STAT 510

EXAM 2

SOLUTIONS

2017

1. 17
2. 20

3.a) 15

3.b) 18

4.a) 15

4.b) 15

1.

A \ B	1	2
1	3.0 2	5.0 8
2	7.0 6	3.0 4

$$SS(A \mid 1) + SS(B \mid 1, A) + SS(A \times B \mid 1, A, B) = SS(A, B, A \times B \mid 1)$$

$\Rightarrow$ Type II SS for B

$$= SS(B \mid 1, A)$$

$$= SS(A, B, A \times B \mid 1) - SS(A \mid 1) - SS(A \times B \mid 1, A, B)$$

$SS(A \mid 1)$:

$$\bar{y}_{1\cdot} = \frac{2 \times 3.0 + 8 \times 5.0}{10} = 4.6$$

$$\bar{y}_{2\cdot} = \frac{6 \times 7.0 + 4 \times 3.0}{10} = 5.4$$

$$\bar{y}_{\cdot\cdot} = \frac{2 \times 3.0 + 8 \times 5.0 + 6 \times 7.0 + 4 \times 3.0}{20} = 5.0$$

$$SS(A \mid 1) = \| P_A \underset{\sim}{y} - P_1 \underset{\sim}{y} \|^2$$

$$= 10(4.6 - 5.0)^2 + 10(5.4 - 5.0)^2 = 3.2$$

$$SS(A, B, A\times B \mid 1) = 2(3.0-5.0)^2 + 8(5.0-5.0)^2 + 6(7.0-5.0)^2 + 4(3.0-5.0)^2$$

$$= 2\times 4 + 6\times 4 + 4\times 4 = 48$$

$$SS(A\times B \mid 1, A, B) = (C\hat{\underline{\beta}})'(C(X'X)^{-}C')^{-1}C\hat{\underline{\beta}}$$

$$= (3-5-7+3)\left(\tfrac{1}{2}+\tfrac{1}{8}+\tfrac{1}{6}+\tfrac{1}{4}\right)^{-1}(3-5-7+3)$$

$$= 36\left(\frac{12+3+4+6}{24}\right)^{-1} = 36\times 24/25$$

$\therefore$ TYPE II SS FOR B IS

$$48 - 3.2 - 24\times 36/25$$

2. LET $A_1 = \underset{\sim}{1}_{48}$

$A_2 = \underset{4\times 4}{I} \otimes \underset{\sim}{1}_{12}$ BLOCK

$A_3 = \underset{\sim}{1}_4 \otimes \underset{3\times 3}{I} \otimes \underset{\sim}{1}_4$ GENO

$A_4 = \underset{12\times 12}{I} \otimes \underset{\sim}{1}_4$ BLOCK x GENO

$A_5 = \underset{\sim}{1}_{12} \otimes \underset{4\times 4}{I}$ FERT

$A_6 = \underset{\substack{\sim \\ 4\times 4}}{1} \otimes \underset{12\times 12}{I}$ GENO x FERT

NOW, TO GUARANTEE NESTED COLUMN SPACES,

SET $X_1 = A_1$, $X_2 = [X_1, A_2]$, $X_3 = [X_2, A_3]$,

$X_4 = [X_3, A_4]$, $X_5 = [X_4, A_5]$, $X_6 = [X_5, A_6]$.

COULD ALTERNATIVELY USE

$X_1 = A_1$, $X_2 = A_2$, $X_3 = A_3$, $X_4 = A_4$

$X_5 = [A_4, A_5]$, $X_6 = [A_4, A_6]$

3. A = foot (1 = L, 2 = R)

B = OM

C = FT

A = 1

	C = 1	C = 2
B = 1	μ	$\mu + C2$
B = 2	$\mu + B2$	$\mu + B2 + C2 + B2C2$

A = 2

	C = 1	C = 2
B = 1	$\mu + A2$	$\mu + A2 + C2 + A2C2$
B = 2	$\mu + A2 + B2 + A2B2$	$\mu + A2 + B2 + C2 + A2B2 + A2C2 + B2C2 + A2B2C2$

a) LSMEAN FOR OM1 IS ESTIMATE OF

$$\frac{\mu + (\mu + C_2) + (\mu + A2) + (\mu + A2 + C2 + A2C2)}{4}$$

$$= \mu + \frac{A_2}{2} + \frac{C_2}{2} + \frac{A2C2}{4}.$$

THUS, LSMEAN IS

$$6.44 - \frac{3.08}{2} + \frac{1.22}{2} - \frac{1.32}{4}$$

b) BASED ON THE TABLE OF CELL MEANS, WE CAN SEE THAT

$$\text{footR} = \widehat{\mu + A2} - \hat{\mu}$$

$$= \hat{\mu}_{211} - \hat{\mu}_{111}$$

$$= \bar{y}_{211\cdot} - \bar{y}_{111\cdot}$$

$$\text{VAR}(\bar{y}_{211\cdot} - \bar{y}_{111\cdot}) = 2(\sigma_s^2 + \sigma_e^2)/5$$

