Preliminary Problems

These are some basic problems in introductory physics. The length of this problem set is representative of those in the course, though the questions are easier. The purpose of this problem set is to assess your general physics knowledge, and also to help guide your learning if you're still picking up the basics. Email your solutions by **8/23** to kzhou7@gmail.com.

1 Mechanics

The following mechanics problems can be solved using the material in chapters 1 through 17 of Halliday and Resnick.

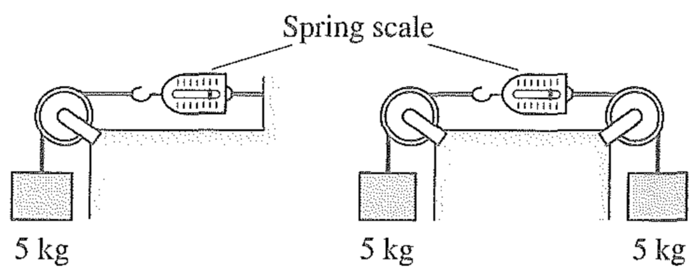
[2] **Problem 1.** As a warmup, take the Force Concept Inventory test. This should take no longer than 30 minutes. You do not have to justify your answers; just list them.

[1] **Problem 2.** A sewage worker is using a ladder inside a large, frictionless, horizontal circular aqueduct. The ladder is of the same length as the diameter of the aqueduct.

(a) First the ladder is placed perfectly vertically and the worker climbs to the midpoint. Draw a free body diagram indicating all forces on the ladder, and their names. Do the forces balance?

(b) Now suppose the ladder is placed perfectly horizontally and the worker hangs from the midpoint. Draw a free body diagram indicating all forces on the ladder, and their names. Do the forces balance?

[1] **Problem 3.** Consider the two following setups involving pulleys and spring scales. Treat the ropes and spring scales as massless and the pulleys as frictionless. Model the pulleys as uniform discs with masses of 5 kg which are fixed by a rigid support.



(a) Draw a free-body diagram for the second setup, showing all external forces on the spring scale.

(b) What are the readings on the two spring scales?

(c) Draw a free-body diagram for the first setup, showing all external forces on the pulley.

(d) What is the magnitude of the force that must be provided by the support?

[1] **Problem 4.** A projectile is thrown upward and passes a point A and a point B a height h above. Let T_A be the time interval between the two times the projectile passes point A, and define T_B similarly.

(a) Show that g can be measured as

$$g = \frac{8h}{T_A^2 - T_B^2}.$$

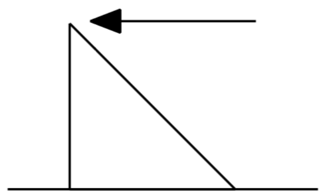
(b) This procedure probably looks a little contrived. Why is it better than doing something simpler, such as just dropping the ball and using $\Delta y = gt^2/2$?

[2] **Problem 5.** Consider a projectile launched with speed v at an angle θ from the horizontal on a flat plane.

(a) Find $y(x)$ and the ratio of the range to the maximum height.

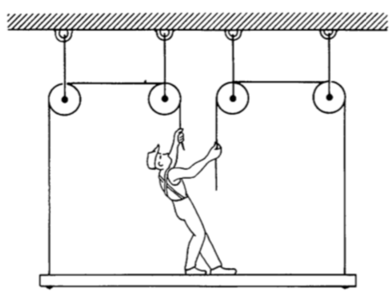
(b) What is the maximum θ for which the projectile is always moving away from the thrower?

[2] **Problem 6.** A wooden isosceles right triangle with uniform mass density is placed on a table, and a force is applied as shown.



The force is gradually increased until the triangle begins to tip over without sliding. The force is then removed. Next, the surface is inclined with angle θ. For what range of θ can you be certain the triangle will not slide down the incline?

[1] **Problem 7.** A painter of mass M stands on a platform of mass m as shown.



He pulls each rope down with force F, and accelerates upward with acceleration a. Find a.

[2] **Problem 8.** A small block lies at the bottom of a frictionless spherical bowl of radius R.

