Performance Analysis and Optimization of M-ary Code Shifted Differential Chaos Shift Keying System

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***Abstract*—In this paper, a power allocation strategy is applied to improve the bit-error-rate (BER) performance of M-ary code shifted differential chaos shift keying system (GCS-MDCSK) system. Particularly, different power coefficients are allocated for the reference and information bearing signals of GCS-MDCSK system, respectively. Moreover, the theoretical BER expressions of GCS-MDCSK system with power allocation strategy are derived over additive white Gaussian noise (AWGN) and multipath Rayleigh fading channels. Then, the optimal power ratio of reference and information bearing signals is deduced by finding the minimum of BER expression. Finally, the BER performance of the optimized GCS-MDCSK system is compared to ordinary GCS-MDCSK and other chaotic communication systems. Numerical results confirm the preferable BER performance of the optimized GCS-MDCSK system.**

***Index Terms*—Bit-error-rate (BER), M-ary code shifted differential chaos shift keying system (GCS-MDCSK), power allocation strategy.**

I. Introduction

As a kind of non-coherent chaotic modulation schemes, differential chaos shift keying (DCSK) [1] can recover the transmitted bits without requiring complicated chaotic synchronization and channel state information (CSI) at receiver. In addition, DCSK system not only has the benefit to be an excellent candidate for spread spectrum (SS) communication, but also shows strong robustness to mitigate degradation even in severe multipath fading environments [2]–[6]. However, DCSK system suffers from two main drawbacks inherently, i.e., relatively low data rate and spectral efficiency. Both defects are derived from the transmitted-reference (TR) structure of DCSK system, namely half of symbol period is used to transmit the non-information-bearing reference signal. Therefore, lots of researchers have proposed many improved schemes to overcome the two drawbacks.

In order to enhance spectral efficiency, code shifted differential chaos keying (CS-DCSK) was proposed in [7], where reference and information bearing signals are overlapping in time domain, but orthogonal in code domain by using different Walsh code sequences. Then the generalized code shifted differential chaos keying (GCS-DCSK) was proposed by the same authors [8], where its data rate is enhanced by adding more information bearing signals in a symbol period. As a binary modulation scheme, multilevel code shifted differential chaos shift keying (MCS-DCSK) [9] can obtain higher data rate and better BER performance than GCS-DCSK system at the expense of hardware complexity, because its receiver introduces more delay lines. A high-data-rate CS-DCSK scheme (HCS-DCSK) was presented in [10] where different chaotic codes are chosen to separate reference and information bearing signals rather than the limited Walsh code sequences. Furthermore, orthogonal multilevel DCSK (OM-DCSK), which can achieve higher data rate and spectral efficiency, was presented in [11]. In [12], M-ary code shifted differential chaos shift keying scheme (GCS-MDCSK) was proposed, which takes the advantages of both M-ary constellation and multilevel modulations to increase the data rate and spectral efficiency.

To increase the data rate, quadrature chaos shift keying (QCSK) was proposed in [13], which obtains double data rate with the same bandwidth occupation in contrast to DCSK. And then a circle-constellation-based M-ary DCSK was proposed in [14], which offers preferable BER performance than M-ary PSK-DCSK system [15]. In [16], in order to improve BER performance of circle-constellation-based M-ary DCSK, a square-constellation-based M-ary DCSK system with high energy efficiency was proposed. A new multi-carrier DCSK (MC-DCSK) was presented in [17], in which its energy efficiency and data rate are improved, but it requires the parallel matched filters. In addition, lots of researchers have proposed many new non-coherent chaotic communication systems to further increase the data rate of DCSK system, such as high efficiency DCSK (HE-DCSK) [18], reference modulated DCSK (RM-DCSK) [19] and multi-carrier chaos shift keying (CS-CSK) [20].

