Electromagnetism III: Magnetostatics

Chapters 4 and 6 of Purcell cover DC circuits and magnetostatics, as does chapter 5 of Griffiths. For advanced circuits techniques, see chapter 9 of Wang and Ricardo, volume 2. Chapter 5 of Purcell famously derives magnetic forces from Coulomb's law and relativity. It's beautiful, but not required to understand chapter 6; we will cover relativistic electromagnetism in depth in **R3**. For further discussion, see chapters II-12 through II-15 of the Feynman lectures. There is a total of **79** points.

1 Static DC Circuits

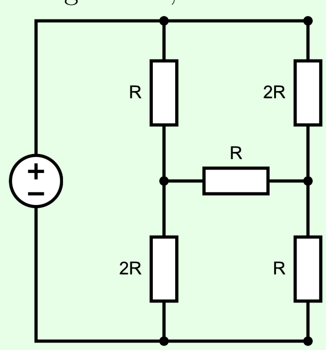
We continue with DC circuits, in more complex setups than in **E2**.

Idea 1

When analyzing circuits, it is sometimes useful to parametrize the currents in the circuits in terms of the current in each independent loop. This is typically more efficient, because it enforces Kirchoff's junction rule automatically, leading to fewer equations.

Example 1: Wheatstone Bridge

Find the current through the following circuit, if the battery has voltage V.



Solution

This circuit can't be simplified using series and parallel combinations, so instead we use Kirchoff's rules directly. From the diagram, we see the circuit has three loops. Let I_1 be the clockwise current on the left loop, I_2 be the clockwise current through the top-right loop, and I_3 be the clockwise current through the bottom-right loop. For instance, this means that the current flowing downward through the top-left resistor is $I_1 - I_2$.

The three Kirchoff's loop rule equations are

$$3I_1R - I_2R - 2I_3R = V,$$
$$4I_2R - I_1R - I_3R = 0,$$
$$4I_3R - 2I_1R - I_2R = 0.$$

Adding the last two equations shows that

$$I_1 = I_2 + I_3$$

and plugging this back in shows that $3I_2 = 2I_3$, so we have

$$I_2 = \frac{2}{5} I_1, \quad I_3 = \frac{3}{5} I_1.$$

Since the answer to the question is just I_1, we can now plug this back into the first equation,

$$\frac{V}{R} = 3I_1 - I_2 - 2I_3 = \left(3 - \frac{2}{5} - \frac{6}{5}\right) I_1 = \frac{7}{5} I_1.$$

This gives the answer, $5V/7R$.

Incidentally, the circuit above is also called a Wheatstone bridge. We note that the current through the middle resistor is zero when the ratios between the top and bottom resistances match on both sides of it. Hence if three of these outer resistances are known, we can adjust one of them until the current through the middle resistor vanishes, thereby measuring the fourth resistor.

Idea 2

Since Kirchoff's loop equations are linear, currents and voltages in a DC circuit with multiple batteries can be found by superposing the currents and voltages due to each battery alone.

Idea 3: Thevenin's Theorem and Norton's Theorem

Consider any system of batteries and resistors, with two external terminals A and B. Suppose that when a current I is sent into A and out of B, then a voltage difference $V = V_A - V_B$ appears. From an external standpoint, the function $V(I)$ is all we can measure.

Now, by the linearity of Kirchoff's rules, $V(I)$ is a linear function, so we can write

$$V(I) = V_{\text{eq}} + IR_{\text{eq}}.$$

In other words, $V(I)$ is exactly the same as if the entire system were a resistor R_{eq} in series with a battery with emf V_{eq} (with the positive end pointing towards A). This generalizes the idea of replacing a system of resistors with an equivalent resistance, and is known as Thevenin's theorem.

We can also flip this around. Note that $I(V)$ must also be a linear function, and we can write

$$I(V) = I_{\text{eq}} + \frac{V}{R_{\text{eq}}}.$$

This is precisely the $I(V)$ of an ideal current source I_{eq} (sending current towards B) in parallel with a resistor R_{eq}. (An ideal current source makes a fixed current flow through it, just like a battery creates a fixed voltage across it.) This is known as Norton's theorem.

Since these functions are inverses of each other, you can see that the R_{eq}'s in both equations above are the same (both are equal to the ordinary equivalent resistance), and $V_{\text{eq}} = -I_{\text{eq}} R_{\text{eq}}$.

Example 2

Consider some batteries connected in parallel, with emfs $\mathcal{E}_i$ and internal resistances R_i. What is the Thevenin equivalent of this circuit?

