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Week02

Problem 1

a.

$$\hat{\mu_1} = E[X] = \frac{1}{n}\sum_{i}^{n} x_i = 1.0489703904839582$$

$$\hat{\mu_2} = E[(X-\hat{\mu_1})^2] = \frac{1}{n-1}\sum_{i}^{n}(x_i-\hat{\mu_1})^2 = 5.4272206818817255$$

$$\sigma = \frac{1}{n}\sum_{i}^{n}(x_i-\hat{\mu_1})^2 = 5.421793461199844$$

$$\hat{\mu_3} = E[(\frac{X-\hat{\mu_1}}{\sigma})^3] = \frac{1}{n}\sum_{i}^{n}(\frac{X-\hat{\mu_1}}{\sigma})^3 = 0.8806086425277379$$

$$\hat{\mu_4} = E[(\frac{X-\hat{\mu_1}}{\sigma})^4] - 3 = \frac{1}{n}\sum_{i}^{n}(\frac{X-\hat{\mu_1}}{\sigma})^4 - 3 = 23.122200789989723$$

b.

Mean: 1.048970

Var: 5.427221

Skew: 0.881932

Kurt: 23.244253

c.

Simulate a list of 100 data and use formular and package to calculate the statistics. Repeat this process 1000 times. Mean and Var have no difference. So these are unbiased. Use Student T to test whether Skew and Kurt are biased or not. Hypotheses:

$$H_0 : Skew = \hat{\mu_3}$$
$$H_1 : Skew \neq \hat{\mu_3}$$
$$t_{Skew} = \frac{Skew - \hat{\mu_3}}{\sqrt{Var(Skew)/n}}$$
$$H_0 : Kurt = \hat{\mu_4}$$
$$H_1 : Kurt \neq \hat{\mu_4}$$
$$t_{Kurt} = \frac{Kurt - \hat{\mu_4}}{\sqrt{Var(Kurt)/n}}$$

Result:

1st Moment: 0.495990

2nd Moment: 0.082549

3th Moment: 0.020703

4th Moment: -1.120182

Mean: 0.495990

Var: 0.082549

Skew: 0.021809

Kurt: -1.101909

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t_skew: 0.130940

p_skew: 0.895849

t_kurt: 1.903562

p_kurt: 0.057254

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if $\alpha = 0.05, p > \alpha$, so we can't refuse H_0.

Problem 2

a.

OLS Beta: [-0.08738446427005078, 0.7752740987226112]
OLS std: 1.003756319417732

MLE Beta: [-0.0873844781126185, 0.7752740776445967]
MLE std: 1.003756309651628
R^2: 0.3456068835648125

β_{MLE} is a little smaller than β_{OLS} because minimum optimization. MLE's standard deviation is greater than OLS's.

Respectively, for 2 methods calculate:

$$\epsilon = \hat{\beta}_1 \bar{X} + \hat{\beta}_0 - \bar{Y}$$
$$\epsilon_{OLS} = 0.363009317747427706$$
$$\epsilon_{MLE} = 0.3630092926962777$$