Although GCS-MDCSK system can achieve higher data rate and better spectral efficiency, its BER performance is deteriorating with the increasing of the number of parallel data streams, which is an undesired result. This phenomenon can be explained by the fact that when the overall number of parallel data streams increases, the interference within different data streams is reinforced which results in poor BER performance. Therefore, to overcome the performance loss and obtain the high data rate and good BER performance in the same time, we apply power allocation strategy to GCS-MDCSK system and propose the optimized GCS-MDCSK system. The main contributions of this paper are summarized as follows:

- To further improve the BER performance of GCS-MDCSK system, a power allocation strategy is applied to its reference and information bearing signals. The theoretical BER expressions of the power allocated GCS-MDCSK system are derived over AWGN and multipath Rayleigh fading channels.
- Through minimizing the theoretical BER expression of the power allocated GCS-MDCSK, we derive the optimal power ratio of reference and information bearing signals, and thus the optimized GCS-MDCSK system is designed. Numerical results confirm the excellent BER performance of the optimized GCS-MDCSK system.

The remainder of this paper is organized as follows. The next section presents GCS-MDCSK system with power allocation strategy. Performance analysis is provided in Section III. In Section IV, numerical results and discussions are given. Section V concludes this paper.

II. GCS-MDCSK System with Power Allocation Strategy

Combining M-ary modulation with GCS-DCSK system, GCS-MDCSK system can transmit more than one M-ary constellation signals within a symbol duration [12]. Although GCS-MDCSK system can achieve higher data rate, its BER performance improvement seems not apparent compared to GCS-DCSK system. The above slightly poor BER performance of GCS-MDCSK system enlightens us to further optimize this system. Therefore, the power allocation strategy is applied to GCS-MDCSK system to improve its BER performance, where different power coefficients are allocated to reference and information bearing signals, respectively. Therefore the transmitted signal of the power allocated GCS-MDCSK system is written in a vector form as

$$\mathbf{e} = \sqrt{\lambda_R}\mathbf{w}_R \otimes \mathbf{x} + \sqrt{\lambda_I}\mathbf{u}, \tag{1}$$

where $\otimes$ is Kronecker product operator, $\sqrt{\lambda_R}$ and $\sqrt{\lambda_I}$ are the power coefficients of reference and information bearing signals, respectively. $\mathbf{w}_R = [w_{R,1}, w_{R,2}, \cdots, w_{R,P}]$ denotes one row of P-order Walsh code matrix applied to carry reference signal. Moreover, vector $\mathbf{x} = [x_1, x_2, \cdots, x_\theta]$ represents a θ-length chaotic sequence generated by logistic map $x_{j+1} = 1 - 2x_j^2$. Vector $\mathbf{e}$ is the transmitted signal of GCS-MDCSK and $\mathbf{u}$ is the summation of all information bearing signals described as

$$\mathbf{u} = \sum_{n=1}^{N} \left(a_n \mathbf{w}_{I_{2n-1}} \otimes \mathbf{x} + b_n \mathbf{w}_{I_{2n}} \otimes \mathbf{x}\right), \tag{2}$$

where N denotes the number of transmitted M-ary constellation symbols within a GCS-MDCSK symbol duration. $\mathbf{w}_{I_{2n-1}} = [w_{I_{2n-1},1}, w_{I_{2n-1},2}, \cdots, w_{I_{2n-1},P}]$ and $\mathbf{w}_{I_{2n}} = [w_{I_{2n},1}, w_{I_{2n},2}, \cdots, w_{I_{2n},P}]$ are two different rows of P-order Walsh code matrix for information bearing signals. a_n and b_n represent the real and imaginary parts of constellation point. And the spreading factor of this system is $\beta = P\theta$.

III. Performance Analysis

Assuming the transmitted signal is polluted by multipath fading and additive white Gaussian noise, the received discrete baseband signal is expressed as

$$r_k = \sum_{l=1}^{L} \alpha_l e_{k-\tau_l} + n_k, \tag{3}$$

where $e_{k-\tau_l}, 1 \le k \le P\theta$ denotes k^{th} discrete sample value of vector $\mathbf{e}$ with time delay τ_l. L is the number of path, then α_l and τ_l are the channel propagation coefficient and the appropriate time delay of the l^{th} path, respectively. Besides, n_k is additive white Gaussian noise with zero mean and variance $\frac{N_0}{2}$. Particularly, the channel degrades into AWGN channel when $L = 1$, $\alpha_1 = 1$ and $\tau_1 = 0$.