Solution

The equivalent resistance is simply

$$R_{\text{eq}} = \left(\sum_i \frac{1}{R_i} \right)^{-1}.$$

To infer V_{eq}, we just need one more $V(I)$ value. The most convenient is to set $V = 0$, shorting all of the batteries. Each battery alone would produce a current of $\mathcal{E}_i / R_i$, so

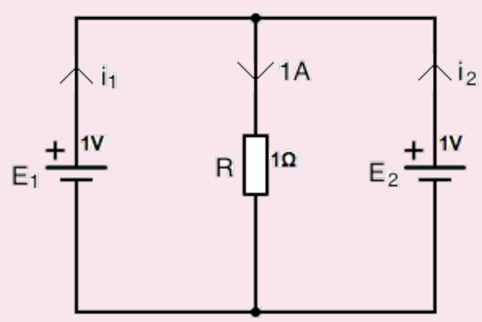
$$0 = V_{\text{eq}} + \left(\sum_i \frac{\mathcal{E}_i}{R_i} \right) R_{\text{eq}}.$$

Thus, we have

$$V_{\text{eq}} = \left(\sum_i \frac{\mathcal{E}_i}{R_i} \right) \left(\sum_j \frac{1}{R_j} \right)^{-1}.$$

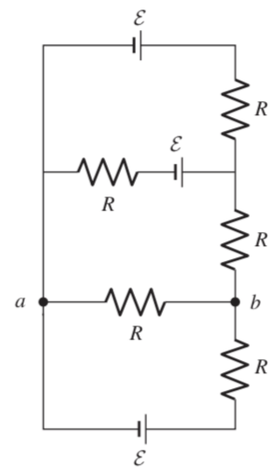
Remark

With ideal batteries, it's easy to set up circuits that don't make any sense.



For example, in the above circuit, Kirchoff's rules don't determine the currents; they only say that $i_1 + i_2 = 1\,\text{A}$. If the emfs of the batteries were different, the situation would be even worse: the equations would be contradictory, with no solution at all! In real life, this is avoided because all batteries have some internal resistance. Adding such a resistance to each battery, no matter how small, resolves the problem and gives a unique solution.

[2] **Problem 1** (Purcell 4.12)**.** Consider the circuit below.



(a) Find the potential difference between points a and b.

(b) Find the equivalent Thevenin resistance and emf between points a and b.

[2] **Problem 2** (Wang)**.** A circuit containing batteries and resistors has two terminals. When an ideal ammeter is connected between them, the reading is I_1. When a resistor R is connected between them, the current through the resistor is I_2, in the same direction. What would be the reading V of an ideal voltmeter connected between them?

[3] **Problem 3.** USAPhO 2015, problem A2.

Now we give a few problems on current flow through continuous objects. Fundamentally, all one needs for these problems is the definition $\mathbf{J} = \sigma \mathbf{E}$, and superposition.

Example 3

Consider two long, concentric cylindrical shells of radii $a < b$ and length L. The volume between the two shells is filled with material with conductivity $\sigma(r) = k/r$. What is the resistance between the shells, and the charge density?

Solution

To find the resistance, we compute the current I when a voltage V is applied between the shells. By symmetry, in the steady state the current density must be

$$\mathbf{J}(\mathbf{r}) = \frac{I}{2\pi r L}\,\hat{\mathbf{r}}.$$

On the other hand, we also know that

$$V = \int \mathbf{E} \cdot d\mathbf{r} = \int_a^b \frac{I}{2\pi r L \sigma}\, dr = \frac{I(b-a)}{2\pi k L}$$

from which we conclude

$$R = \frac{b-a}{2\pi k L}.$$

Note that the radial electric field between the shells is constant, so

$$\mathbf{E}(\mathbf{r}) = \frac{V}{b-a}\,\hat{\mathbf{r}}.$$

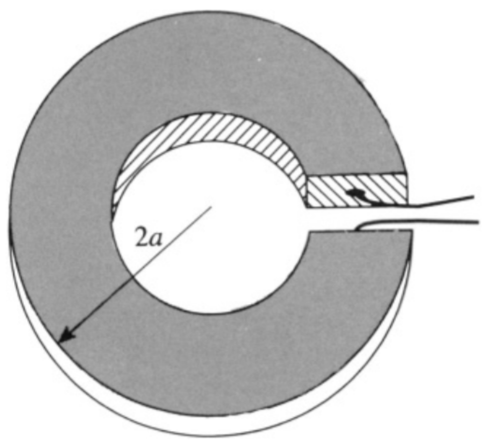
This means that in the steady state, there must be a nonzero charge density between the shells. (If there weren't, then we would have $E(r) \propto 1/r$, rather than a constant.)