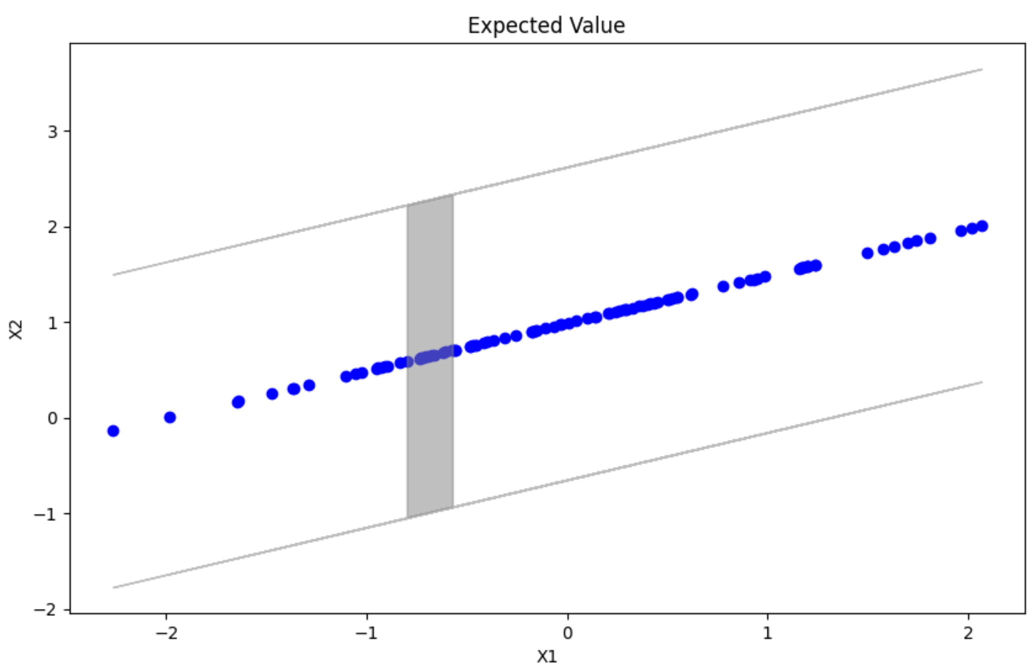
OLS's expected residual is also greater than MLE's.

b.

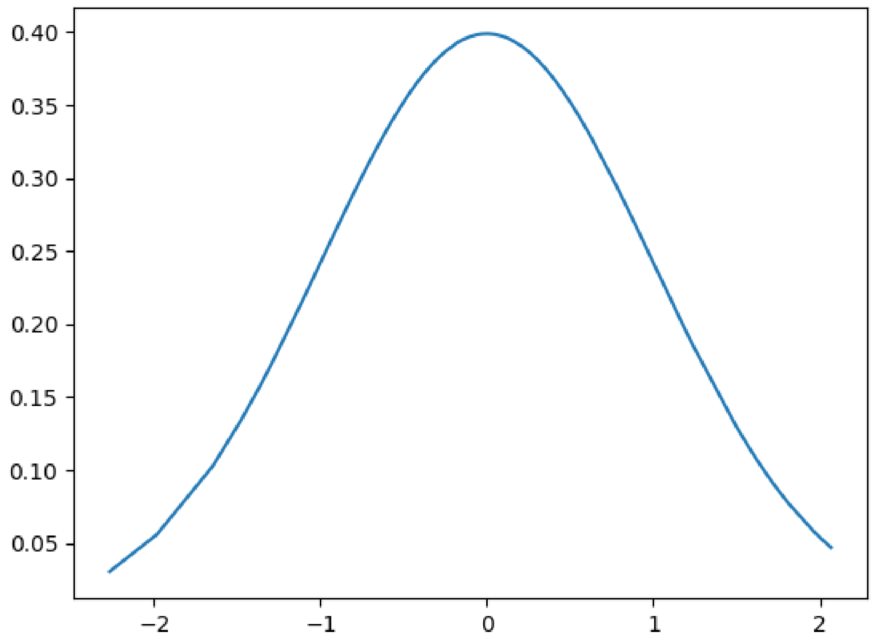
MLE_t Beta: [-0.0972694029223096, 0.6750091823157014]
MLE_t std: 7.159786463810651
R^2: 0.3396547079914969

