T1: (1) $>$ (2) $=$

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T2: (a): all condition: $C_{52}^{4}$

there are 4 possibilities: 3-7 ... 6-10

$$\therefore \frac{C_4^1 (C_4^1 C_4^1 C_4^1 C_4^1)}{C_{52}^{4}}$$

(b): choose 2 with 2: $C_{13}^{2} C_4^2 C_4^2$

choose 1 with 1: $C_{11}^{1} C_4^1$

$$\therefore \frac{C_{13}^{2} C_4^2 C_4^2 + C_{11}^{1} C_4^1}{C_{54}^{4}}$$

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T3: 110 step up & 111 step right

$$\therefore C_{221}^{110}$$

(b): $$\frac{C_{221}^{110} C_{220}^{110}}{C_{421}^{210}}$$

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T4: numbers for $n$ $(n \le 26)$ letters: $C_{26}^{n} n! = \frac{26!}{n!(26-n)!} n! = \frac{26!}{(26-n)!}$

$$\therefore \frac{26! C_{26}^{26}}{\sum_{n=1}^{26} n! C_{26}^{n}} = \frac{26!}{\sum_{n=1}^{26} \frac{26!}{(26-n)!}} = \frac{1}{\sum_{n=1} \frac{1}{(26-n)!}} = \frac{1}{1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots} \approx \frac{1}{e}$$

T5: there are n people and everyone may be choose. it I choose one people, $C_n^1$, choose two $C_n^2$ ......, all possiblities are $\sum_{k=0}^{n} C_n^k$, and for each people they may be choose or not, so $2^n$

T6:

$\boxed{\text{algebra:}}$ $C_{n+1}^{k} = \frac{(n+1)!}{k!\,(n+1-k)!}$

$$C_n^k + C_n^{k-1} = \frac{n!}{k!\,(n-k)!} + \frac{n!}{(k-1)!\,(n-k+1)!} = \frac{n!\cdot(n+1-k)}{k!\,(n-k+1)!} + \frac{n!\,(k)}{k!\,(n-k+1)!} = \frac{(n+1)!}{k!\,(n-k+1)!}$$

$\boxed{\text{story:}}$ we're going to choose k people from (n+1), with me in it. if choosing me, then $C_n^{k-1}$, if not, $C_n^k$

T1: just $$\frac{C_3^1 C_2^1 C_1^1 \; C_3^1 C_2^1 C_1^1}{6!} = \frac{1}{20}$$

T2: (a) $C_{12}^2 \cdot \left(\frac{1}{2} C_{10}^5\right) = \frac{1}{2} C_{10}^5 C_{12}^2$

(b): $\frac{1}{3!}\left(C_{12}^4 C_8^4 C_4^4\right)$

T3 $$\frac{10 \times 9 \times 8}{10 \times 10 \times 10} = 0.72$$

T4 $$1 - \frac{6!}{6^6}$$

T5: $$\frac{C_n^k \; C_{N-m}^{m-k}}{C_N^m}$$

T6:

(a) so trivial

(b): A: the first is red  B: the second is red

C the first is green  D: the second is green

$P(C) = \frac{g}{r+g}$,  $P(D) = A \cdot P(D|A) + C \cdot P(D|C)$

$= \frac{r}{r+g} \cdot \frac{g}{r+g-1} + \frac{g}{r+g} \cdot \frac{g-1}{(r+g-1)}$

$= \frac{g}{r+g}\left(\frac{r}{r+g-1} + \frac{g-1}{r+g-1}\right) = \frac{g}{r+g}$

(c): $P(AD) + P(BC) = \frac{1}{2}$ :

$P(AD) = P(A)\ P(D|A) = \frac{rg}{(r+g)(r+g-1)}$

$P(BC) = P(C)\ P(B|C) = \frac{gr}{(r+g)(r+g-1)}$

$\therefore 4rg = (r+g)(r+g-1)$ $(r+g \leq 16,\ r \geq 0,\ g \geq 0)$

$\begin{cases} r=1 \\ g=3 \end{cases}$ $\begin{cases} r=3 \\ g=1 \end{cases}$ $\begin{cases} r=6 \\ g=3 \end{cases}$ $\begin{cases} r=3 \\ g=6 \end{cases}$ $\begin{cases} r=10 \\ g=6 \end{cases}$ $\begin{cases} r=6 \\ g=10 \end{cases}$

T7: (1) Image a group of people by age and then think about the oldest person in a group of $(k+1)$ if he is oldest of all: $C_n^k$, if second, $C_{n-1}^k$ …… $C_k^k$ ✓

(2): $\sum_{i=30}^{50} C_{i+5-1}^{4} = \sum_{i=30}^{50} C_{i+4}^{4} = \sum_{i=34}^{54} C_i^{4}$

$= \sum_{i=4}^{54} C_i^4 - \sum_{i=4}^{33} C_i^4 = C_{55}^5 - C_{34}^5$