Xi'an Jiaotong-Liverpool University

西交利物浦大学

PAPER CODE	EXAMINER	DEPARTMENT	TEL
DTS203TC		School of AI and Advanced Computing	

2nd SEMESTER 2022/23 RESIT EXAMINATION

Undergraduate - YEAR 3

DESIGN AND ANALYSIS OF ALGORITHMS

TIME ALLOWED: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. **There are a total of 3 pages numbered 1 through 3, please ensure that your copy of the examination is complete.**
2. **This is a closed-book examination, which is to be attempted without books or notes.**
3. **Total marks available are 100.**
4. **Only the university approved calculator – Casio FS82ES/83ES – can be used.**
5. **This exam consists of SIX questions. You are required to answer ALL questions.**
6. **Answer should be written in the answer booklet(s) provided.**
7. **Only English solutions are accepted.**
8. **All materials must be returned to the exam supervisor upon completion of the exam. Failure to do so will be deemed academic misconduct and will be dealt with accordingly**.

Question 1. [2+2+3+3=10 MARKS]

Give an asymptotically tight bound (Θ notation) to each of the following functions and recurrences. You need not justify your answers.

i. $n^3 + 10n^2 - 100n + 1000$ is Θ (____). **[2 marks]**
ii. $logn + loglogn$ is Θ (____). **[2 marks]**
iii. $T(n) = 4T(n/2) + \Theta(n)$ is Θ (____). **[3 marks]**
iv. $T(n) = 7T(n/2) + n^2$ is Θ (____). **[3 marks]**

Question 2. [4+6+14= 24 MARKS]

Given an array representation of a min heap A = [4,5,6,15,9,7,20,16,25,14,12,11,8]

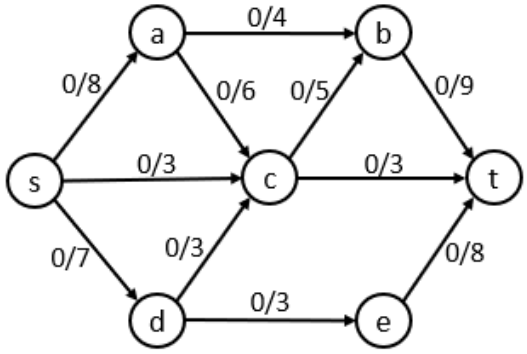
i. Draw the corresponding binary tree representation. **[4 marks]**
ii. Show the steps for replacing 5 with 18 in the given min heap A. **[6 marks]**
iii. Let T be a min heap storing n keys. Give an efficient algorithm for reporting all the keys in T that are smaller than or equal to a given query key x (which is not necessarily in T). For example, given the heap A and query key x =7, the algorithm should report 4, 5, 6, 7. Note that the keys do not need to be reported in sorted order. Your algorithm should run in O(k) time, where k is the number of keys reported. Describe your algorithm using pseudo-code and explain why it runs in O(k) time. **[14 marks]**

Question 3. [14 MARKS]

Given two lists X and Y, each containing *n* numbers already in sorted order. Design an efficient algorithm to find the median of all *2n* elements in arrays X and Y, and analyse the time complexity of your algorithm. **[14 marks]**

Question 4. [4+10+6 = 20 MARKS]

The figure below describes a flow network.



i. Apply breadth-first search and depth-first search to traverse the given network starting from node 's'. Show the order of exploration. **[4 marks]**
ii. Use Ford-Fulkerson method to find the maximum flow of the network. For each step of the algorithm, report the augmenting path by listing its vertices and its residual capacity. **[10 marks]**
iii. What is the value of the maximum flow? Show the flow network that gives the maximum flow and draw the minimum cut. **[6 marks]**

Question 5. [4+6+6 = 16 MARKS]

The RSA Encryption Scheme is often used to encrypt and decrypt electronic communications. Suppose Alice wants her friends to encrypt email messages before sending them to her. Alice chooses two prime numbers p=13 and q=17, and publishes her public key (e,n)=(35,221).

i. Use Euler's totient function ϕ to count the number of integers between 1 and 220 which are relatively prime to 221. **[4 marks]**
ii. Use the extended Euclidean algorithm to find Alice's private key (d,n). d is the multiplicative inverse of e in mod ϕ(221) **[6 marks]**
iii. Alice received an encrypted number 13 from Bob. What is the original integer chosen by Bob? Use repeated squaring method to find the integer. **[6 marks]**

Question 6. [10+6 = 16 MARKS]

i. Consider the following problems:
Subset Sum problem: Given non-negative integer numbers $s_1, s_2, \cdots, s_n$ and a target value T. Is there a subset of these numbers that adds up to T?
0-1 knapsack problem: Given non-negative weights $w_1, w_2, \cdots, w_n, W$, and profits $v_1, v_2, \cdots, v_n, V$. Is there a subset of weights with total weight at most W, such that the corresponding profit is at least V?
Show that 0-1 knapsack problem is NP-complete using a reduction from Subset Sum problem.
[10 marks]

ii. 0-1 Knapsack problem can be solved in O(nW) time by dynamic programming (n is the number of items and W is the maximum weight). This would mean P = NP. Is this statement true? Justify your answer. **[6 marks]**

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