Managing Operations at Golf-Sport

Zeyu Xing
Xiangyi Fan
Zhe Fu

Ⅰ. Consulting Report

Ⅰ. Executive summary

We have simplified this problem into a linear programming problem with objective function and constraints. By solving it in AMPL, we could get the optimal solution of the problem.

For the objective function, our goal is to maximize the profit of company Golf-Sport. The profit equals to the revenue of products (both components and sets) minus the total costs, including materials, production and assembly cost, as well as the cost of storage.

As for the constraints, there are eight parts of constraints in our model.

- Assembly times for the sets;
- Labor availability;
- Packing availability;
- Advertising availability;
- Demand of each products;
- Graphite supply limitation;
- Relationship between produced components and assembled products.
- Relationship between inventory, assembled products and products to be sold.

After solving the problem with AMPL, not only did we obtain the optimal plan for Golf-Sport Company in the following two months, we also applied sensitivity analysis into the final results. Besides, we used AMPL to make further sensitivity analysis. Combining objective solution with sensitivity analysis, we finally got the comprehensive and exhaustive conclusions to this problem.

Ⅱ. Solution

To get the maximum profit of company Golf-Sport, the **plan** of producing, selling and storing for each month are listed below:

a. Production: the amounts of components each plant should produce in 2 months are:

	Month1			**Month2**		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
Steel shafts	2000	0	0	2000	2000	0
Graphite shafts	1886.36	100	2413.64	1100	128.75	1250
Forged iron heads	1046.15	200	418.182	1746.57	222.115	1023.08
Metal wood heads	30	30	195.455	283.846	36.6346	562.821
Titanium insert heads	1377.81	2723	2000	1219.65	1277	2000

b. Assembly: the amounts of sets each plant should assemble in 2 months are:

	Month1			Month2		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
Steel metal	0	0	0	0	0	0
Steel insert	0	0	0	0	0	0
Graphite metal	0	0	31.8182	84.6154	2.21154	92.3077
Graphite insert	84.6154	0	0	0	0	0

c. Inventory: the amounts of products (parts or sets) each plant should store for the second month are:

	Month1		
	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	100	0	0
Forged iron heads	0	0	0
Metal wood heads	0	0	0
Titanium insert heads	0	723	0
Steel metal	0	0	0
Steel insert	0	0	0
Graphite metal	0	0	0
Graphite insert	0	0	0

d. Sales: the amounts of products (parts or sets) each plant should sell in 2 months are:

	Month1			Month2		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
Steel shafts	2000	0	0	2000	2000	0
Graphite shafts	686.364	100	2000	100	100	50
Forged iron heads	200	200	100	900.42	200	100
Metal wood heads	30	30	100	30	30	285.897
Titanium insert heads	1123.96	2000	2000	1219.65	2000	2000
Steel metal	0	0	0	0	0	0
Steel insert	0	0	0	0	0	0

Graphite metal	0	0	31.8182	84.6154	2.21154	92.3077
Graphite insert	84.6154	0	0	0	0	0

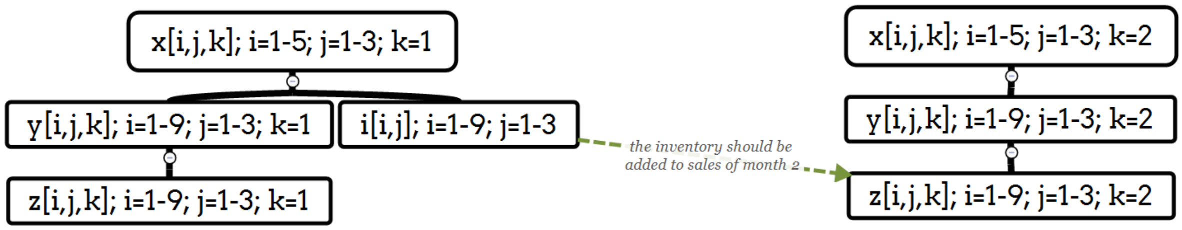
If company Golf-Sport follow this plan, the **profi**t will reach the maximum which is $249292.1541.