To find the charge density explicitly, it's easiest to use Gauss's law in differential form in cylindrical coordinates. We use the form of the divergence derived in **E1**,

$$\nabla \cdot \mathbf{E} = \frac{1}{r}\frac{\partial(rE_r)}{\partial r} + (\text{other terms}) = \frac{1}{r}\frac{V}{b-a} = \frac{\rho}{\epsilon_0}$$

thus showing that the charge density is proportional to $1/r$. Of course, we could also get this result by applying Gauss's law in integral form, to concentric spheres.

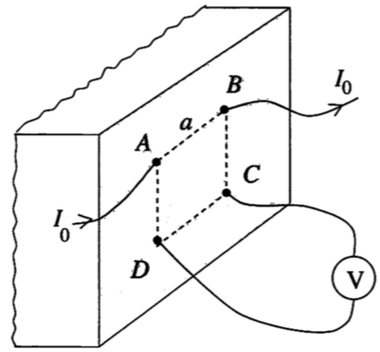
[2] **Problem 4** (Grad)**.** A washer is made of a material of resistivity ρ. It has a square cross section of length a on a side, and its outer radius is $2a$. A small slit is made on one side and wires are connected to the faces exposed.



Find the resistance of the washer. (Hint: first argue that no current flows radially.)

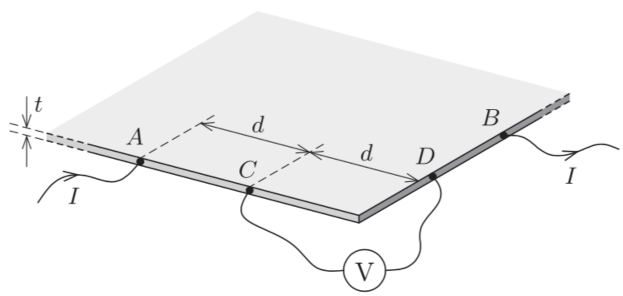
[3] **Problem 5** (BAUPC 1995)**.** An electrical signal can be transferred between two metallic objects buried in the ground, where the current passes through the Earth itself. Assume that these objects are spheres of radius r, separated by a horizontal distance $L \gg r$, and suppose both objects are buried a depth much greater than L in the ground. If the Earth has uniform resistivity ρ, find the approximate resistance between the terminals. (Hint: consider the superposition principle.)

[3] **Problem 6** (PPP 162)**.** A plane divides space into two halves. One half is filled with a homogeneous conducting medium, and physicists work in the other. They mark the outline of a square of side a on the plane and let a current I_0 in and out at two of its neighboring corners. Meanwhile, the measure the potential difference V between the two other corners.



Find the resistivity ρ of the medium.

[3] **Problem 7** (MPPP 174)**.** We aim to measure the resistivity of the material of a large, thin, homogeneous square metal plate, of which only one corner is accessible. To do this, we chose points A, B, C and D on the side edges of the plate that form the corner.



Points A and B are both $2d$ from the corner, whereas C and D are each a distance d from it. The length of the plate's sides is much greater than d, which, in turn, is much greater than the thickness t of the plate. If a current I enters the plate at point A, and leaves it at B, then the reading on a voltmeter connected between C and D is V. Find the resistivity ρ of the plate material.

Remark

Setups like those in the previous two problems are commonly used to measure resistivities, but why do they use a complicated "four terminal" setup? Wouldn't it have been easier to just attach two terminals, send a current I through them, and measure the voltage drop V? The problem with this is that it also picks up the resistance R of the contacts between the terminals and the material, along with the resistances of the wires. By having a pair of terminals measure voltage alone, drawing negligible current, we avoid this problem.

[4] **Problem 8.** [A] This problem is just for fun; the techniques used here are too advanced to appear on Olympiads. We will prove Rayleigh's monotonicity law, which states that increasing the resistance of any part of a resistor network increases the equivalent resistance between any two points. This may seem obvious, but it's actually tricky to prove. The following is the slickest way.

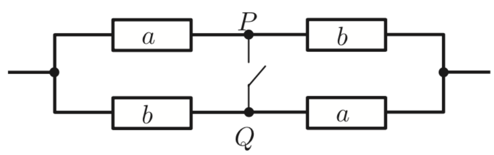
(a) Consider a graph of resistors, where a battery is attached across two of the vertices, fixing their voltages. Write an expression for the total power dissipated, assuming the voltages at each vertex are V_i and the resistances are R_{ij}.

