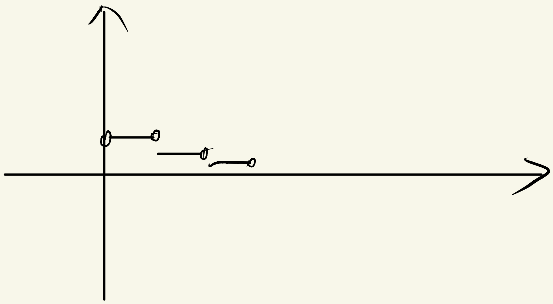
Q1: throw a die

X: 1, 2, 3 $\quad$ Y: 4, 5, 6

Q2: Yes, $P(X<Y) = \frac{6}{7}$

Q3: $P(X=j) = (\frac{1}{2})^j$



Q4: $X = N(1000, 100)$ $\quad$ $Y = N(1, 0.00000000001)$

Q5: $\sum_{n=0}^{\infty} (1-F(n)) = (1-F(0)) + (1-F(1)) + \cdots + (1-F(\infty))$

$= (P(X>0) + P(X>1) + P(X>2)$

$= (\sum_{n=1}^{\infty} P(X=n)) + (\sum_{n=2}^{\infty} P(X=n)) \cdots$

$= P(X=1) + 2P(X=2) + \cdots + nP(X=n) = E(X)$

Q6: $E(X) = \sum_{n=0}^{\infty} P(X>n) = P(X>0) + \sum_{n=1}^{\infty} P(X>1) = 1 + \sum_{n=1}^{\infty} \frac{1}{n} = \infty$

Q1: $X_{ij} = \begin{cases} 1 & \text{if } i \text{ have the same birthday with } j \\ 0 & \text{otherwise} \end{cases}$

$$E\left(\sum_{0\le i<j\le 50} X_{ij}\right) = \sum_{0\le i<j\le 50} E(X_{ij}) = \binom{50}{2}\frac{1}{365} = \frac{1225}{365} = \frac{245}{73}$$

$$E(Y) = 365\left(1-\left(\frac{364}{365}\right)^{50} - \binom{50}{1}\frac{1}{365}\left(\frac{364}{365}\right)^{49}\right)$$

Q2 for a student

$$X_j \sim Bin\left(20 \sim \frac{1}{20}\right) \therefore E(X_j) = 1$$

$$E(X_1+X_2+X_3) = 3, \quad E(I_1 + \dots + I_{20}) = 20\,E(I_1) = 20\left(1-\left(\frac{19}{20}\right)^{20}\right)$$

Q3: (a): 100

(b): $I_j = \begin{cases} 1 & \text{if form a loop} \\ 0 & \text{otherwise} \end{cases}$

for each step, $p(\text{form a loop}) = \dfrac{101-j}{\binom{202-2j}{2}} = \dfrac{1}{201-2j}$

$\therefore$ we need $\displaystyle\sum_{j=1}^{100} \frac{1}{201-2j} = \frac{1}{199} + \frac{1}{197} + \dots + \frac{1}{1} = \sum_{j=1}^{100} \frac{1}{2j-1} = \sum_{n=1}^{100} \frac{1}{2n-1}$

Q4 (a): for a simple place

only one is $\binom{k}{1}\frac{1}{n}\left(\frac{n-1}{n}\right)^{k-1} = \frac{k(n-1)^{k-1}}{n^k}$, $E = k\left(1-\frac{1}{n}\right)^{k-1}$

more than one is $1 - \frac{k(n-1)^{k-1}}{n^k} - \left(\frac{n-1}{n}\right)^k \quad E = n - \frac{k(n-1)^{k-1} - (n-1)^k}{n^{k-1}}$

NO! what about Nobody ?

Q1

$$G(x) = \sum_{n=0}^{x-1} F(n)$$

Q2:

$$F(x) = n\cancel{p}$$

$$F(x) = npr$$

Q3: $F(x) = (\frac{1}{2})^x$

$$\therefore E(x) = \sum_{x=1}^{\infty} x(\frac{1}{2})^x = \lim_{n\to\infty} 2 - (\frac{1}{2})^{n-1} - 2n(\frac{1}{2})^{n+1} = 2.$$

Q4:

for each box, denote E: empty

$$P(E) = (\frac{n-1}{n})^k$$

$$\therefore P(nE) = \frac{(n-1)^k}{n^{k-1}}$$

Q5

disease status \ gender	men	women
disease	X	$r-X$
nondisease	$m-X$	$n-r+X$

$$P(X=x \mid X+Y=r) = \frac{P(X+Y=r \mid X=x)\,P(X=x)}{P(X+Y=r)}$$

$$= \frac{\binom{n}{r-x} p^{r-x}(1-p)^{n-r+x}\binom{m}{x} p^{x}(1-p)^{m-x}}{\binom{m+n}{r} p^{r}(1-p)^{m+n-r}}$$

$$= \frac{\binom{m}{x}\binom{n}{r-x}}{\binom{m+n}{r}}$$

(b) just by intuition is OK

Q6: $X_{ij} = \begin{cases} 1 & \text{if } i \text{ is before } j \\ 0 & \text{otherwise} \end{cases}$

$$E\left(\sum_{1\le i<j\le 5} X_{ij}\right) = \sum_{1\le i<j\le 5} E(X_{ij}) = \binom{5}{2}\cdot\frac{1}{2} = 5$$

(b): for left every inversion need a check.

for right, the worst case is $(n-1)$ firstly, $(n-2)$ secondly ...

and $(n-1)+(n-2)+\cdots+1 = \frac{n(n-1)}{2} = \binom{n}{2}$

Q7 (a): Yes

A: the 110th jumper setting a record

B: the 111th jumper jumps second highest

$P(A) = \frac{1}{110}$, $P(B) = \frac{1}{111}$.

$P(AB) = \frac{109!}{111!} = \frac{1}{110\times 111}$

$\therefore P(A)P(B) = P(AB)$

(b): $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{j} \to \infty$