As for **product-resource**, the uses of labor, packing and advertising are listed below:

	Month 1			**Month 2**		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
labor	12000	15000	22000	12000	15000	22000
packing	19069.21	21631.00	32800.00	20000.00	5509.00	19941.00
advertising	21918.30			20857.22		
graphite	1100			619.688		

Ⅱ Technical Part

. Ⅰ variables explanation



Picture1: relationship between different variables

$$x_{ijk} = number\ of\ components\ i\ produced\ in\ plant\ j\ in\ month\ k$$
$$for\ i = 1, \ldots, 5 \quad j = 1,2,3 \quad k = 1,2$$
$$y_{ij1} = number\ of\ components\ \ i\ left\ after\ assembling\ sets$$
$$besides\ inventory\ in\ plant\ j\ in\ month\ 1$$
$$for\ i = 1, \ldots, 5 \quad j = 1,2,3$$
$$y_{ij1} = number\ of\ sets\ \ i\ assembled\ except\ inventory\ in\ plant\ j\ in\ month\ 1$$
$$for\ i = 6, \ldots, 9 \quad j = 1,2,3$$
$$y_{ij2} = number\ of\ components\ \ i\ left\ after\ assembling\ sets\ in\ plant\ j\ in\ month\ 2$$
$$for\ i = 1, \ldots, 5 \quad j = 1,2,3$$
$$y_{ij2} = number\ of\ sets\ \ i\ \ assembled\ \ in\ plant\ j\ in\ month\ 2$$
$$for\ i = 6, \ldots, 9 \quad j = 1,2,3$$
$$I_{ij} = number\ of\ components\ and\ sets\ i\ inventoried\ in\ plant\ j$$
$$for\ i = 1, \ldots, 9 \quad j = 1,2,3$$
$$Z_{ijk} = number\ of\ components\ and\ sets\ i\ sold\ in\ plant\ j\ in\ month\ k$$
$$for\ i = 1, \ldots, 9 \quad j = 1,2,3 \quad k = 1,2$$

The meaning of the superscript is shown in the following table:

mark	meaning	mark	Meaning
i=1	Steel shafts	i=7	Set: Steel, insert
i=2	Graphite shafts	i=8	Set: Graphite, metal
i=3	Forged iron heads	i=9	Set: Graphite, insert
i=4	Metal wood shafts	j=1	Plant: Chandler
i=5	Titanium insert heads	j=2	Plant: Glendale
i=6	Set: Steel, metal	j=3	Plant: Tucson

Assembly times for the sets at each plant j：

$$\mathrm{T}_j = [65\ 60\ 65]$$

Monthly Availability times for the sets at each plant j：

$$\mathrm{M}_j = [5500\ 5000\ 6000]$$

Labor times for the components i at each plant j ：

$$\mathrm{l}_{ij} = \begin{bmatrix} 1.0 & 3.5 & 3.0 \\ 1.5 & 3.5 & 3.5 \\ 1.5 & 4.5 & 4.0 \\ 3.0 & 4.5 & 4.5 \\ 4.0 & 5.0 & 5.5 \end{bmatrix}$$

Monthly availability of labor times at each plant j：

$$\mathrm{L}_j = [12000\ 15000\ 22000]$$

Packing times for the components i at each plant j：

$$\mathrm{p}_{ij} = \begin{bmatrix} 4.0 & 7.0 & 7.5 \\ 4.0 & 7.0 & 7.5 \\ 5.0 & 8.0 & 8.5 \\ 6.0 & 9.0 & 9.5 \\ 6.0 & 7.0 & 8.0 \end{bmatrix}$$

Monthly availability of packing times at each plant j：

$$\mathrm{P}_j = [20000\ 40000\ 35000]$$

Advertising fees for the components i at each plant j：

$$\mathrm{a}_{ij} = \begin{bmatrix} 1.0 & 1.1 & 1.3 \\ 1.5 & 1.1 & 1.3 \\ 1.1 & 1.1 & 1.3 \\ 1.5 & 1.2 & 1.3 \\ 1.9 & 1.9 & 1.9 \end{bmatrix}$$