(b) The voltages V_i at all the other vertices are determined by Kirchoff's rules. But suppose you didn't know that, or didn't want to set up those equations. Remarkably, it turns out that you can derive the exact same results by simply treating the voltages V_i as free to vary, and setting them to minimize the total power dissipated! Show this result. (This is an example of a variational principle, like the principle of least action in mechanics.)

(c) For any network of resistors, show that $P = V^2/R$ when V is the battery voltage applied across two vertices, R is the equivalent resistance between them, and P is the total power dissipated in the resistors. (This is intuitive, but it's worth showing in detail to assist with the next part.)

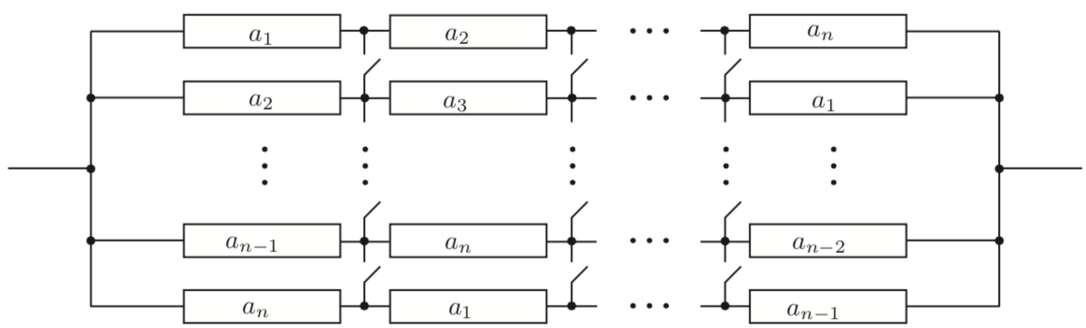
(d) By combining all of these results, prove Rayleigh's monotonicity law.

(e) We can use Rayleigh's monotonicity law to prove some mathematical results. Consider the resistor network shown below, where the variables label the resistances.



By considering the resistances before and after closing the switch PQ, show that the arithmetic mean of two numbers is at least the geometric mean.

(f) Consider the resistor network shown below.



By closing all the switches, show that the arithmetic mean of n numbers is at least the harmonic mean.

Remark

You might think that Rayleigh's monotonicity law is too obvious to require a proof; if you decrease a resistance, how could the net resistance possibly go up? In fact, this kind of non-monotonicity occurs very often! For example, Braess's paradox is that fact that adding more roads can slow down traffic, even when the total number of cars stays the same. A U.S. Physics Team coach has argued that allowing more team strategies can make a basketball team score less. For more on this subject, see the paper *Paradoxical behaviour of mechanical and electrical networks* or this video.

Remark

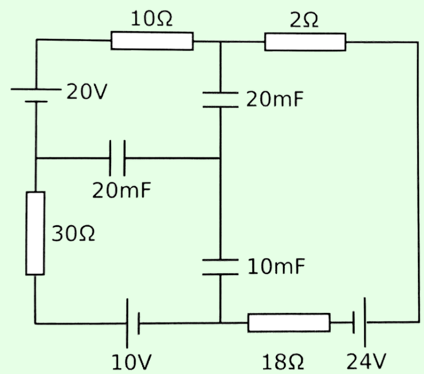
Circuit questions can get *absurdly* hard, but at some point they start being more about mathematical tricks than physics. As a result, I haven't included any such problems here; they tend not to appear on the USAPhO or IPhO, or in college physics, or in real life, or really anywhere besides a few competitions. On the other hand, you might find such questions fun! For some examples, see the Physics Cup problems 2013.6, 2017.2, 2018.1, and 2019.4.

2 RC Circuits

Next we'll briefly cover RC circuits, our first exposure to a situation genuinely changing in time.

Example 4: CPhO

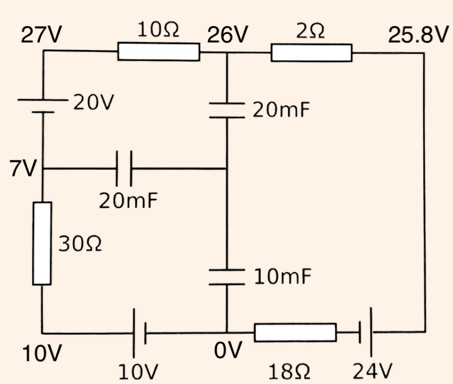
The capacitors in the circuit shown below were initially neutral. Then, the circuit is allowed to reach the steady state.



