Exercise 3 FFT Convolution

March 16, 2018

Xiangyi Cheng (xxc283)

1 Concept and Background

Convolution between an image and a Gaussian kernel is one of the most crucial calculations in image blurring. Specically, each pixel in the image gets multipiled by the value of the Guassian kernel in the corresponding position. The result value replaces the center pixel of the image where the kernel overlaps.

However, this approach requires much time and calculation which is not efficient. Fourier transform shows advantage on this field to do convolution between the image and kernel. Fourier transform convolution states the principle that multiplication in the frequency domain corresponds to convolution in the time domain. With this basic idea, calculations are simplified. Also, efficiency are obviously improved.

To prove the Fourier transformation convolution theorem, the image will be processed by both Fourier transformation approach and traditional approach then compare them.

2 Implement

2.1 Fourier Transform Convolution

The useful libraries are loaded first.

```
In [2]: import numpy as np
        import matplotlib.pyplot as plt
        from scipy import fftpack
        import cv2
```

Then the image should be read in and its size should be identified.

```
In [3]: img=cv2.imread('sydney.jpg',cv2.IMREAD_GRAYSCALE)
        img_blurred=img.copy()
        h,w=img.shape[:2]
```

The Gaussian filter I chose is a 5x5 kernel, as defined below. The matrix is generated by a column matrix times a row matrix due to its symmetry and separability.

$$kernel = \frac{1}{256} * \begin{pmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix}$$

```
In [4]: # set the Gaussian filter
        Gaussian=1/256.*np.matrix('1;4;6;4;1')*np.matrix('1 4 6 4 1')
```

Since the dimensions of the image are different from that of the kernel. Both of the them should be padded to get the same size. The dimensions of padded matrix should at least be $(M_{img} + m_{kernel} - 1) * (N_{img} + n_{kernel} - 1)$.

```
In [5]: kernel_pad_m=h-1
        kernel_pad_n=w-1
        img_pad_width=len(Gaussian)-1

        kernel_pad=np.pad(Gaussian,((0,kernel_pad_m),(0,kernel_pad_n)),'constant')
        img_pad=np.pad(img,((0,img_pad_width),(0,img_pad_width)),'constant')
```

Do the fourier transform for both the padded image and kernel.

```
In [9]: # do fourier transformation
        img_frq=fftpack.fft2(img_pad)
        kernel_frq=fftpack.fft2(kernel_pad)
```

Once getting the Fourier transform of the image and kernel, multiply them as described before to get the convolution in frequency domain.

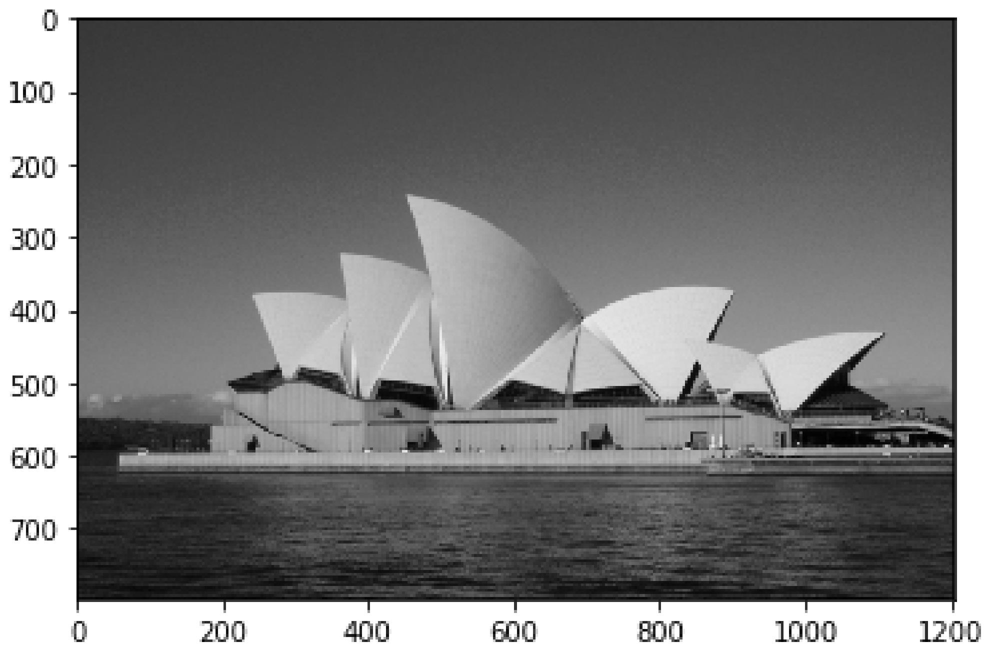
```
In [10]: # compute the production of kernel_frq and img_frq
         pro_frq= np.multiply(img_frq,kernel_frq)
```