By referring to GCS-MDCSK receiver in [12], we can extract reference and information bearing signals from the received signal by using corresponding rows of Walsh code matrix, respectively. Then, we multiply reference signal with different information bearing signals, thus the decision variables z_a^n and $z_b^n, n = 1, \cdots, N$ are obtained. For example, the decision variable z_a^n is described as (4), as shown at the bottom of this page. Thus, the mean and variance of z_a^n are calculated as

$$\begin{aligned}
\mathrm{E}[z_a^n] =& 2a_n P\theta \sqrt{\lambda_R \lambda_I} \sum_{l=1}^{L} \alpha_l^2 \mathrm{E}\left[x_{j-\tau_l}^2\right] \\
=& \sum_{l=1}^{L} \alpha_l^2 \frac{2\sqrt{\lambda_R \lambda_I} a_n E_s}{(\lambda_R + N\lambda_I)},
\end{aligned} \tag{5}$$

$$\begin{aligned}
z_a^n =& \sum_{p=1}^{P}\sum_{j=1}^{\theta} w_{R,p} w_{I_{2n-1},p} \left[\sum_{l=1}^{L} \alpha_l \left(\sqrt{\lambda_R} w_{R,p} x_{j-\tau_l} + \sqrt{\lambda_I} \sum_{n=1}^{N} \left(a_n w_{I_{2n-1},p} + b_n w_{I_{2n},p}\right) x_{j-\tau_l}\right) + n_{p\theta+j}\right]^2 \\
=& \sum_{p=1}^{P}\sum_{j=1}^{\theta}\sum_{l=1}^{L} \alpha_l^2 w_{R,p} w_{I_{2n-1},p} \left(\sqrt{\lambda_R} w_{R,p} + \sqrt{\lambda_I} \sum_{n=1}^{N} \left(a_n w_{I_{2n-1},p} + b_n w_{I_{2n},p}\right)\right)^2 x_{j-\tau_l}^2 + \sum_{p=1}^{P}\sum_{j=1}^{\theta} w_{R,p} w_{I_{2n-1},p} (n_{p\theta+j})^2 \\
&+ \sum_{p=1}^{P}\sum_{j=1}^{\theta}\sum_{l=1}^{L} 2\alpha_l w_{R,p} w_{I_{2n-1},p} \left(\sqrt{\lambda_R} w_{R,p} + \sqrt{\lambda_I} \sum_{n=1}^{N} \left(a_n w_{I_{2n-1},p} + b_n w_{I_{2n},p}\right)\right) x_{j-\tau_l} n_{p\theta+j}
\end{aligned} \tag{4}$$

$$\begin{aligned}\mathrm{Var}[z_a^n] =&4\left(\lambda_R+N\lambda_I\right)P\theta\sum_{l=1}^{L}\alpha_l^2\mathrm{E}\left[x_{j-\tau_l}^2\right]\frac{N_0}{2}+P\theta\frac{N_0^2}{2}\\ =&2\sum_{l=1}^{L}\alpha_l^2E_sN_0+\beta\frac{N_0^2}{2},\end{aligned}\tag{6}$$

where $E_s=\left(\lambda_R+N\lambda_I\right)P\theta\mathrm{E}\left[x_{j-\tau_l}^2\right]$ is the symbol energy. In addition, the relationship of symbol energy and bit energy is expressed as $E_s=\left(N\log_2M\right)E_b$, where E_b denotes bit energy and M is modulation order. Similarly, the mean and variance of z_b^n are obtained as

$$\mathrm{E}[z_b^n]=\sum_{l=1}^{L}\alpha_l^2\frac{2\sqrt{\lambda_R\lambda_I}b_nE_s}{\left(\lambda_R+N\lambda_I\right)},\tag{7}$$

$$\mathrm{Var}[z_b^n]=2\sum_{l=1}^{L}\alpha_l^2E_sN_0+\beta\frac{N_0^2}{2}.\tag{8}$$