After a long time, what is the charge stored on the 10 mF capacitor?

Solution

After a long time, no current flows through the capacitors, so there is effectively a single loop in the circuit. It has a total resistance $60\,\Omega$ and a total emf $6\,\mathrm{V}$, so the current is $I = 0.1\,\mathrm{A}$. Using this, we can straightforwardly label the voltages everywhere on the outer loop.



To finish the problem, we need to know the voltage V_0 of the central node, so we need one more equation. That equation is charge conservation: the fact that the central part of the circuit, containing the inner plates of the three capacitors, begins and remains uncharged. Suppressing units, this means

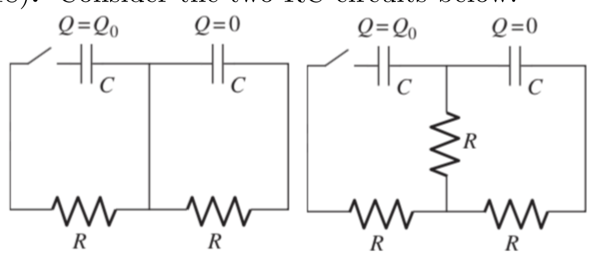
$$20(26 - V_0) + 20(7 - V_0) + 10(0 - V_0) = 0, \quad V_0 = \frac{66}{5}\,\mathrm{V}$$

from which we read off the answer,

$$Q = CV = 0.132\,\mathrm{C}.$$

[3] **Problem 9.** USAPhO 1997, problem A3.

[3] **Problem 10** (Purcell 4.18)**.** Consider the two RC circuits below.

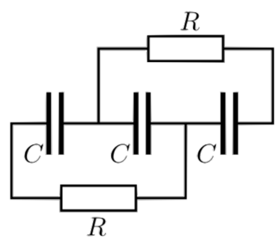


(a) The circuit shown below contains two identical capacitors and two identical resistors, with initial charges as shown above at left. If the switch is closed at $t = 0$, find the charges on the capacitors as functions of time.

(b) Now consider the same setup with an extra resistor, as shown above at right. Find the maximum charge that the right capacitor achieves. (Hint: the methods of **M4** can be useful.)

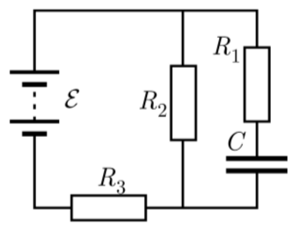
[3] **Problem 11.** USAPhO 2004, problem A1.

[3] **Problem 12** (Kalda)**.** Three identical capacitors are placed in series and charged with a battery of emf $\mathcal{E}$. Once they are fully charged, the battery is removed, and simultaneously two resistors are connected as shown.



Find the heat dissipated on each of the resistors after a long time.

[3] **Problem 13** (Kalda)**.** Find the time constant of the RC circuit shown below.



[3] **Problem 14** (MPPP 175/176). A metal sphere of radius R has charge Q and hangs on an insulating cord. It slowly loses charge because air has a conductivity σ. In all cases, neglect any magnetic or radiation effects.

(a) Find the time for the charge to halve.

(b) You should have found that the time is independent of the radius R of the sphere, which follows directly from dimensional analysis. Can you show that, in fact, it is completely independent of the shape? (This doesn't just follow from dimensional analysis, because the shape might be described by dimensionless numbers, such as the eccentricity of an ellipsoid.)

(c) Air has a conductivity of $\sigma \sim 10^{-13}\,\Omega^{-1}\text{m}^{-1}$, while water has a conductivity of $\sigma \sim 10^{-2}\,\Omega^{-1}\text{m}^{-1}$. About how long does the charge on an object last, if it is in air or water?

This problem generalizes USAPhO 2010, problem A2, which you can compare.

[5] **Problem 15.** IPhO 1993, problem 1. A really neat question with real-world relevance.

[5] **Problem 16.** IPhO 2007, problem "orange". A combination of mechanics and RC circuits.