(a) Find the period of small oscillations. Can you give an intuitive explanation of the simplicity of your answer?

(b) What is the period if the block is replaced with a small uniform ball, which rolls without slipping?

[3] **Problem 9.** A cue ball is a uniform sphere of radius R.

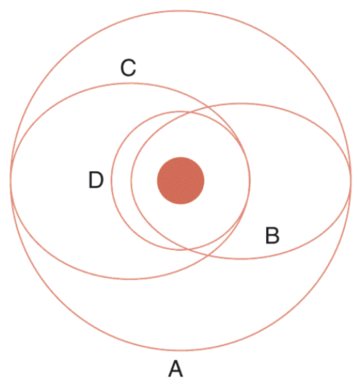
(a) Find the height at which the cue ball must be hit horizontally so that it immediately begins rolling without slipping.

(b) If the cue ball is hit below this height, it will decelerate due to friction. Skillful players can hit the cue ball so that it ends up with a *backwards* velocity. What is the general criterion for a hit to be able to do this? Justify your answer carefully.

[2] **Problem 10.** A car accelerates uniformly from rest. Initially, its door is slightly ajar. Calculate how far the car travels before the door slams shut. Assume the door has a frictionless hinge, a uniform mass distribution, and a length L from front to back.

[2] **Problem 11.** A baseball player holds a baseball bat, modeled as a uniform rod, horizontally at one of its ends. Usually, when the baseball hits the bat, the player will momentarily feel a "sting" in their hands. However, there is no sting if the baseball hits the "sweet spot". Where is it?

[2] **Problem 12.** Several possible elliptical orbits of a satellite are shown below.

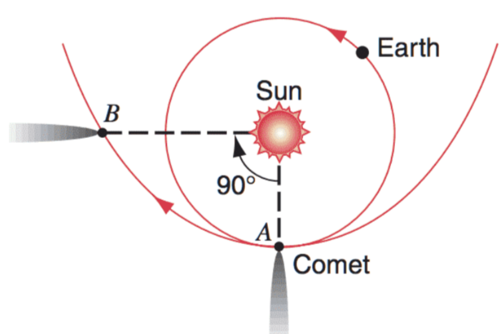


(a) Which orbit has the largest angular momentum?

(b) Which orbit has the largest total energy?

(c) On which orbit is the largest speed acquired?

[2] **Problem 13.** A comet passes by Sun as shown, in a parabolic path.



How long, in years, does the comet take to get from point A to point B? (Hint: if you apply Kepler's laws and properties of conics, this problem can be done with almost no computation.)

[2] **Problem 14.** Because of the rotation of the Earth, the line of a plumb bob will not quite align with the local gravitational field. Find the angle of deviation between them as a function of the latitude θ.

[2] **Problem 15.** An entrepreneur proposes to propel the Earth through space by attaching many balloons to one side of it with ropes. The balloons will experience a buoyant force, which will create a tension in the ropes, which will pull on the Earth. Will this scheme work? If you think it does, explain why it doesn't violate conservation of momentum. If you think it doesn't, show explicitly how the tension force on the Earth is cancelled.

2 Problem Solving Skills

You should be able to answer these questions without any background reading. However, for some fun background on estimation, see *Guesstimation: Solving the World's Problems on the Back of a Napkin*. For practical tips for real experiments, see chapter 7 of *Physics Olympiad: Basic to Advanced Exercises*.

[2] **Problem 16.** Argon atoms are special because they stay in the atmosphere for a very long time. They are not recycled like oxygen and nitrogen. An average breath inhales around 0.5 L of air and people breath on average around once every five seconds. Air is about 1% argon and has density $1.2\,\text{kg/m}^3$. Assume all air particles have a mass of approximately 5×10^{-26} kg. Take the atmosphere to have constant density and be around 20 km thick. The radius of the Earth is 6.4×10^6 m.

(a) Estimate the total number of distinct argon atoms inhaled by Galileo throughout their life.

(b) Assuming the atmosphere has been uniform mixed since then, estimate the number of argon atoms in each of your breaths that were once in Galileo's lungs.

