Q1:

if X has average λ, then $P(X=k) = e^{-\lambda} \frac{(\lambda)^k}{k!}$

$P(\lambda t) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$

when $\lambda = 100$, $t = \frac{1}{20}$, $k=0$

$P = e^{-5}$

Q2 E_j: the jth student have the same seat in the two room

$P(N=0) = 1 - P(\bigcup_{j=1}^{100} E_j)$

$= 1 - [(P(E_1) + P(E_2) + \cdots + P(E_{100}) - \binom{100}{2} P(E_i E_j) + \binom{100}{3} P(E_i E_j E_k) \cdots + P(E_1 E_2 \cdots E_{99})]$

$= 1 - 100 P(E_1) + \binom{2}{100} P(E_1 E_2) - \binom{3}{100} P(E_1 E_2 E_3) \cdots$

$= 1 + \sum_{j=1}^{100} (-1)^j \binom{100}{j} \frac{(100-j)!}{100}$

(b): I_i: student i has the same seat

$P((I_i = 1) \cap (I_j = 1)) = \left(\frac{1}{100}\right)\left(\frac{1}{99}\right) \approx \frac{1}{100} \frac{1}{100} = P(I_i = 1) P(I_j = 1)$

$\therefore \text{Pois}(\lambda) \quad \lambda = 1$

$\therefore P(X=0) = e^{-1}$

(c): $P(X=1) = e^{-1}$

$\therefore P(X \geq 2) = 1 - \frac{2}{e}$

Q3:

(a): $E(e^{-3X}) = \sum_{k=0}^{\infty} e^{-3k} P(X=k) = \sum_{k=0}^{\infty} e^{-3k} \frac{\lambda^k e^{-\lambda}}{k!} = e^{-\lambda} e^{\lambda e^{-3}}$

$\therefore E(e^{-3X}) - e^{-3\lambda} = e^{-\lambda} e^{\lambda e^{-3}} - e^{-3\lambda} \neq 0$

NO

(b): $E((-2)^X) = \sum_{k=0}^{\infty} (-2)^k P(X=k) = \sum_{k=0}^{\infty} (-2)^k \frac{\lambda^k e^{-\lambda}}{k!} e^{-\lambda} = e^{-3\lambda}$

Yes

Q1

$P(SZ \leq X) = P(SZ \leq x \mid S=1) P(S=1) + P(SZ \leq x \mid S=-1) P(S=-1)$

$= \frac{1}{2}(P(Z \leq x) + P(Z \geq -x)) = P(Z \leq 1)$

Q2: Yes, because X & Y is symmetric

NO, it depends on $P(X=Y)=0$?

Q3

$P((n-X)=m) = P(X=n-m) = \binom{n}{n-m} p^{n-m} (1-p)^m = \binom{n}{m} (1-p)^m p^{n-m}$

Q4:

$\frac{1}{2}$: there won't be 2~99 seats because if it is empty then somebody will sit there.

Q1:

$$P(Y \le y) = P(e^X \le y) = P(X \le \ln y) = \Phi\left(\frac{\ln y - u}{\sigma}\right)$$

Q2: $X = -\frac{1}{\lambda}\ln U$

Q3: $F(X) \sim \text{Unif}(0,1)$

$$E(\Phi(Z)) = \frac{1}{2}$$

Q4: $F_L(l) = P(L<l) = P(-l < U < l) = 2l - 1$

Q1 $$E(X!) = e^{-\lambda}\sum_{k=0}^{\infty} k!\,\frac{\lambda^k}{k!} = \frac{e^{-\lambda}}{1-\lambda}$$

Q2: $$E|Z| = \int_{-\infty}^{\infty} |z| f(z)\,dz = 2\int_0^{\infty} z f(z)\,dz = \int_0^{\infty} \frac{2z e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\,dz = \sqrt{\frac{2}{\pi}}$$

Q3 $$E(e^{tX}) = p\sum_{k=0}^{\infty} e^{tk} q^k = p\sum_{k=0}^{\infty} (qe^t)^k = \frac{p}{1-qe^t}$$