The decision variables z_a^n and z_b^n can be regarded as independent Gaussian variables with means $m_1=a_nE_m$ and $m_2=b_nE_m$ respectively, where $E_m=\sum_{l=1}^{L}\alpha_l^2\frac{2\sqrt{\lambda_R\lambda_I}E_s}{(\lambda_R+N\lambda_I)}$. Furthermore, both decision variables have the same variance, i.e., $\sigma^2=2\sum_{l=1}^{L}\alpha_l^2E_sN_0+\beta\frac{N_0^2}{2}$. Therefore the BER of the power allocated GCS-MDCSK system is expressed as [21]

$$\begin{aligned}P_e\approx&\frac{2}{\sqrt{2\pi}\left(\log_2M\right)}\int_{\mu\sin\frac{\pi}{M}}^{+\infty}\exp\left(-\frac{\nu^2}{2}\right)\mathrm{d}\nu\\ \approx&\frac{2}{\log_2M}Q\left(\mu\sin\frac{\pi}{M}\right),\end{aligned}\tag{9}$$

where

$$\mu=\frac{E_m}{\sigma}=\frac{4\sqrt{\lambda_R\lambda_I}\gamma_s}{\left(\lambda_R+N\lambda_I\right)\sqrt{8\gamma_s+2\beta}},\tag{10}$$

$$Q\left(x\right)=\frac{1}{\sqrt{2\pi}}\int_x^{+\infty}\exp(-\frac{t^2}{2})\mathrm{d}t,\ x\geq0,\tag{11}$$

where $\gamma_s=\sum_{l=1}^{L}\alpha_l^2\frac{E_s}{N_0}$ is the instantaneous symbol SNR. When $\sqrt{\lambda_R}=\sqrt{\lambda_I}=1$, the power allocated GCS-MDCSK system degrades into the ordinary GCS-MDCSK system. In this case, $\mu=\frac{E_m}{\sigma}=\frac{4\gamma_s}{(1+N)\sqrt{8\gamma_s+2\beta}}$. Generally, we define $\kappa=\sqrt{\frac{\lambda_R}{\lambda_I}}$ as the power ratio of reference and information bearing signals, thus we have $\mu=\frac{4\gamma_s}{f(\kappa)\sqrt{8\gamma_s+2\beta}}$, where $f\left(\kappa\right)=\frac{\lambda_R+N\lambda_I}{\sqrt{\lambda_R\lambda_I}}=\kappa+\frac{N}{\kappa}$. When $\frac{df(\kappa)}{d\kappa}=0$, the BER of power allocated GCS-MDCSK system achieves its minimum, i.e., $1-\frac{N}{\kappa^2}=0$. Therefore, the optimal value of κ is calculated as $\kappa^*=\sqrt{N}$, namely $\frac{\lambda_R}{\lambda_I}=N$. At this point, $\mu=\frac{E_m}{\sigma}=\frac{2\sqrt{N}\gamma_s}{N\sqrt{8\gamma_s+2\beta}}$.

Considering L independent and identically distributed (i.i.d) Rayleigh-fading channels, the probability density function of instantaneous symbol SNR is written as [14], [17]

$$f\left(\gamma_s\right)=\frac{\gamma_s^{L-1}}{(L-1)!\bar{\gamma}_c^L}\exp\left(-\frac{\gamma_s}{\bar{\gamma}_c}\right),\tag{12}$$

where $\bar{\gamma}_c$ is the average symbol SNR per channel defined as $\bar{\gamma}_c=\frac{E_s}{N_0}\mathrm{E}\left[\alpha_j^2\right]=\frac{E_s}{N_0}\mathrm{E}\left[\alpha_l^2\right],j\neq l$ with $\sum_{l=1}^{L}\mathrm{E}\left[\alpha_l^2\right]=1$.

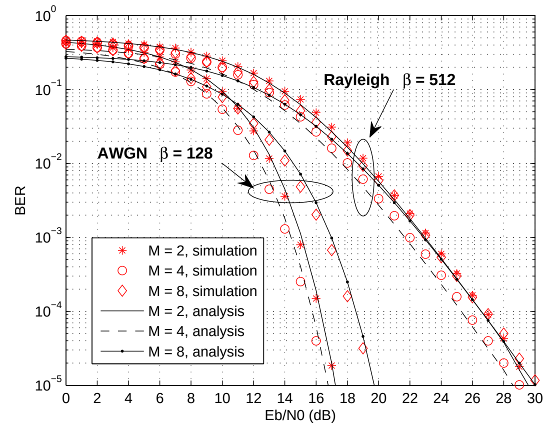

Fig. 1. Comparisons of the simulated and theoretical results for GCS-MDCSK system ($\kappa=1$) over AWGN and multipath Rayleigh fading channels with $P=8$, $N=2$ and $M=2,4,8$.