Do the inverse Fourier transform to get the convolution in the time domain so that the image is reconstructed.

```
In [11]: # inverse the fourier transformation
         img_processed=abs(fftpack.ifft2(pro_frq))
```

Show the blurred image with Fourier transform convolution.

```
In [12]: #show processed image
         cv2.imwrite('sydney_fft.jpg',img_processed)
         img_fft=cv2.imread('sydney_fft.jpg')
         img_fft1=cv2.cvtColor(img_fft,cv2.COLOR_BGR2RGB)
         plt.imshow(img_fft1)
         plt.show()
```

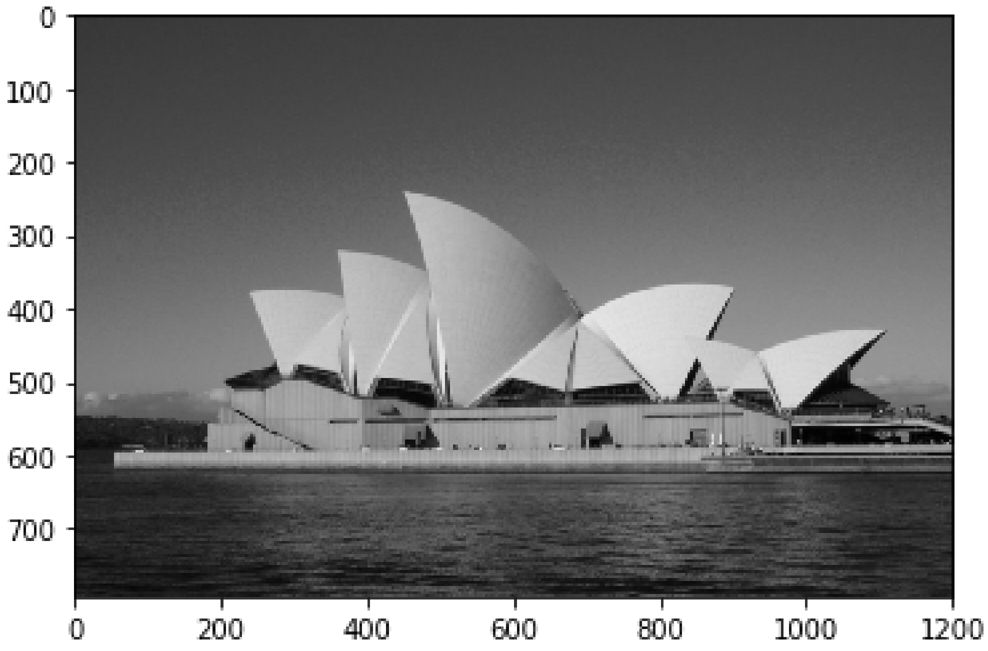


2.2 Traditional Convolution

The traditional approach to do convolution was described in Exercise 1 in A1. The code is as shown at below:

```
In [13]: # traditional convolution
         img_blurred=img.copy()

         center_poc=(len(Gaussian)-1)/2

         # add two rows above and below the image respectively. add two colums on left and right
         # reshape a zero matrix in (h+2,w+2) dimension.
         img_reshape=np.zeros((h+2*center_poc,w+2*center_poc))

         # give the original pixel values from the original image to the corresponding position
         for k in range(0,h):
                 for l in range(0,w):
                         img_reshape[k+center_poc,l+center_poc]=img_blurred.item(k,l)

         # do the blurring process
         for i in range(0,h):
                 for j in range(0,w):
                         sum=0
                         for h1 in range(0,len(Gaussian)):
                                 for w1 in range(0,len(Gaussian)):
                                         pixel_bg=img_reshape.item(i+h1,j+w1)
```

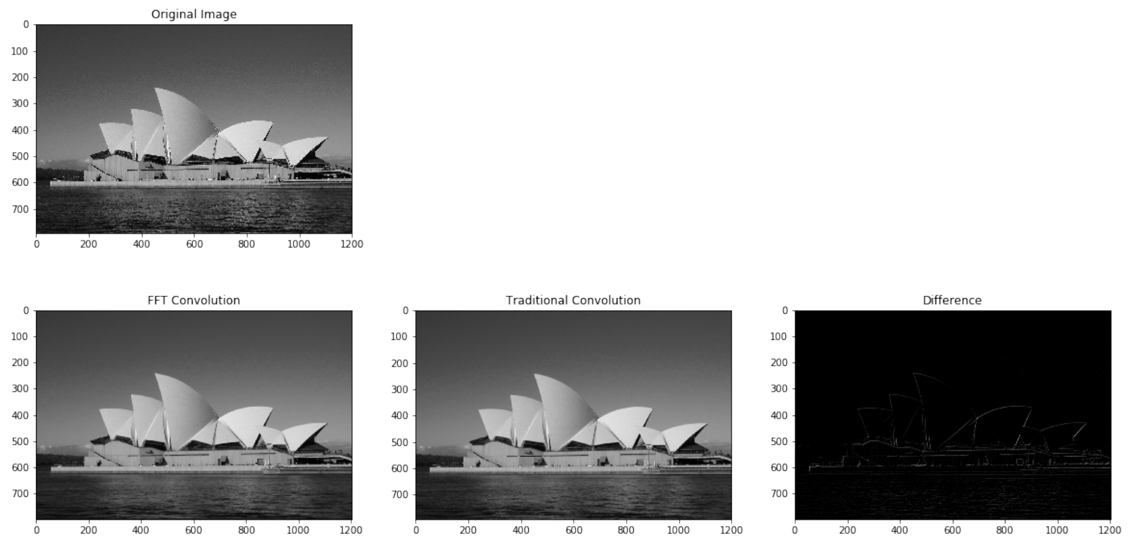
```
                                pixel_kernel=Gaussian[h1,w1]
                                sum=sum+(pixel_bg*pixel_kernel)
                img_blurred.itemset((i,j),sum)

cv2.imwrite('sydney_blurred.jpg',img_blurred)
img_trad=cv2.imread('sydney_blurred.jpg')
img_trad1=cv2.cvtColor(img_trad,cv2.COLOR_BGR2RGB)
plt.imshow(img_trad1)
plt.show()
```