3 Computing Magnetic Fields

Idea 4

The Biot–Savart law is

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\mathbf{s} \times \mathbf{r}}{r^3}.$$

As a consequence, we have Ampere's law,

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

as well as Gauss's law for magnetism,

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0, \quad \nabla \cdot \mathbf{B} = 0.$$

Idea 5

The force on a stationary wire carrying current I in a magnetic field $\mathbf{B}$ is

$$\mathbf{F} = I \int d\mathbf{s} \times \mathbf{B}.$$

The energy of a magnetic field is

$$U = \frac{1}{2\mu_0} \int B^2 \, dV.$$

The magnetic dipole moment of a planar current loop of area A and current I is $m = IA$, with $\mathbf{m}$ directed perpendicular to the loop by the right-hand rule.

You should have already seen basic examples of using the Biot–Savart law in Halliday and Resnick, such as the field of a circular ring of current on its axis. We'll start with some problems that are similarly straightforward, but more technically complex.

[3] **Problem 17** (Purcell 6.11). A spherical shell with radius R and uniform surface charge density σ spins with angular frequency ω about a diameter.

(a) Find the magnetic field at the center.

(b) Find the magnetic dipole moment of the sphere.

(c) Sketch the magnetic field.

[2] **Problem 18** (Purcell 6.12). A ring with radius R carries a current I. Show that the magnetic field due to the ring, at a point in the plane of the ring, a distance r from the center, is given by

$$B = \frac{\mu_0 I}{2\pi} \int_0^\pi \frac{(R - r\cos\theta)R \, d\theta}{(r^2 + R^2 - 2rR\cos\theta)^{3/2}}.$$

In the $r \gg R$ limit, show that

$$B \approx \frac{\mu_0}{4\pi} \frac{m}{r^3}$$

where $m = IA$ is the magnetic dipole moment of the ring.

[3] **Problem 19** (Purcell 6.14). Consider a square loop with current I and side length a centered at the origin, with sides parallel to the x and y axes. Show that the magnetic field at $r\hat{\mathbf{x}}$ is $B \approx (\mu_0/4\pi)(m/r^3)$ for $r \gg a$, just like the previous problem. Be careful with factors of 2!

Idea 6

The results you have found above, for the fields far from currents, are special cases of the general magnetic dipole field: far from a magnetic dipole with magnetic moment m, its magnetic field is just the same as the electric field of an electric dipole,

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}) = \frac{\mu_0}{4\pi r^3}(3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}).$$

As with the electric dipole field, you don't need to memorize this result, but you should remember that it's proportional to the dipole moment, falls off as $1/r^3$, and be able to sketch it. Of course, static electric and magnetic fields behave differently; when you get inside an electric dipole the field reverses direction, but this isn't true for a magnetic dipole.

[3] **Problem 20.** 🕓 USAPhO 2012, problem A3.

We now give a few arguments for computing fields using symmetry.

Example 5: PPP 31

An electrically charged conducting sphere "pulses" radially, i.e. its radius changes periodically with a fixed amplitude. What is the net pattern of radiation from the sphere?

Solution

There is no radiation. By spherical symmetry, the magnetic field can only point radially. But then this would produce a magnetic flux through a Gaussian sphere centered around the pulsing sphere, which would violate Gauss's law for magnetism. So there is no magnetic field at all, and since radiation always needs both electric and magnetic fields (as you'll see in **E7**), there is no radiation at all. In fact, outside the sphere the electric field is always exactly equal to $Q/4\pi\epsilon_0 r^2$, in accordance with Coulomb's law.

Example 6

Find the magnetic field of a very long cylindrical solenoid, of radius R and n turns per unit length, carrying current I.

Solution

Orient the solenoid along the vertical direction and use cylindrical coordinates. By symmetry, the field must be independent of z. Now consider the radial component of the magnetic field B_r. Turning the solenoid upside-down is equivalent to reversing the current. But the former does not flip B_r while the latter does, so we must have $B_r = 0$.

Now, by rotational symmetry, the tangential component B_ϕ must be uniform. But then Ampere's law on any circular loop gives $B_\phi(2\pi r) = 0$, so we must have $B_\phi = 0$ as well.

The only thing left to consider is B_z. By applying Ampere's law to small vertical rectangles, we see that B_z is constant unless that rectangle crosses the surface of the solenoid. Furthermore, B_z must be zero far from the solenoid, so it must be zero everywhere outside the solenoid. Now, for a rectangle of height h that does cross the surface, Ampere's law gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B_z^{\text{in}} h = \mu_0 I_{\text{enc}} = \mu_0 n I h$$

which tells us that $B_z^{\text{in}} = \mu_0 n I$.

Remark

The above analysis is not quite how real solenoids behave for several reasons. First, we didn't account for the discreteness of the wires. We just treated them as forming a uniform current per length $K = nI$, which is how we wrote $I_{\text{enc}} = nIh$. This is valid when you don't care about looking too close to the wires themselves.