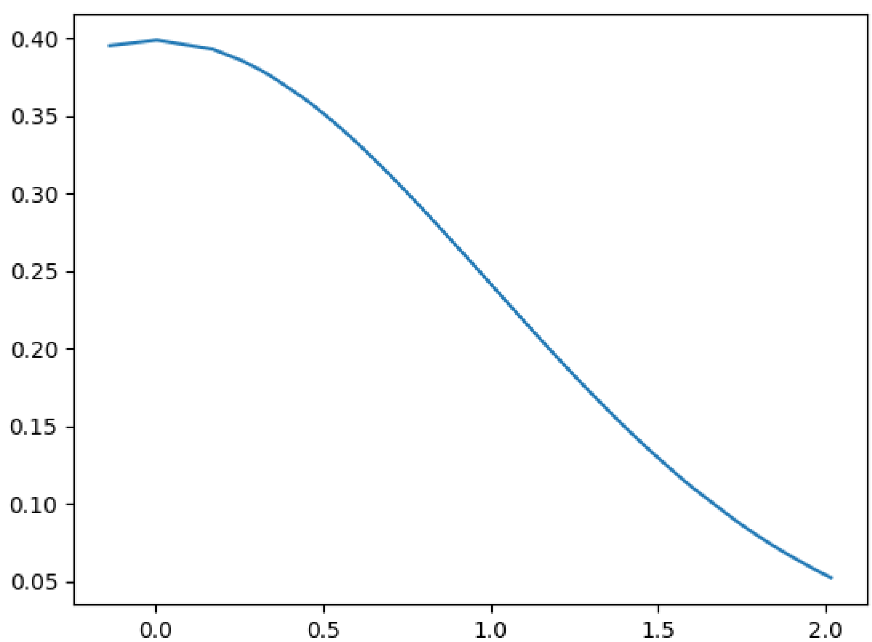
MLE under normality assumption is the best of fit.

c.



$$X_2 = \hat{\beta} X_{1obs} + \hat{\beta}_0$$





X_2 has a linear relation with observed X_1. X_1 has a normal distribution. So X_2 will have the same type of distribution with X_1.

d.

$$l = \prod_{i=1}^{n} f(Y|X;\beta)$$

$$ll = \sum_{i=1}^{n} ln(f(Y|X;\beta))$$

$$ll = \sum_{i=1}^{n} ln(f(x_i)) = \sum_{i=1}^{n}[-\frac{1}{2}ln(\sigma^2 2\pi) - \frac{1}{2}(\frac{x_i - \mu}{\sigma})^2]$$

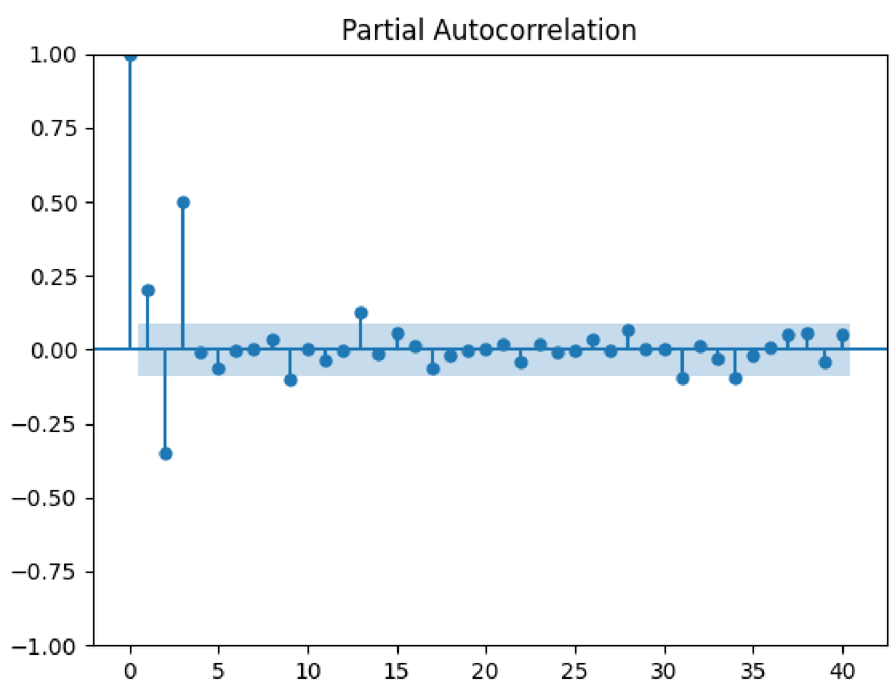
$$= -\frac{n}{2}ln(\sigma^2 2\pi) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$$