Minimum product demand per month for the components i at each plant j：

$$\mathrm{min}_{ij} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 100 & 100 & 50 \\ 200 & 200 & 100 \\ 30 & 30 & 100 \\ 100 & 100 & 100 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Maximum product demand per month for the components i at each plant j：

$$\max_{ij} = \begin{bmatrix} 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 200 & 200 & 200 \\ 100 & 100 & 100 \\ 300 & 300 & 300 \\ 400 & 400 & 400 \end{bmatrix}$$

Material, Production and Assembly costs per part for the components i at each plant j：

$$c_{ij} = \begin{bmatrix} 6 & 5 & 7 \\ 19 & 18 & 20 \\ 4 & 5 & 5 \\ 10 & 11 & 12 \\ 26 & 24 & 27 \\ 178 & 175 & 180 \\ 228 & 220 & 240 \\ 350 & 360 & 370 \\ 420 & 435 & 450 \end{bmatrix}$$

Revenue per part for the components i at each plant j：

$$r_{ij} = \begin{bmatrix} 10 & 10 & 12 \\ 25 & 25 & 30 \\ 8 & 8 & 10 \\ 18 & 18 & 22 \\ 40 & 40 & 45 \\ 290 & 290 & 310 \\ 380 & 380 & 420 \\ 560 & 560 & 640 \\ 650 & 650 & 720 \end{bmatrix}$$

Ⅱ Model formulation

We can formulate the problem as:

Maximum $\sum_{k=2}^{2}\sum_{j=1}^{3}\sum_{i=1}^{9} z_{ijk} \times r_{ij} - \sum_{j=1}^{3}\sum_{i=1}^{9}\left(y_{ij1} + I_{ij} + y_{ij2} \times 1.25\right) \times c_{ij} -$
$- \sum_{j=1}^{3}\sum_{i=1}^{9} I_{ij} \times c_{ij} \times 0.05$

Subject to:

Constraint 1: Assembly hours for the sets

For j={1,2,3}

$$T_j \times \sum_{i=6}^{9}(y_{ij1} + I_{ij}) \le M_j$$

$$T_j \times \sum_{i=6}^{9} y_{ij2} \le M_j$$

Constraint 2：Labor availability

For j={1,2,3}, k={1,2}

$$\sum_{i=1}^{5} x_{ijk} \times l_{ij} \le L_j$$

Constraint 3：Packing availability

For j={1,2,3}

$$\sum_{i=1}^{5}(y_{ij1} + I_{ij}) \times p_{ij} \le P_j$$

$$\sum_{i=1}^{5} y_{ij2} \times p_{ij} \le P_j$$

Constraint 4：Advertising availability

For k={1,2}

$$\sum_{j=1}^{3}\sum_{i=1}^{5} a_{ij} \times x_{ijk} \le 22000$$

Constraint 5：Product demand of each products

For i={1,2,3,4,5,6,7,8,9},j={1,2,3},k={1,2}

$$z_{ijk} \ge min_{ij}$$
$$z_{ijk} \le max_{ij}$$

Constraint 6: Graphite supply limitation

For k= {1,2}

$$\sum_{j=1}^{3} x_{2jk} \leq 1100 \times 16 \div 4$$

Constraint 7：Relationship between produced components, assembled sets and Inventory
For j= {1,2,3}
Month1:

$$x_{1j1} = y_{1j1} + I_{1j} + 13y_{6j1} + 13I_{6j} + 13y_{7j1} + 13I_{7j}$$
$$x_{2j1} = y_{2j1} + I_{2j} + 13y_{8j1} + 13I_{8j} + 13y_{9j1} + 13I_{9j}$$
$$x_{3j1} = y_{3j1} + I_{3j} + 10\sum_{i=6}^{9}(y_{ij1} + I_{ij})$$
$$x_{4j1} = y_{4j1} + I_{4j} + 3y_{6j1} + 3I_{6j} + 3y_{8j1} + 3I_{8j}$$
$$x_{5j1} = y_{5j1} + I_{5j} + 3y_{7j1} + 3I_{7j} + 3y_{9j1} + 3I_{9j}$$

