Practice:

一、

T1: only 3 season: $\binom{4}{3}3^7$

only 2 season: $\binom{4}{2}2^7$

only 1 season: $\binom{4}{1}1^7$

$\therefore \dfrac{4^7-4\cdot3^7-6\cdot2^7-4}{4^7}$

T2: $\dfrac{\binom{5}{1}\binom{6}{2}\binom{6}{1}\binom{6}{1}\binom{6}{1}\binom{6}{1}}{\binom{30}{6}}$

二、

T1: for naive it can't

for proof: suppose: $P(A)\cdot P(A)=P(A)$.

1°: $P(A)=0$, 2°: $P(A)=1$

$1-\frac{1-mn}{1-n}=\frac{1-n-1+mn}{1-n}=\frac{mn-n}{1-n}$

$mn-n$.

T2: Yes

$P(A)=m$, $P(B)=n$, $P(AB)=mn$.

$P(A^c)=1-m$, $P(B^c)=1-n$, $P(A^cB^c)=P(B^c)P(A^c|B^c)$

$P(A^c|B^c)=1-P(A|B^c)$ | $P(A^cB^c)=1-P(A\cup B)=1-(m+n-mn)=mn-m-n+1$

HW.

Q1: A: throw a die with 3/4/5 upside
B throw a die with 2/3/4 upside.

$$Q2: \frac{C_4^3 C_{39}^{13}}{C_{52}^{13}} = \frac{4\binom{39}{13}}{\binom{52}{13}}$$

Q3: (1) No

(2) A: A>B B: A>C

we just (A, B, C), (A, C, B), (B, A, C), (B, C, A)
(C, A, B), (C, BA)

P(A): (A, B, C), (A, C, B), (C, A, B)

$P(C|A) = \frac{2}{3}$

(a) A: choose first B: choose second C: up twice.

$P(A|C)$?

$P(C|A)=\frac{1}{4}$, $P(C|B)=\frac{1}{16}$, $P(C)=\frac{1}{8}+\frac{1}{32}=\frac{5}{32}$

$P(A|C)=\frac{P(AC)}{P(C)}=\frac{P(A)\cdot P(C|A)}{P(C)}=\frac{4}{5}$

(b): NO! C_1: first up C_2: second up

$P(C_1)=\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{4}=\frac{3}{8}$, $P(C_2)=\frac{3}{8}$

$P(C_1C_2)=\frac{5}{32}\neq P(C_1)P(C_2)$

(c) first: $\frac{1}{2}\cdot C_{10}^{3}\left(\frac{1}{2}\right)^{10}$

second: $\frac{1}{2}\cdot C_{10}^{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{7}$ $\frac{C_{10}^{3}}{2}\left(\left(\frac{1}{2}\right)^{10}+\frac{3^7}{4^{10}}\right)$

A: abuse wife B wife be murderd C: wife --

$P(A|BC^c)=\frac{1}{10}$, $P(C|B)=0.2$ $P(A|C)=\frac{1}{2}$.

$$P(C|AB)=\frac{P(ABC)}{P(AB)}=\frac{P(B)\cdot P(AC|B)}{P(B)\cdot P(A|B)}=\frac{P(AC|B)}{P(A|B)}$$

$$P(A|B)=P(A|BC^c)P(C^c|B)+P(A|BC)P(C|B)$$

$$=\frac{1}{10}\times 0.8+\frac{1}{2}\times 0.2=\frac{9}{50}$$

$$P(AC|B)=P(C|B)\cdot(P A|BC)=0.2\times 0.5=\frac{5}{50}$$

$$\therefore P(C|AB)=\frac{5}{9}.$$

Q6: $P = \frac{1}{2}$.

(2): A: both are girl B: at least one is girl

$P(AB) = P(A) = \frac{1}{4}$, $P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

$\therefore \frac{1}{3}$