Show the orginal image, processed image applied Fourier transform convolution, processed image applied traditional convolution and the difference between two blurred images.

```
In [22]: # show the difference between fft and traditional approach doing convolution
         img_trad=img_blurred
         img_fft=img_processed
         img_trad=np.pad(img_trad,((0,img_pad_width),(0,img_pad_width)),'constant')
         fft_trad_diff=img_trad-img_fft
         cv2.imwrite('diff_fft_trad.jpg',fft_trad_diff)
         fft_trad_diff=cv2.imread('diff_fft_trad.jpg')
         fft_trad_diff=cv2.cvtColor(fft_trad_diff,cv2.COLOR_BGR2RGB)
         original=cv2.imread('gray_sydney.jpg')
         original1=cv2.cvtColor(original,cv2.COLOR_BGR2RGB)
         ##############################################################################
         ##############################################################################
         plt.figure(figsize=(20,10))
         plt.subplot(2,3,1)
```

```
    plt.imshow(original1)
    plt.title('Original Image')
    plt.subplot(2,3,4)
    plt.imshow(img_fft1)
    plt.title('FFT Convolution')
    plt.subplot(2,3,5)
    plt.imshow(img_trad1)
    plt.title('Traditional Convolution')
    plt.subplot(2,3,6)
    plt.imshow(fft_trad_diff)
    plt.title('Difference')
    plt.show()
```