Month2:

$$x_{1j2} = y_{1j2} + 13y_{6j2} + 13y_{7j2}$$
$$x_{2j2} = y_{2j2} + 13y_{8j2} + 13y_{9j2}$$
$$x_{3j2} = y_{3j2} + 10\sum_{i=6}^{9} y_{ij2}$$
$$x_{4j2} = y_{4j2} + 3y_{6j2} + 3y_{8j2}$$
$$x_{5j2} = y_{5j2} + 3y_{7j2} + 3y_{9j2}$$

Constraint 8：Relationship between sold products and Inventory
For i= {1,…,9} j= {1,2,3}

$$z_{ij1} = y_{ij1}$$
$$z_{ij2} = y_{ij2} + I_{ij}$$

Ⅲ. Sensitivity Analysis

For sensitivity questions:

a) More graphite or advertising cash
If more graphite is available, I would like 25.94 more ounces of graphite in month 1 and I am willing to pay $3 for one additional ounce of graphite. Besides, I would like to use the additional graphite to produce 103.752 more graphite shafts in Chandler.
As for advertising cash, I would not like more cash.

It is because from the information extracted from AMPL, for graphite used in month 1 the "_con" is 3 and "_con.up" is 1125.94. It means the shadow price of graphite in month 1 is $3 and the maximum amount of graphite I can get to remain the objective solution is 1125.94 ounce. To find the use of additional graphite, I changed the original availability of graphite in month 1 to 1125.94 and re-solved the model. Comparing to the original solution, the new solution will produce 103.752 more graphite shafts in Chandler and all these graphite shafts are sold in the first month as components.

Yet for graphite used in month 2 and advertising cash, the shadow prices are all 0, so I would not to get more of them.

b) Extra packing machine hours, Extra assembly hours, Extra labor hours

	_con	**_con.up**
assembly_set1[1]	0.884615	6870.44
assembly_set1[2]	0	0
assembly_set1[3]	0	0
assembly_set2[1]	0.698252	8512.26
assembly_set2[2]	0	0
assembly_set2[3]	0.361538	6549.24
labor[1,1]	3.5	12172
labor[1,2]	1.77273	13284.1
labor[2,1]	2.01769	15215
labor[2,2]	1.05769	19112.4
labor[3,1]	2.62879	22343.3
labor[3,2]	1.55556	25955.8
packing1[1]	0	0
packing1[2]	0	0
packing1[3]	0	0
packing2[1]	0.0681818	23023.8
packing2[2]	0	0
packing2[3]	0	0

From "_con", we can see the shadow price for each constraint, for those whose shadow price is 0, that means the constraint is not binding, and we would not add extra hours to it. For those whose shadow price isn't 0, they are binding and worth us to add extra hours.

Thus, the extra hours and the maximum extra money we are willing to pay for each plant each month is:

	Plants	**Month**	**extra hours**	**maximum extra money**	**original constraints**
Assemble Time	Chandler	month1	1370.44	0.884615	5500
		month2	3012.26	0.698252	5500
	Glendale	month1	0	0	5000
		month2	0	0	5000
	Tucson	month1	0	0	6000
		month2	549.24	0.361538	6000
	Chandler	month1	172	3.5	12000

Labor Time		month2	1284.1	1.77273	12000
	Glendale	month1	215	2.01769	15000
		month2	4112.4	1.05769	15000
	Tucson	month1	343.3	2.62879	22000
		month2	3955.8	1.55556	22000
Packing Time	Chandler	month1	0	0	20000
		month2	3023.8	0.0681818	20000
	Glendale	month1	0	0	40000
		month2	0	0	40000
	Tucson	month1	0	0	35000
		month2	0	0	35000

c) 50% increase in products' minimum demand

Yes, we can handle the 50% increase in their minimum demand.

By put the minimum demand time 1.5 and then resolving the model, the optimal value decreases by 1779.6854, from 249292.1541 to 247512.4687. After introducing another auxiliary variable b as the cost from production process only, we find that, after taking the additional demand, the cost actually decreases by 26, from 525570 to 5245544.

In conclusion, it would cost $1779.6854 more from the profit to cover the demand.