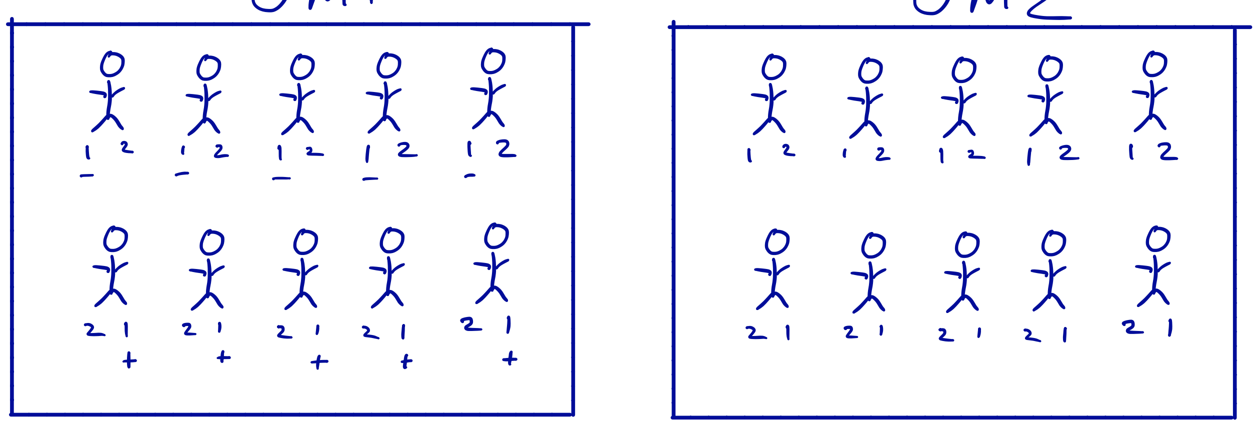
$$\text{SE}(\bar{y}_{211\cdot} - \bar{y}_{111\cdot}) = \sqrt{2(\hat{\sigma}_s^2 + \hat{\sigma}_e^2)/5}$$

$$= \sqrt{2(1.13231^2 + 0.39418^2)/5}$$

FROM R OUTPUT

TO SEE WHERE VARIANCE COMES FROM . . .

OM1 OM2



VARIANCE OF EACH OBSERVATION IS $\sigma_s^2 + \sigma_e^2$. OBSERVATIONS FROM DIFFERENT SUBJECTS ARE INDEPENDENT. WE ARE COMPARING AVERAGE OF 5 OBSERVATIONS MARKED WITH + TO AVERAGE OF 5 OBSERVATIONS MARKED WITH −.

4. LET $\underset{\sim}{y}_i = (y_{i1}, \dots, y_{in_i})'$ FOR $i = 1, 2, 3$.

$$\underset{\sim}{y} = \begin{bmatrix} \underset{\sim}{y}_1 \\ \underset{\sim}{y}_2 \\ \underset{\sim}{y}_3 \end{bmatrix}$$ FOLLOWS THE AITKEN MODEL

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}, \quad \underset{\sim}{\varepsilon} \sim N(0, \sigma^2 V),$$

WHERE $X = \begin{bmatrix} \underset{\sim}{1}_{n_1} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{1}_{n_2} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{1}_{n_3} \end{bmatrix}$, $\underset{\sim}{\beta} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}$,

AND $V = \text{DIAG}(\underset{\sim}{1}'_{n_1}, 4\underset{\sim}{1}'_{n_2}, 9\underset{\sim}{1}'_{n_3})$

$$\hat{\underset{\sim}{\beta}}_V = (X'V^{-1}X)^{-1}X'V^{-1}\underset{\sim}{y} = \begin{bmatrix} \frac{1}{n_1} & 0 & 0 \\ 0 & \frac{4}{n_2} & 0 \\ 0 & 0 & \frac{9}{n_3} \end{bmatrix} \begin{bmatrix} \underset{\sim}{1}'\underset{\sim}{y}_1 \\ \frac{1}{4}\underset{\sim}{1}'\underset{\sim}{y}_2 \\ \frac{1}{9}\underset{\sim}{1}'\underset{\sim}{y}_3 \end{bmatrix}$$

$$= \begin{bmatrix} \overline{y}_{1\cdot} \\ \overline{y}_{2\cdot} \\ \overline{y}_{3\cdot} \end{bmatrix}$$

a)

$$\hat{\sigma}^2_V = \frac{(\underset{\sim}{y} - X\hat{\underset{\sim}{\beta}})'V^{-1}(\underset{\sim}{y} - X\hat{\underset{\sim}{\beta}})}{n_1 + n_2 + n_3 - 3} = \frac{\sum_{i=1}^{3} \frac{1}{i^2} \sum_{j=1}^{n_i} (y_{ij} - \overline{y}_{i\cdot})^2}{n_1 + n_2 + n_3 - 3}$$

4 b) IN THIS REDUCED MODEL WHERE

$\mu = \mu_1 = \mu_2 = \mu_3$,

$\overline{Y}_{1.}$, $\overline{Y}_{2.}$, AND $\overline{Y}_{3.}$ ARE INDEPENDENT BLUEs OF μ COMPUTED FROM DATA IN TREATMENT GROUPS 1, 2, AND 3, RESPECTIVELY. TO COMBINE THESE ESTIMATORS OPTIMALLY, WE USE INVERSE VARIANCE WEIGHTING:

$$\frac{\frac{n_1}{\sigma^2}\overline{Y}_{1.} + \frac{n_2}{4\sigma^2}\overline{Y}_{2.} + \frac{n_3}{9\sigma^2}\overline{Y}_{3.}}{\frac{n_1}{\sigma^2} + \frac{n_2}{4\sigma^2} + \frac{n_3}{9\sigma^2}}$$

$$= \frac{36n_1\overline{Y}_{1.} + 9n_2\overline{Y}_{2.} + 4n_3\overline{Y}_{3.}}{36n_1 + 9n_2 + 4n_3}$$