西交利物浦大学 Xi'an Jiaotong-Liverpool University

Pattern Recognition

Lecture 10. Eigenfaces and Fisherfaces in Face Recognition

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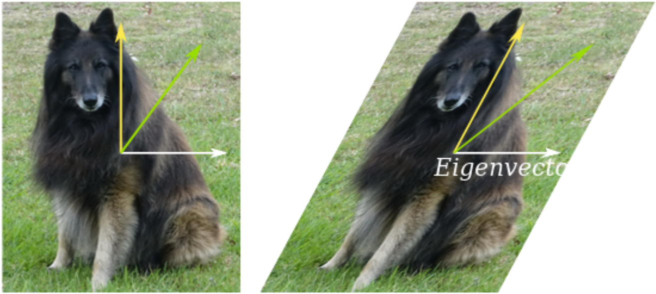
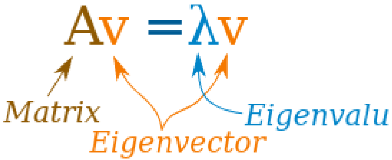
Eigenvalues and Eigenvectors

An **eigenvector** does not change direction in a transformation:



Mathematics

For a square matrix A, an Eigenvector and Eigenvalue make this quation true:



Example

For this matrix

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$$

and **eigenvetor** is

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

with corresponding **eigenvalue** of 6

Example

Check if $A\boldsymbol{v} = \lambda\boldsymbol{v}$ is true:

- $A\boldsymbol{v}$ gives us:

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -6 + 3 \times 4 \\ 4 \times 1 + 5 \times 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \end{bmatrix}$$

- $\lambda\boldsymbol{v}$ gives us

$$6 \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 24 \end{bmatrix}$$

Computing Eigenvalues and Eigenvectors

We can rewrite the condition $A\boldsymbol{v} = \lambda\boldsymbol{v}$ as

$$(A - \lambda I)\boldsymbol{v} = 0$$

where I is the $n \times n$ identity matrix. Now, in order for a non-zero vector $\boldsymbol{v}$ to satisfy this equation, $A - \lambda I$ must not be invertible. If $\boldsymbol{v}$ is non-zero then we can solve for λ using the determinant:

$$A - \lambda I = 0$$

Example

Let $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$, then

$$\begin{aligned} p(\lambda) &= det \begin{bmatrix} 2-\lambda & -4 \\ -1 & -1-\lambda \end{bmatrix} \\ &= (2-\lambda)(-1-\lambda) - (-4)(-1) \\ &= \lambda^2 - \lambda - 6 \\ &= (\lambda - 3)(\lambda + 2) \end{aligned}$$

Thus,$\lambda_1 = 3$ and $\lambda_2 = -2$ are eigenvalues of A.

Example

The matrix $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$ of previous example has eigenvalues $\lambda_1 = 3$ and $\lambda_2 = -2$.

Let's find the eigenvectors corresponding to $\lambda_1 = 3$.

Let $\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ Then, $(A - 3I)\boldsymbol{v} = 0$ gives us

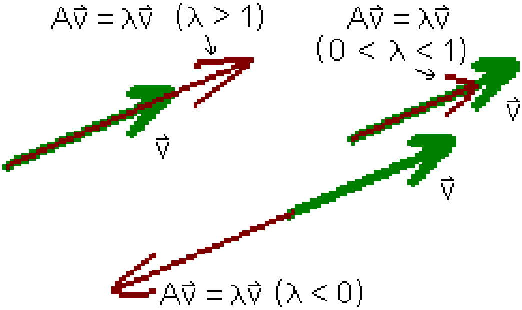
$$\begin{bmatrix} 2-3 & -4 \\ -1 & -1-3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

from which we obtain the duplicate equations

$$-v_1 - 4v_2 = 0$$
$$-v_1 - 4v_2 = 0$$

If we let $v_2 = t$, then $v_1 = -4t$. All eigenvectors corresponding to $\lambda_1 = 3$ are multiples of $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$, and thus the eigenspace corresonding to $\lambda_1 = 3$ is given by a span of $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$.