[2] **Problem 17.** The acceleration due to gravity can be measured by measuring the time period of a simple pendulum. However, it can be challenging to get an accurate result.

(a) Suppose you constructed a pendulum using regular household materials. Name at least five sources of possible experimental error in your calculated value of g. How would you make the pendulum and perform the measurements to minimize these sources of error?

(b) Make an actual pendulum yourself and carry out the measurement. Describe your experimental procedure, show your data, and give a value of g with a reasonable uncertainty. Formal error analysis is not required.

[2] **Problem 18.** Blackbody radiation is an electromagnetic phenomenon, so the radiation intensity depends on the speed of light c. It is also a thermal phenomenon, so it depends on the thermal energy $k_B T$, where T is the object's temperature and k_B is Boltzmann's constant. And it is a quantum phenomenon, so it depends on Planck's constant h.

(a) Using the relation $E = hf$, find the dimensions of h.

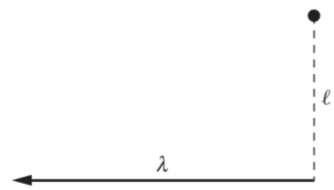
(b) Using dimensional analysis, show that the power emitted by the blackbody per unit area, called the radiation intensity I, obeys $I \propto T^4$, and find the constant of proportionality up to a dimensionless constant.

(c) How would the result change in a world with d spatial dimensions?

3 Electromagnetism

The following electromagnetism problems can be solved using the material in chapters 25 through 38 of Halliday and Resnick.

[2] **Problem 19.** A half-infinite line has linear charge density λ.



(a) Find the electric field at a point that is "even" with the end, a distance ℓ from it, as shown.

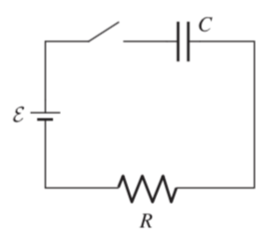
(b) You should find the direction of the field is independent of ℓ. Explain why.

[2] **Problem 20.** A parallel plate capacitor of capacitance C is placed in a region of zero electric field. The first plate is given total charge Q_1 and the second plate is given total charge Q_2.

(a) Each of the plates has an inner and outer surface. Using Gauss's law, find the total charge on each of these four surfaces.

(b) Find the potential difference between the plates.

[2] **Problem 21.** A battery is connected to an RC circuit as shown.



The switch is initially open, and the charge on the capacitor is initially zero. The switch is closed at $t = 0$.

(a) Solve for the charge on the capacitor as a function of time.

(b) Solve for the power dissipated in the resistor as a function of time.

(c) What is the total power dissipated in the resistor over all time? Can you find a simple way to derive this result?

[2] **Problem 22.** In this problem we estimate the maximum firing speed of a human neuron. Model a human cell simply as a sphere of radius 10^{-6} m.

(a) It has been measured that $1\,\text{cm}^2$ of cell membrane has a resistance of $1000\,\Omega$. Estimate the resistance of a single human cell.

(b) Estimate the capacitance of a single human cell, treating the two sides of the membrane as capacitor plates.

(c) By modeling the cell as an RC circuit, estimate the maximum firing speed of a human neuron. Is this a reasonable result? If yes, how do you know? If not, how could this model be refined?

[2] **Problem 23.** An infinite solenoid with radius b has n turns per unit length. The current varies in time according to $I(t) = I_0 \cos \omega t$. A ring with radius $r < b$ and resistance R is centered on the solenoid's axis, with its plane perpendicular to the axis.

(a) What is the induced current in the ring?

(b) A given little piece of the ring will feel a magnetic force. For what values of t is this force maximum?

(c) What is the effect of the force on the ring? That is, does the force cause the ring to translate, spin, etc.?

[1] **Problem 24.** A 120 V rms, 60 Hz line provides power to a 40 W light bulb. By what factor will the brightness decrease if a 10 μF capacitor is connected in series with the light bulb?

4 Thermodynamics

The following thermodynamics problems can be solved using the material in chapters 21 through 24 of Halliday and Resnick.