Finally, the BER of GCS-MDCSK system over multipath Rayleigh fading channel is given by

$$P_{\text{fading}}=\int_0^{+\infty}P_{\mathrm{e}}\cdot f\left(\gamma_s\right)\mathrm{d}\gamma_s.\tag{13}$$

IV. Numerical Results and Discussions

To verify the correctness of our theoretical derivations for GCS-MDCSK system ($\kappa=1$), the simulated and theoretical results over AWGN and multipath Rayleigh fading channels are drawn in Fig. 1. It is clearly observed that there is a good match between simulation and theoretical results for different modulation orders. Note that a major disagreement appears when SNR is small, which is caused by the approximation made in (9). In AWGN channel, GCS-MDCSK system achieves the best BER performance when $M=4$. And then the BER performance of GCS-MDCSK system is deteriorating with the increasing M which is due to that the Euclidean distance of adjacent constellation point becomes smaller when M increases which degrades its BER performance. For multipath Rayleigh fading channel, 3-path channel with equal average power gain is used to perform our simulations. The parameters of 3-path Rayleigh fading channel are given as $\mathrm{E}(\alpha_1^2)=\mathrm{E}(\alpha_2^2)=\mathrm{E}(\alpha_3^2)=1/3$ and $\tau_1=0$, $\tau_2=T_c$, $\tau_3=2T_c$. Similar to AWGN channel, the best performance also appears in $M=4$ for fading channel.

In order to confirm the excellent BER performance of the optimized GCS-MDCSK system, we make a comparison for BER performance between the optimized GCS-MDCSK system and GCS-MDCSK system, as shown in Fig. 2. In this figure, "optimized" denotes the optimized GCS-MDCSK system and "ordinary" represents GCS-MDCSK system. Apparently, the simulation results almost match theoretical results for both optimized GCS-MDCSK system and GCS-MDCSK system which verifies our derivations. As observed in Fig. 2, the BER performance of optimized GCS-MDCSK system outperforms that of GCS-MDCSK system over both AWGN

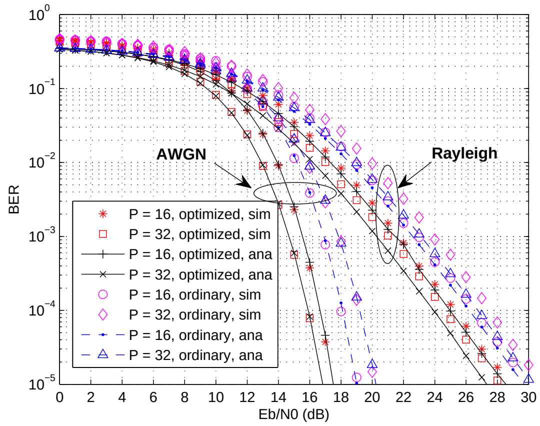

Fig. 2. Performance comparison between the optimized GCS-MDCSK system ($\kappa^* = \sqrt{N}$) and GCS-MDCSK system ($\kappa = 1$) over AWGN and multipath Rayleigh fading channels with $\beta = 1024$, $M = 4$, $P = 16, 32$ and $N = 4, 8$.