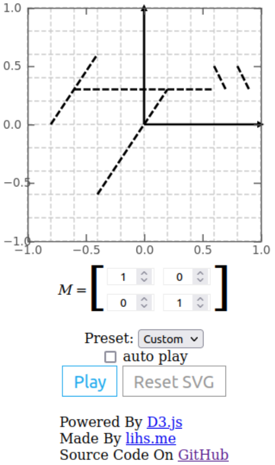
Example



$A\vec{v} = \lambda\vec{v}$ $(\lambda > 1)$
$\vec{v}$
$A\vec{v} = \lambda\vec{v}$
$(0 < \lambda < 1)$
$\vec{v}$
$\vec{v}$
$A\vec{v} = \lambda\vec{v}$ $(\lambda < 0)$

Linear Algebra by Animies

https://lihs.me/matrix-animation/



Matrix transformation: Diagonal matrix 1

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

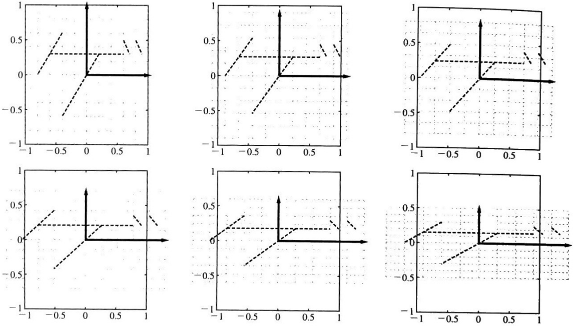


Figure 1: Preset:s0

Matrix transformation: Diagonal matrix 2

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$$

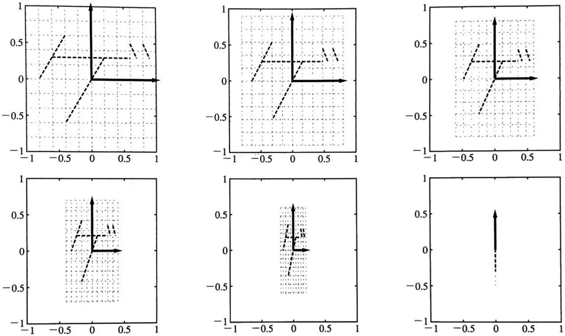


Figure 2: Preset:s1

Matrix transformation: Diagonal matrix 3

$$A = \begin{bmatrix} 1.5 & 0 \\ 0 & -0.5 \end{bmatrix}$$

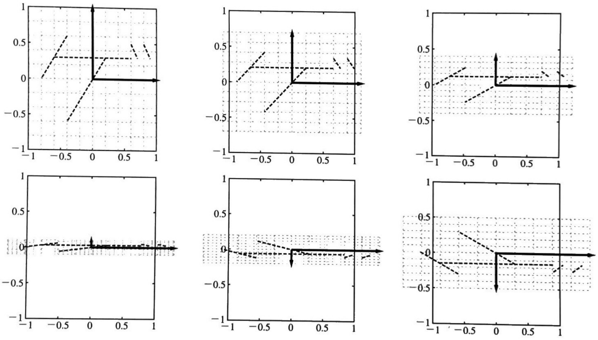


Figure 3: Preset:s2

Matrix transformation: nonDiagonal matrix

$$A = \begin{bmatrix} 1 & -0.3 \\ -0.7 & 0.6 \end{bmatrix}$$

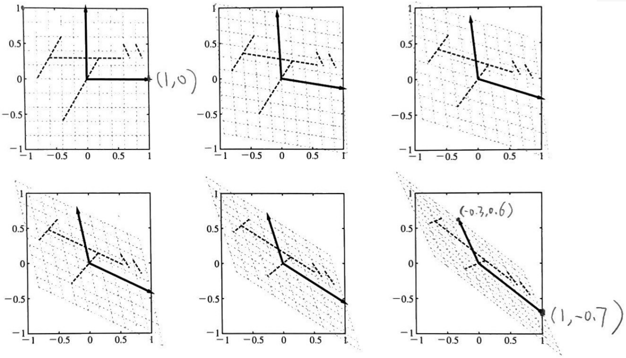


Figure 4: Preset:s3

Matrix transformation: nonDiagonal matrix mapping its eigenvectors

$$A = \begin{bmatrix} 1 & -0.3 \\ -0.7 & 0.6 \end{bmatrix}$$

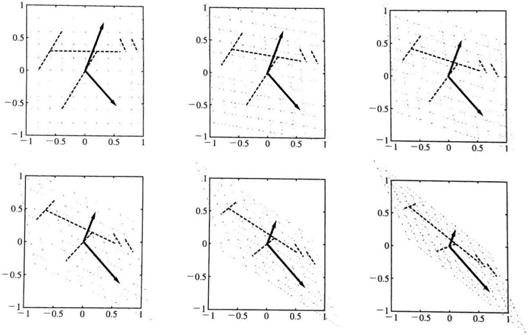


Figure 5: Preset:s4

Matrix transformation: rank-deficient matrix

$$A = \begin{bmatrix} 0.8 & -0.6 \\ 0.4 & -0.3 \end{bmatrix}$$

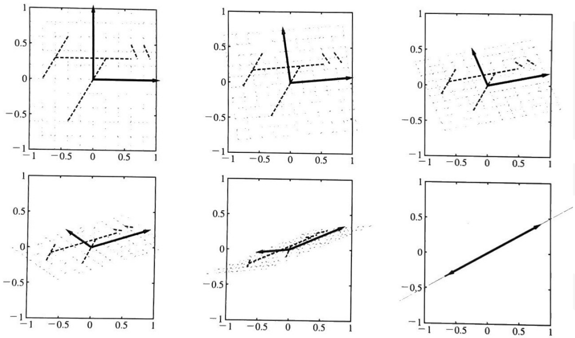


Figure 6: Preset:s6

Eigenvector and Eigenvalue

https://www.youtube.com/watch?v=PFDu9oVAE-g

Eigenfaces

Eigenfaces refers to an appearance-based approach to face recognition that seeks to capture the variation in a collection of face images and use this information to encode and compare images of individual faces in a holistic (as opposed to a parts-based or feature-based) manner.