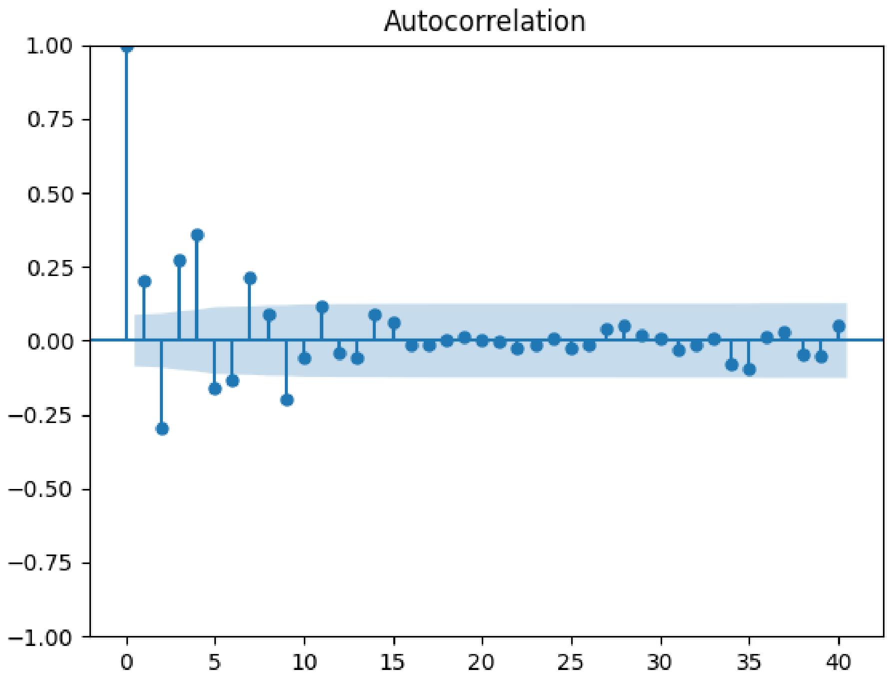
$$= -\frac{n}{2}ln(\sigma^2 2\pi) - \frac{1}{2\sigma^2}(Y - X\hat{\beta})(Y - X\hat{\beta})'$$

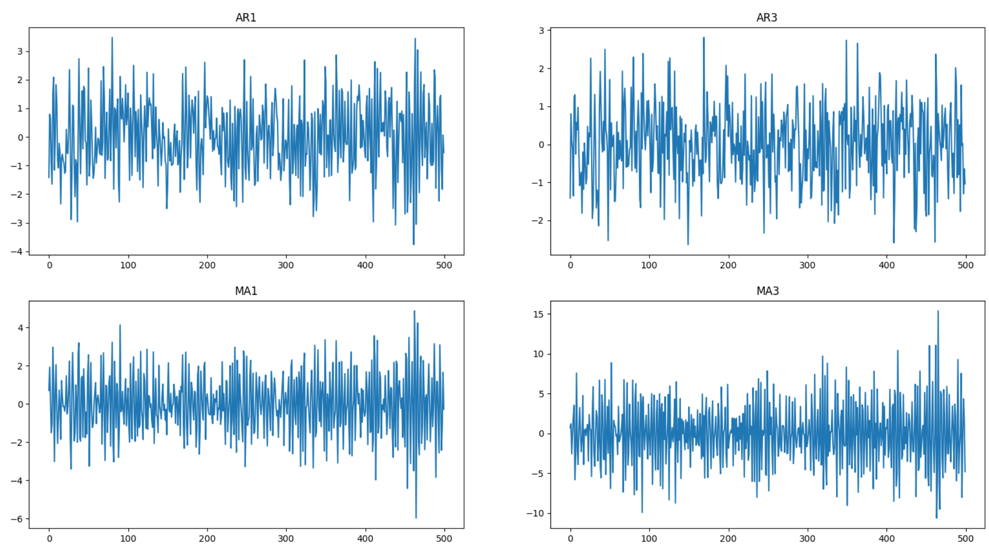
$$\frac{\Delta ll}{\Delta \hat{\beta}} = -\frac{1}{\sigma^2}X'(Y - X\hat{\beta}) = 0$$

$$\therefore \hat{\beta} = (X'X)^{-1}X'Y$$

Problem 3







	AIC	R^2
AR1	1644.655505	0.042621
AR3	1436.659807	0.373768
MA1	1891.667876	-0.591070
MA3	2788.209192	-8.778802

According to plots and information above, AR(3) is best of fit.