The fact that solenoids are made by winding real wires means there is another contribution to the current, even in the limit $n \to \infty$. The wires are wound with a small slope, since a net current I still has to move along the solenoid. Another way of saying this is that the current per length along the solenoid surface is $\mathbf{K} = nI\hat{\boldsymbol{\theta}} + (I/2\pi R)\hat{\mathbf{z}}$. This causes a tangential magnetic field $B_\phi = \mu_0 I/2\pi r$ outside the solenoid.

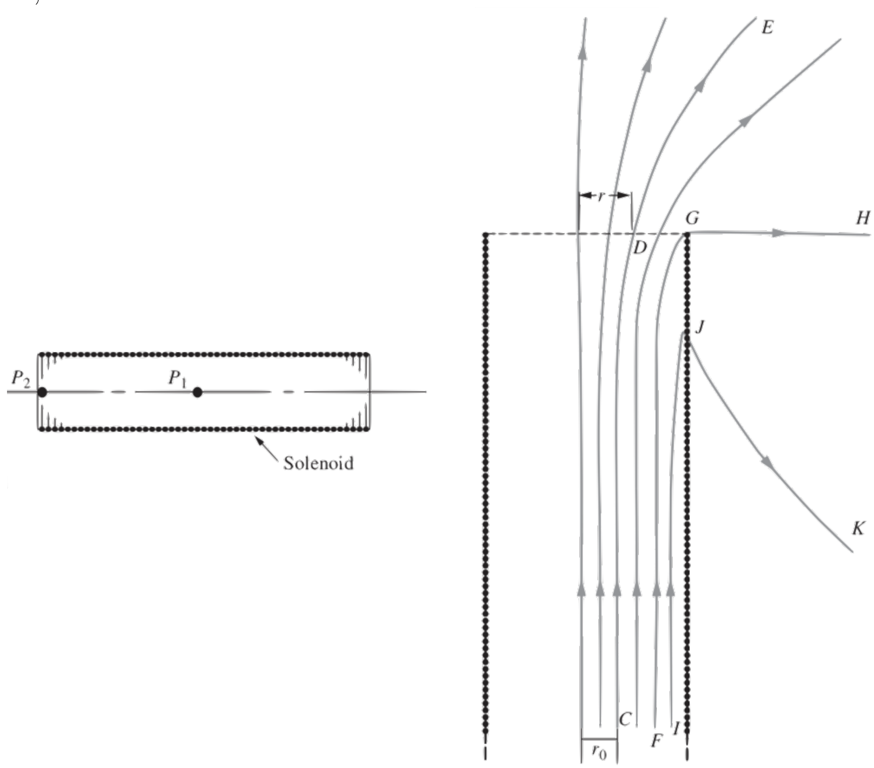
Another factor is that a real solenoid isn't infinitely long, and end effects are important. You'll analyze these in a slick way in problem 22.

[2] **Problem 21.** A toroidal solenoid is created by wrapping N turns of wire around a torus with a rectangular cross section. The height of the torus is h, and the inner and outer radii are a and b.

(a) In the ideal case, the magnetic field vanishes everywhere outside the toroid, and is purely tangential inside the toroid. Find the magnetic field inside the toroid.

(b) There is another small contribution to the magnetic field due to the winding effect mentioned above. Roughly what does the resulting extra magnetic field look like? If you didn't want this additional field, how would you design the solenoid to get rid of it?

[3] **Problem 22** (Purcell 6.63). A number of simple facts about the fields of solenoids can be found by using superposition. The idea is that two solenoids of the same diameter, and length L, if joined end to end, make a solenoid of length $2L$. Two semi-infinite solenoids butted together make an infinite solenoid, and so on.



Prove the following facts.

(a) In the finite-length solenoid shown at left above, the magnetic field on the axis at the point P_2 at one end is approximately half the field at the point P_1 in the center. (Is it slightly more than half, or slightly less than half?)

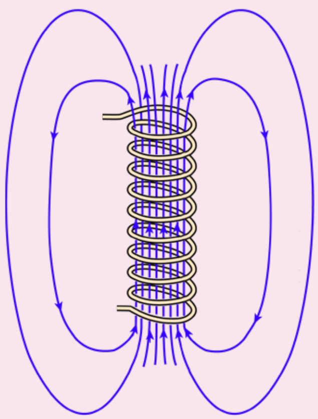
(b) In the semi-infinite solenoid shown at right above, the field line FGH, which passes through the very end of the winding, is a straight line from G out to infinity.