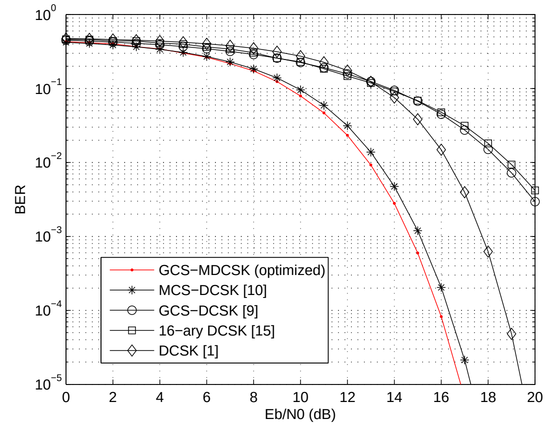

Fig. 3. BER performance comparisons between the optimized GCS-MDCSK and other non-coherent chaotic communication systems over AWGN channel with $\beta = 256$.

and multipath Rayleigh fading channels. In AWGN channel, for example, the optimized GCS-MDCSK system performs over 1dB better than GCS-MDCSK system at BER level of 10^{-5} in AWGN channel when $P = 16$. Additionally, the BER performance gain between the optimized GCS-MDCSK system and GCS-MDCSK system is about 3dB at the same BER level above when $P = 32$. From another perspective, the performance of GCS-MDCSK system worsens gradually when P increases. On the contrary, as the value of P increases, the BER performance of optimized GCS-MDCSK system improves. In other words, the BER performance gain for optimized GCS-MDCSK system over GCS-MDCSK system becomes larger with the increasing of P.

In Fig. 3 and Fig. 4, we make a BER performance comparison between the optimized GCS-MDCSK system and other non-coherent chaotic communication systems over AWGN and multipath Rayleigh fading channels. For a fair comparison, the overall number of bits per symbol is set to 4 for all systems above (except for DCSK system). In addition, the order of the Walsh code is $P = 8$ for the optimized GCS-MDCSK, MCS-DCSK and GCS-DCSK systems. Other simulation parameters are set as follow: $M = 4$, $N = 2$ and $\kappa^* = \sqrt{2}$ are used for the optimized GCS-MDCSK simulation. With respect to MCS-DCSK and GCS-DCSK systems, the number of parallel data streams is set to 4. As shown in Fig. 3, in AWGN channel, the optimized GCS-MDCSK system achieves the best BER performance compared to its competitors. Explicitly, although the BER performance of MCS-DCSK system is quite closed to the optimized GCS-MDCSK system, the optimized GCS-MDCSK system has lower hardware complexity than MCS-DCSK system. This is due to the fact that the optimized GCS-MDCSK system removes all delay lines in its receiver, while MCS-DCSK system introduces more delay lines in its receiver. In multipath Rayleigh fading channel, as illustrated in Fig. 4, we find that the BER performance of the proposed optimized GCS-MDCSK system is also superior to its rivals.

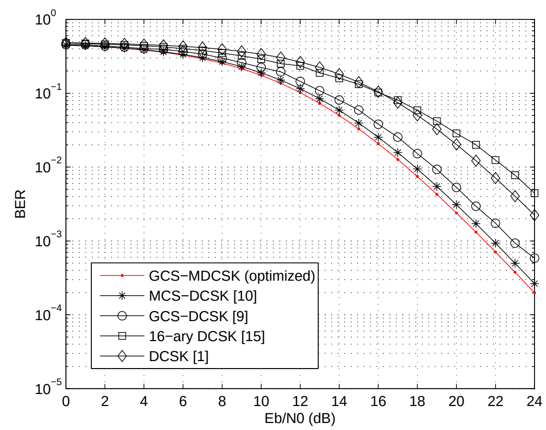

Fig. 4. BER performance comparisons between the optimized GCS-MDCSK and other non-coherent chaotic communication systems over multipath Rayleigh fading channel with $\beta = 512$.

V. Conclusion

In this paper, a power allocation strategy is applied to GCS-MDCSK system. Then, the theoretical BER expressions for the power allocated GCS-MDCSK system are derived over AWGN and multipath Rayleigh fading channels. In addition, based on the theoretical BER expressions, we derive the optimal power ratio for its reference and information bearing signals. According to our numerical results, the optimized GCS-MDCSK system shows $1 \sim 3$dB BER performance gain compared to ordinary GCS-MDCSK system at BER level 10^{-5} over AWGN and multipath Rayleigh fading channels. Furthermore, the performance gain of the optimized GCS-MDCSK system over ordinary GCS-MDCSK system becomes more significant when P increases. Moreover, the optimized GCS-MDCSK system has the best BER performance in contrast to other non-coherent chaotic communication systems.

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