1.

Q1: A: choose door 1 firstly B choose door 2. firstly C: choose door 3 firstly

But by intuitive, I think preference makes no difference.

$\therefore \frac{2}{3}$

(b): $\frac{1}{2}$. E: open door 2

if A: $P(S) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}$

if not A: $P(S) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}$

(c): $\frac{1}{2}$ again.

Q2:

1. False: suppose $X = Z$
2. False X represents the first die showing 3 faces up. Y represents the second die with 3 faces up, and Z represents the total of the two dice being 5
3. False change T_2 into Z represents the total of the two dice is 10000
4. False: X: 1,5,6 Y: 1,5. Z: 1,2,3

2.

Q1: NO

$P(A) = P(E)P(A|E) + P(E^c)P(A|E^c) \leq P(E)P(B|E) + P(E^c)P(B|E^c) = P(B)$

(b): NO: $P(A|B) = P$ B

Q2

(a) A: a man has many ivory

B: a man doesn't have much ivory

C: hurt the elephant

(b): $P(C|A)$ $P(C|B)$

(c): just kidding. disire $\neq$ doing

3

Q: lets return to what we learn in class.

Another Gambler have $(N-i)$ dollar ($N=1\times10^6$, $i=2$) and we need him to lose all

$P(i) = qP(i-1) + pP(i+1)$ ($p=\frac{1}{3}$, $q=\frac{2}{3}$) with $P(0)=0$, $P(1\times10^6+2)=1$

guess $P(i)=x^i$

$x^i = qx^{i-1} + px^{i+1}$, $px^2+q-x=0$, $x=\frac{1\pm\sqrt{1-4pq}}{2q} = \frac{1\pm\sqrt{(2p-1)^2}}{2p} \in \{1, \frac{q}{p}\}$

$\therefore P_i = A1^i + B(\frac{q}{p})^i$, $P(0)=0 \Rightarrow A+B=0$ $P(N)=1 \Rightarrow A+B(\frac{p}{q})^N=1$ $\therefore B=-A=\frac{1}{(\frac{p}{q})^N-1}$

$\therefore P_i = \frac{-1}{(\frac{q}{p})^N-1} + (\frac{q}{p})^i \frac{1}{(\frac{q}{p})^N-1}$

$\therefore P_{1\times10^6} = \frac{-1}{(2)^{100002}-1} + (2)^{10000}\frac{1}{(2)^{100002}-1} = ((2)^{10000}-1)\cdot\frac{1}{(2)^{10002}-1} = \frac{2^{10000}-1}{2^{10002}-1} < \frac{2^{100001}}{2^{100002}-4} = \frac{1}{4}$

4

Q1: 4:0: $\binom{3}{0}p^4$ 4:1: $\binom{4}{1}p^4q$ 4:2 $\binom{5}{2}p^4q^1$ 4:3: $\binom{6}{3}p^4q^3$

$\therefore \binom{3}{0}p^4 + \binom{4}{1}p^4q + \binom{5}{2}p^4q^2 + \binom{6}{3}p^4q^3$

Q2: No of course

we've calculate so many time before Gaokao

Q2: suppose success a time:

for a possibility: $P(i) = p^a(1-p)^{n-a}$

$P(X=a) = \binom{n}{a} p^a (1-p)^{n-a}$, there are $\binom{n}{a}$ outcomes

$P(i \mid X=a) = \frac{P(i, X=a)}{P(X=a)} = \frac{P(i)}{P(X=a)} = \frac{1}{\binom{n}{a}}$

Q3:

(a): just two experience is OK.

firstly, n time, secondly, m time, each P,

(b): X-Y may be less than 0

HW:

Q1: WLOG, we assume he chooses the first door initially

A: the gift is in the first door B: switch C: success

if switch: $P(S)=P(A)\cdot P(S|A)+P(A^c)\,P(S|A^c)=0+\frac{6}{7}\times\frac{1}{3}=\frac{2}{7}$

if not switch: $P(S)=P(A)\cdot P(S|A)+P(A^c)\,P(S|A^c)=\frac{1}{7}$ $\therefore$ Switch

(b):

if switch

$P(S)=P(A)\,P(S|A)+P(A^c)\,P(S|A^c)= \quad +\frac{n-1}{n}\cdot\frac{1}{n-1-m}=\frac{n-1}{n(n-1-m)}$

if not switch: $\frac{1}{n}$

if $\frac{1}{n}>\frac{n-1}{n(n-1-m)}$, then $m>0$

$\therefore$ always switch.