Figure 7: The leftmost in the first row is the average face, the others are top two eigenfaces; the second row shows eigenfaces with least three eigenvalues.

Eigenfaces

The motivation of Eigenfaces is twofold:

- Extract the relevant facial information, which may or may not be directly related to human intuition of face features such as the eyes, nose, and lips. One way to do so is to capture the statistical variation between face images.
- Represent face images efficiently. To reduce the computation and space complexity, each face image can be represented using a small number of parameters.

Figure 8: training images

[1]

[1] https://www.geeksforgeeks.org/ml-face-recognition-using-eigenfaces-pca-algorithm/

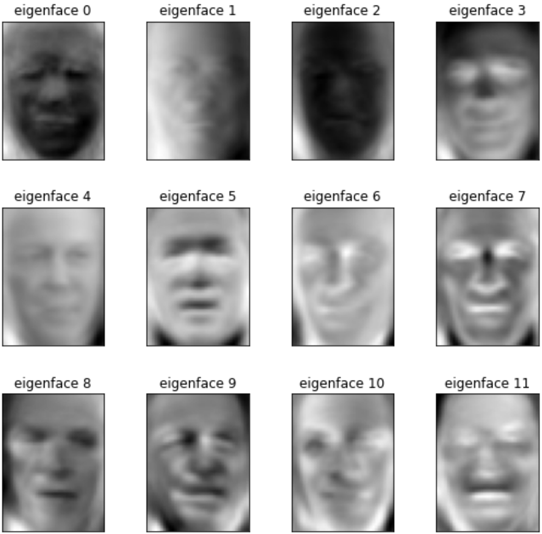


Figure 9: EigenFaces

Eigenfaces

Using Eigenfaces in Face Processing

- Face detection
- Face recognition

[2]

[2] http://www.scholarpedia.org/article/Eigenfaces

Fisherfaces

- Eigenfaces are the eigenvectors associated to the largest eigenvalues of the covariance matrix of the training data.
- When the goal is classification rather than representation, the LS solution may not yield the most desirable results.
- In such cases, one wishes to find a subspace that maps the sample vectors of the same class in a single spot of the feature representation and those of different classes as far apart from each other as possible. The techniques derived to achieve this goal are known as discriminant analysis (DA).

Fisherfaces

Figure 10: Shown here are the first four Fisherfaces from a set of 100 classes (subjects).

Fisherfaces

Algorithms? [4]

[4] https://docs.opencv.org/4.x/da/d60/tutorial_face_main.html

Face recognition with opencv

https://github.com/informramiz/opencv-face-recognition-pyth
blob/master/README.md

Thank You !

Q & A