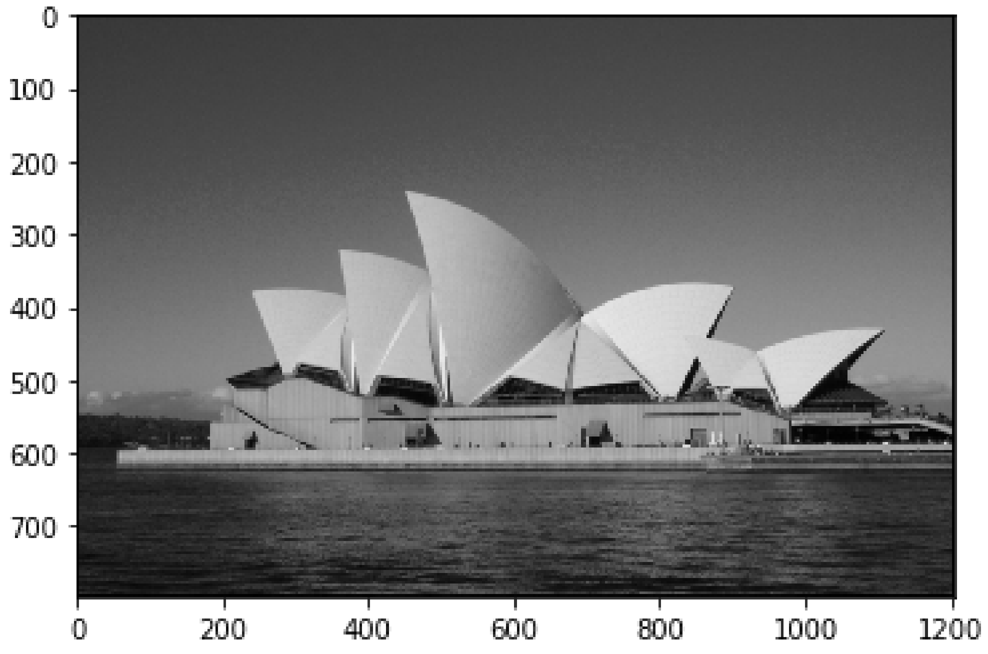


3 Extension

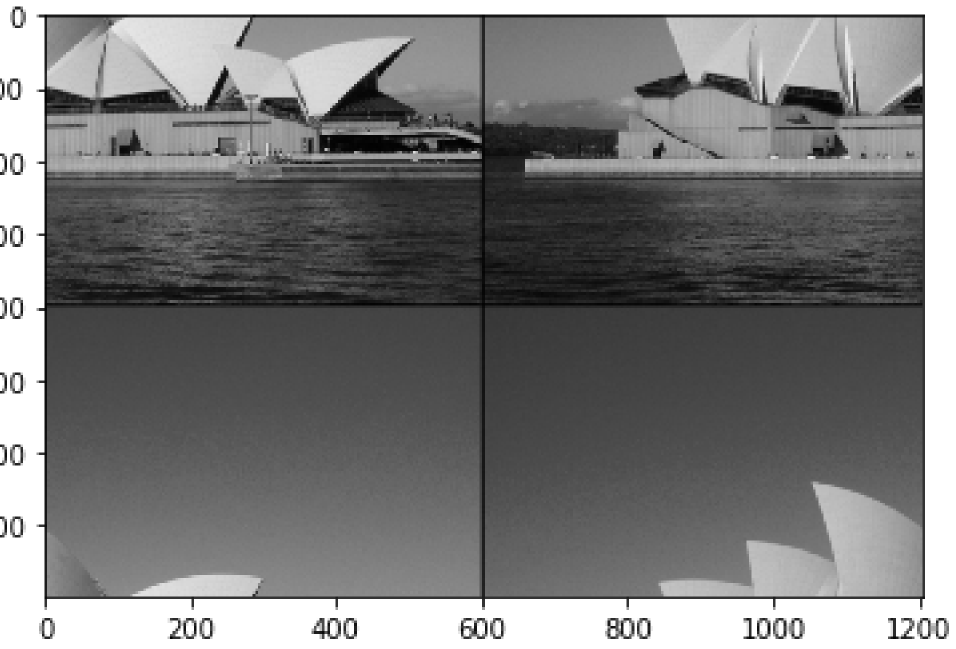
Try to pad the images with different ways to see if there are differences exist.

```
In [7]: kernel_pad=np.pad(Gaussian,((kernel_pad_m,0),(kernel_pad_n,0)),'constant')
        img_pad=np.pad(img,((img_pad_width,0),(img_pad_width,0)),'constant')
        # do fourier transformation
        img_frq=fftpack.fft2(img_pad)
        kernel_frq=fftpack.fft2(kernel_pad)
        # compute the production of kernel_frq and img_frq
        pro_frq= np.multiply(img_frq,kernel_frq)
        # inverse the fourier transformation
        img_processed=abs(fftpack.ifft2(pro_frq))
        #show processed image
        cv2.imwrite('sydney_fft.jpg',img_processed)
        img_fft=cv2.imread('sydney_fft.jpg')
```

```
img_fft1=cv2.cvtColor(img_fft,cv2.COLOR_BGR2RGB)
plt.imshow(img_fft1)
plt.show()
```



```
In [19]: kernel_pad=np.pad(Gaussian,((kernel_pad_m/2,kernel_pad_m/2),(kernel_pad_n/2,kernel_pad_
         img_pad=np.pad(img,((img_pad_width/2,img_pad_width/2),(img_pad_width/2,img_pad_width/2)
         # do fourier transformation
         img_frq=fftpack.fft2(img_pad)
         kernel_frq=fftpack.fft2(kernel_pad)
         # compute the production of kernel_frq and img_frq
         pro_frq= np.multiply(img_frq,kernel_frq)
         # inverse the fourier transformation
         img_processed=abs(fftpack.ifft2(pro_frq))
         #show processed image
         cv2.imwrite('sydney_fft.jpg',img_processed)
         img_fft=cv2.imread('sydney_fft.jpg')
         img_fft1=cv2.cvtColor(img_fft,cv2.COLOR_BGR2RGB)
         plt.imshow(img_fft1)
         plt.show()
```



From above, the padding 0 should be at the edge of the matrix other than padding the matrix in the center.

4 Conclusion and Analysis

Fourier transform convolution is applied to do blurring compared to the traditional approach to do convolution.

From the results, two convolution approaches both achieve the goal to blur the image. And we cannot tell any differences from them with naked eyes. However, some distinctions show up once do the substract. Theoretically, they are equivalent while it shows error in practical. Although the error is tiny, the reasons behind the error should depend on how we do the padding to the image and kernel. The way I developed the traditional convolution is called "full convolution" mathematically. It may be differences between the full convolution and the FFT convolution with certain padding way.

Besides the differences in images, the advantage about computional time costing is revealed clearly in FFT convolution. Compared to traditional convolution, it took much less time in computing FFT convolution. Therefore, FFT convolution is more efficient than traditional convolution, especially in blurring image.