Q2

(a):

proof: $\frac{P(H|D)}{P(H^c|D)}=\frac{P(H)}{P(H^c)}\,\frac{P(D|H)}{P(D|H^c)}$

$$\frac{P(H|D)}{P(H^c|D)}=\frac{\frac{P(D|H)\cdot P(H)}{P(D)}}{\frac{P(D|H^c)\cdot P(H^c)}{P(D)}}=\frac{P(H)}{P(H^c)}\cdot\frac{P(D|H)}{P(D|H^c)}$$

(b): pri

Q3: Yes: a 12-face dice $B: 1\sim 6$, $C: 7\sim 12$

$A_1: 4\sim 8$ $A_2: 4, 5, 6, 9, 10$

$P(A_1|B) = \frac{1}{2}$, $P(A_2|B) = \frac{1}{2}$, $P(A_1|C) = \frac{1}{3}$, $P(A_2|C) = \frac{1}{3}$

$P(A_1 \cup A_2|B) = \frac{1}{2}$, $P(A_1 \cup A_2|C) = \frac{2}{3}$

Q4:

(a): $P(C) = 2pq\,P(C) + p^2$ $\therefore P(C) = \frac{p^2}{1-2pq}$

(b): $p_i = \frac{-1}{(\frac{q}{p})^N - 1} + (\frac{q}{p})^i \frac{1}{(\frac{q}{p})^N - 1}$

$\therefore p_2 = \frac{-1}{(\frac{1-p}{p})^4 - 1} + (\frac{1-p}{p})^2 \frac{1}{(\frac{1-p}{p})^4 - 1} = \left[(\frac{1-p}{p})^2 - 1\right] \frac{1}{(\frac{1-p}{p})^4 - 1}$

Q5:

(a): $P_K = \frac{1}{6}\left(P(K-1)+P(K-2)+P(K-3)+P(K-4)+P(K-5)+P(K-6)\right)$ $(K>0)$

$P_0 = 1$

$P_K = 0$ for $K<0$

(b): $P_1 = \frac{1}{6}$, $P_2 = \frac{7}{36}$, $P_3 = \frac{49}{216}$ $P_4 = \frac{343}{1296}$

$P_5 = \frac{781}{3888}$ $P_6 = \frac{2641}{11664}$

$P_7 = \frac{16819}{69984}$

(c) The average number thrown by the die is $\left(\frac{\text{total of dot}}{6}\right) = \frac{7}{2}$ so that every throw adds on an average of $\frac{7}{2}$. we can therefore expect to land on 2 out of 7

Q6:

(a): $P(X=0) = \binom{m}{0}(1-P_1)^m$

$P(X=j) = \binom{m}{j} p^j (1-P_1)^{m-j}$

(b): $P(X=j) = \text{- - - -}$ Yes:

(c) $r = P_1 + (1-P_1)(1-P_2)r$

$r = \frac{-P_1}{P_1P_2 - P_1 - P_2} = \frac{P_1}{P_1 + P_2 - P_1P_2}$

Q7:

(a) 1°: the last one is right:

$P_1 = \binom{4}{2} 0.1 \times 0.1 \times 0.9 \times 0.9 \times 0.9 + \binom{4}{4} \times (0.1)^4 \times 0.9$

2°: the last one is not right

$P_2 = \binom{1}{4}(0.1)(0.9)^3(0.1) + \binom{3}{4}(0.1)^3(0.9)(0.1)$

$\therefore P = \binom{4}{1}(0.1)^2(0.9)^3 + \binom{4}{2}(0.1)^2(0.9)^3 + \binom{4}{3}(0.1)^4(0.9) + \binom{4}{4}(0.1)^4(0.9)$

(b)

$$\sum_{k=1}^{\lfloor \frac{n}{2} \rfloor} \binom{n-1}{2k} (0.1)^{2k} (0.9)^{n-2k} + \sum_{k=1}^{\lfloor \frac{n}{2} \rfloor} \binom{n-1}{2k-1} (0.1)^{2k} (0.9)^{n-2k},$$