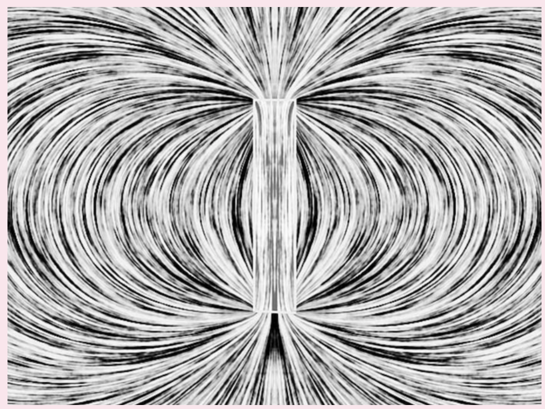
(c) The flux through the end face of the semi-infinite solenoid is half the flux through the coil at a large distance back in the interior.

(d) Any field line that is a distance r_0 from the axis far back in the interior of the coil exits from the end of the coil at a radius $r_1 = \sqrt{2}r_0$, assuming $\sqrt{2}r_0$ is less than the solenoid radius.

Remark: Solenoid Fringe Fields

Introductory physics resources usually have diagrams like the one at left below.





They suggest that all the magnetic flux comes neatly out the ends of the solenoids, in straight lines. But the real story is subtler. For a long, thin solenoid, most of the magnetic flux makes it to the end, but at that point it sprays out almost spherically symmetrically. As you saw in problem 22, half the flux goes up out through the end face, while the rest exits *downward* through the sides. That's shown in the accurate diagram shown at right above, which is taken from this paper.

Note that near the endcaps, the field of the solenoid looks a lot like that of a point charge, which can be understood by symmetry. The solenoid itself carries some magnetic flux Φ towards the end, and by Gauss's law for magnetism, all of that flux must exit somehow. In the limit of a very thin solenoid, the situation looks spherically symmetric, yielding an outgoing field like that of a point charge. Indeed, once you get more than a distance r away from the solenoid axis or ends (where r is the radius of the solenoid), its field looks a lot like that of two opposite magnetic point charges. This is an example of the correspondence between "Ampere" and "Gilbert" dipoles, which we'll revisit in **E4**.

Misleading diagrams are a common problem in introductory textbooks. A general rule is that the more basic a textbook is, the more pictures it'll have, but the less useful they'll be.

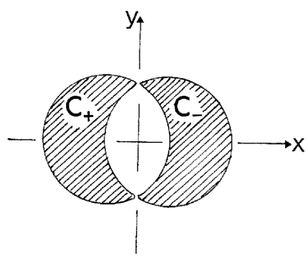
[3] **Problem 23** (MPPP 160)**.** Two infinite parallel wires, a distance d apart, carry electric currents with equal magnitudes but opposite directions. In this problem, we find the shape of the magnetic field lines using a neat trick.

(a) Argue that if we rotated $\mathbf{B}$ by $90°$ at each point, it would produce a valid electrostatic field $\mathbf{E}$. (Hint: consider what happens when we rotate the $\mathbf{B}$ field of a single wire, first.)

(b) Argue that the field lines of $\mathbf{B}$ are the same as the equipotentials of this artificial $\mathbf{E}$, and use this to find the field lines.

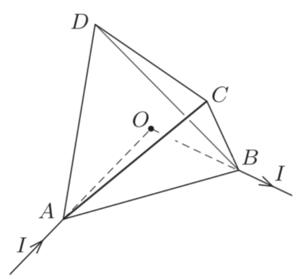
This is a common trick used when working with vortices in two dimensions, where it converts vortices to sources and sinks.

[2] **Problem 24** (IPhO 1996)**.** Two straight, long conductors C_+ and C_-, insulated from each other, carry current I in the positive and the negative $\hat{\mathbf{z}}$ direction respectively. The cross sections of the conductors are circles of diameter D in the xy plane, with a distance $D/2$ between the centers.



The current in each conductor is uniformly distributed. Find the magnetic field in the space between the conductors.

[3] **Problem 25** (MPPP 157)**.** A regular tetrahedron is made of a wire with constant resistance per unit length. A current I is sent into one vertex and removed from another vertex, as shown.



Find the magnetic field at the center of the tetrahedron.

[5] **Problem 26.** APhO 2013, problem 1. A neat question on a cylindrical RC circuit that uses many of the techniques we've covered so far.