.0311036 0.0621772 -0.0931905 0.124114 -0.154917 0.18557 -0.216044 0.246309 -0.276335 0.306094 -0.335557 0.364696 -0.393481 0.421886 -0.449883 0.477444 -0.5
04544 0.531155 -0.557253 0.582811 -0.607806 0.632213 -0.656008 0.679168 -0.701672 0.723496 -0.744621 0.765025 -0.784689 0.803594 -0.821722 0.839054 -0.855575
0.871268 -0.886118 0.900111 -0.913234 0.925472 -0.936816 0.947253 -0.956773 0.965368 -0.97303 0.979749 -0.985521 0.99034 -0.9942 0.997099 -0.999033 1 -1 0.999
033 -0.997099 0.9942 -0.99034 0.985521 -0.979749 0.97303 -0.965368 0.956773 -0.947253 0.936816 -0.925472 0.913234 -0.900111 0.886118 -0.871268 0.855575 -0.839
054 0.821722 -0.803594 0.784689 -0.765025 0.744621 -0.723496 0.701672 -0.679168 0.656008 -0.632213 0.607806 -0.582811 0.557253 -0.531155 0.504544 -0.477444 0.
449883 -0.421886 0.393481 -0.364696 0.335557 -0.306094 0.276335 -0.246309 0.216044 -0.18557 0.154917 -0.124114 0.0931905 -0.0621772 0.0311036
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3 结果分析

1) 可以看出过关 jacobi 方法迭代次数较大，耗时相对较长，我们可以通过并行化方法或优化稀疏矩阵乘法的方法进行算法优化

2) 二分法求矩阵特征值迭代次数较少；在对矩阵特征值有较为合理估计时，用反幂法可以用较低的迭代次数求出特征向量，性能是比较好的。