[2] **Problem 25.** Two moles of a monatomic ideal gas are taken through the following cycle.

- The gas begins at point A with pressure P_0 and volume V_0.
- The gas is heated at constant volume until it doubles its pressure, reaching point B.
- The gas is expanded at constant pressure until it doubles its volume, reaching point C.
- The gas is cooled at constant volume until it halves its pressure, reaching point D.
- The gas is compressed at constant pressure until it halves its volume, returning to point A.

Assume that all processes are quasistatic and reversible.

(a) Draw the process on a PV diagram.

(b) Calculate the net work done by the gas during the cycle.

(c) Calculate the efficiency of the cycle.

(d) Calculate the change in entropy of the gas as the system goes from state A to state D.

[2] **Problem 26.** Deriving some basic results in thermodynamics.

(a) Starting from the first law of thermodynamics, derive the fact that PV^γ is constant in an adiabatic process.

(b) Using the ideal gas law, derive the total work done by a gas as it expands at constant temperature from volume V_1 to V_2, in terms of n, R, T, V_1, and V_2.

(c) Show that if a general gas, not necessarily ideal, satisfies the equation $PV = kU$, where U is the total internal energy, then PV^n is constant in an adiabatic process for some power n, and find n in terms of k.

(d) Does an ideal gas satisfy $PV = kU$? If so, what is k?

[1] **Problem 27.** A monatomic gas is adiabatically compressed to 1/8 of its original volume. For each of the following quantities, indicate by what factor they change.

(a) The rms velocity v_{rms}.

(b) The mean free path λ.

(c) The average time between collisions τ for each gas molecule.

(d) The molar heat capacity C_v.

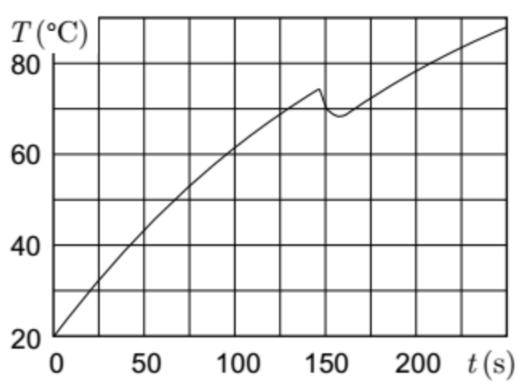
[2] **Problem 28.** A simple heat engine consists of a movable piston in a cylinder filled with an ideal monatomic gas. Initially the gas in the cylinder is at a pressure P_0 and volume V_0. The gas is slowly heated at constant volume until the pressure is $32P_0$. The gas is then adiabatically expanded until its pressure is P_0 again. Finally, the gas is cooled at constant pressure until its volume is V_0 again. Find the efficiency of the cycle.

[2] **Problem 29.** The total mass of a hot-air balloon (envelope, basket, and load) is 320 kg. Initially the air pressure inside and outside the envelope is 1.01×10^5 Pa and its density is $1.29\,\text{kg/m}^3$. In order to raise the hot-air balloon, a gas burner is used to heat the air inside the balloon. The volume of the envelope filled with hot air is $650\,\text{m}^3$. Treat the temperature of the air in the balloon as uniform throughout.

(a) The balloon can either be tightly sealed, so that none of its air mixed with the outside air, or have a hole, so that its pressure equalizes with that of the outside air. For the purposes of generating lift, which is better?

(b) Assuming the better option has been taken, to what temperature must the air inside the balloon be heated to make the balloon begin to rise?

[2] **Problem 30.** Water is heated in an electric kettle. At a certain moment of time, a piece of ice at temperature $T_0 = 0\,°\text{C}$ was put in the kettle. The figure below shows the water temperature as a function of time.



Find the mass of the ice if the heating power of the kettle is $P = 1\,\text{kW}$. The latent heat of melting for ice is $L = 335\,\text{kJ/kg}$, the heat capacity of water is $c = 4.2\,\text{kJ/kg K}$, and the temperature of the room is $T_1 = 20\,°\text{C}$.