Xi'an Jiaotong-Liverpool University

西交利物浦大学

PAPER CODE	EXAMINER	DEPARTMENT	TEL
DTS203TC		School of AI and Advanced Computing	

2nd SEMESTER 2022/23 FINAL EXAMINATION

Undergraduate - YEAR 3

DESIGN AND ANALYSIS OF ALGORITHMS

TIME ALLOWED: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. **There are a total of 4 pages numbered 1 through 4, please ensure that your copy of the examination is complete.**
2. **This is a closed-book examination, which is to be attempted without books or notes.**
3. **Total marks available are 100.**
4. **Only the university approved calculator – Casio FS82ES/83ES – can be used.**
5. **This exam consists of SIX questions. You are required to answer ALL questions.**
6. **Answer should be written in the answer booklet(s) provided.**
7. **Only English solutions are accepted.**
8. **All materials must be returned to the exam supervisor upon completion of the exam. Failure to do so will be deemed academic misconduct and will be dealt with accordingly.**

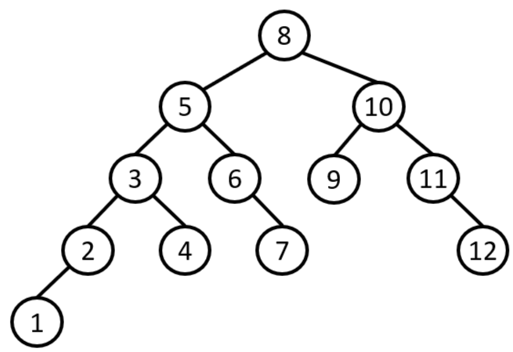
Question 1. [2+2+3+3=10 MARKS]

Give an asymptotically tight bound (Θ notation) to each of the following functions and recurrences. You need not justify your answers.

i. $\sqrt{n} + logn$ is Θ (____). **[2 marks]**
ii. $n + nlogn + nlog^2n$ is Θ (____). **[2 marks]**
iii. $T(n) = 2T(n/4) + \Theta(1)$ is Θ (____). **[3 marks]**
iv. $T(n) = 9T(n/3) + \Theta(n^2)$ is Θ (____). **[3 marks]**

Question 2. [6+8+4=18 MARKS]

Given the following AVL Tree:



i. List the sequence of nodes visited by preorder, inorder, and postorder traversals of the given tree. **[6 marks]**
ii. Draw the resulting AVL tree after 8 is removed. (Replace the node with its successor). Draw a new tree for each rotation that occurs when rebalancing the AVL tree. **[8 marks]**
iii. What are the smallest and largest heights that an AVL tree with 50 nodes might have? **[4 marks]**

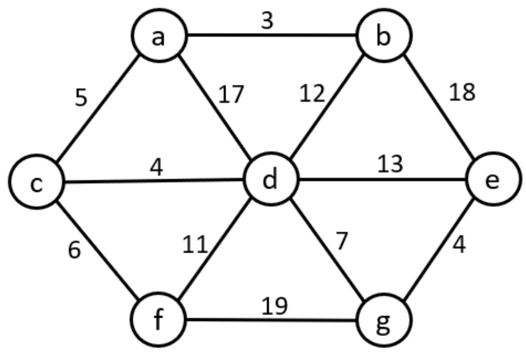
Question 3. [15 MARKS]

Given six items with weights and values shown in the table below.

Item	1	2	3	4	5	6
Weight	1	2	4	5	2	1
Value	12	20	15	35	10	9

Assume we have a knapsack that can hold items with a total weight of 10. What is the best solution to the 0-1 knapsack problem? Solve the problem with dynamic programming. **[15 marks]**

Question 4. [3+9+2+6=20 MARKS]



i. Draw the adjacency matrix of this graph. **[3 marks]**

ii. Step through Dijkstra's Algorithm to calculate the single source shortest path from vertex *a* to every other vertex. You need to show your steps in the table below for full credit. Show your steps by crossing out values that are replaced by a new value. **[9 marks]**

iii. What is the shortest path from a to e? **[2 marks]**

iv. Find the **maximum** spanning tree of the graph above. You can use Kruskal's algorithm if you switch "smallest" with "biggest" when examining edges. List the edges of the maximum spanning tree of the graph in the order that they are added and draw the resulting maximum spanning tree. **[6 marks]**

Question 5. [12+12=24 MARKS]

i. A prefix is a collection of characters at the beginning of a string. For example, "al" is a prefix of "algorithm" and the longest common prefix between "all", "also", "always", and "algorithm" is "al". Use the idea of Divide and Conquer to design an algorithm that find the longest common prefix string amongst an array of strings and analyse its time complexity. **[12 marks]**

ii. A string is a palindrome when it reads the same backward as forward (e.g., "aba" is a palindrome while "abc" is not). Design an efficient algorithm to find the Longest Palindrome Prefix (LPP) of a string (e.g., "ana" is the LPP of the word "analysis") and analyse the complexity of your algorithm. **[12 marks]**

Question 6. [4+9 = 13 MARKS]

i. Suppose we know A≤B (i.e., A reduces to B in linear time) and B≤C where A, B, and C are all problems. Further, suppose we know that B can be solved in Θ(nlogn) time. What can be said about the running time for A and C? **[4 marks]**

ii. In a graph, a cycle is a path that comes back to its initial vertex. A simple cycle is a cycle with no repeated vertices. A Hamiltonian Cycle (HC) is a simple cycle that visits every vertex of the graph. Consider the following two problems; each takes as input a graph G with n vertices.
<u>Problem HC</u>: Does G have a Hamiltonian cycle?

Problem B: Does G have a simple cycle of length exactly floor(n/2)? (The floor function rounds its input down to the nearest integer.)
Problem HC is known to be NP-complete. Show that Problem B is also NP-complete. **[9 marks]**

--- END OF THE EXAM PAPER ---