Fintech (545) **Spring2024**, 1–4

Project Week 05: Advanced VaR and Expected Shortfall

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1 PROBLEM 1

Please see the repo. The *RiskManagement* folder contains all files for tests:

`Cov` – Covariance estimation techniques.
`NonPSD` – Non-PSD fixes for correlation matrices.
`Sim` – Simulation Methods.
`VaR` – VaR calculation methods (all discussed).
`ES` – ES calculation.

1.1 Missing covariance calculations

Two common way yo calculate:

(i) Only use the days on which all markets are open;
(ii) Use pairwise calculations. Find the matching rows for each pair, and build the covariance matrix piece by piece.

1.1.1 Skip missing rows-covariance

	x1	x2	x3	x4	x5
x1	2.148513	-1.389470	-0.516466	-0.129327	-1.056814
x2	-1.389470	1.035342	0.339993	0.193888	0.626876
x3	-0.516466	0.339993	0.942388	0.947887	0.051788
x4	-0.129327	0.193888	0.947887	1.113436	-0.204731
x5	-1.056814	0.626876	0.051788	-0.204731	0.592027

1.1.2 Skip missing rows-correlation

	x1	x2	x3	x4	x5
x1	1.000000	-0.931618	-0.362959	-0.083616	-0.937042
x2	-0.931618	1.000000	0.344202	0.180583	0.800698
x3	-0.362959	0.344202	1.000000	0.925357	0.069333
x4	-0.083616	0.180583	0.925357	1.000000	-0.252163
x5	-0.937042	0.800698	0.069333	-0.252163	1.000000

1.1.3 Pairwise-covariance

	x1	x2	x3	x4	x5
x1	1.173986	-0.629631	-0.278932	-0.081448	-0.735140
x2	-0.629631	1.318197	0.018090	0.446047	0.139309
x3	-0.278932	0.018090	0.918102	0.360836	0.258613
x4	-0.081448	0.446047	0.360836	0.894764	-0.235190
x5	-0.735140	0.139309	0.258613	-0.235190	0.522607

1.1.4 Pairwise-correlation

	x1	x2	x3	x4	x5
x1	1.000000	-0.483199	-0.241787	-0.067767	-0.714761
x2	-0.483199	1.000000	0.015446	0.405660	0.178286
x3	-0.241787	0.015446	1.000000	0.488250	0.336248
x4	-0.067767	0.405660	0.488250	1.000000	-0.322136
x5	-0.714761	0.178286	0.336248	-0.322136	1.000000

1.2 EW covariance

$$w_{t-i} = (1-\lambda)\lambda^{i-1} \tag{1}$$

$$\widehat{w_{t-i}} = \frac{w_{t-i}}{\sum_{j=i}^{n} w_{t-j}} \tag{2}$$

$$\widehat{cov(x,y)} = \sum_{i=1}^{n} \widehat{w_{t-i}}(x_{t-i} - \bar{x})(y_{t-i} - \bar{y}) \tag{3}$$

1.2.1 EW covariance λ=0.97

	x1	x2	x3	x4	x5
x1	0.855911	0.127559	0.186929	0.081415	0.052412
x2	0.127559	1.08735	0.032715	0.112515	-0.432729
x3	0.186929	0.032715	0.744771	0.131065	0.065806
x4	0.081415	0.112515	0.131065	0.86881	0.113836
x5	0.052412	-0.432729	0.065806	0.113836	1.13918

1.2.2 EW correlation λ=0.94

	x1	x2	x3	x4	x5
x1	1.0	0.109711	0.218511	0.116902	0.059677
x2	0.109711	1.0	-0.046716	0.191773	-0.444896
x3	0.218511	-0.046716	1.0	0.184148	0.089927
x4	0.116902	0.191773	0.184148	1.0	0.122028
x5	0.059677	-0.444896	0.089927	0.122028	1.0

1.2.3 EW cov w/EW var(λ=0.94) EW correlation(λ=0.97)

	x1	x2	x3	x4	x5
x1	0.855911	0.10584	0.174461	0.100809	0.058928
x2	0.10584	1.08735	-0.04204	0.186396	-0.495153
x3	0.174461	-0.04204	0.744771	0.148129	0.082832
x4	0.100809	0.186396	0.148129	0.86881	0.121399
x5	0.058928	-0.495153	0.082832	0.121399	1.13918

1.3 Non-psd matrices

$$\Lambda = diag(\lambda_i) \tag{4}$$

$$CS = \Lambda S \tag{5}$$

$$\lambda_i' = max(\lambda_i, 0) \tag{6}$$

$$t_i = [\sum_{j=1}^{n} s_{i,j}^2 \lambda_j']^{-1} \tag{7}$$

$$T = diag(t_i) \tag{8}$$

$$B = \sqrt{T} S \sqrt{\Lambda'} \tag{9}$$

$$BB^T = \hat{C} \approx C \tag{10}$$

1.3.1 Near_psd covariance

	x1	x2	x3	x4	x5
x1	1.173986	-0.617989	-0.284559	-0.065152	-0.688287
x2	-0.617989	1.318197	0.017092	0.445696	0.139176
x3	-0.284559	0.017092	0.918102	0.354147	0.246056
x4	-0.065152	0.445696	0.354147	0.894764	-0.218717
x5	-0.688287	0.139176	0.246056	-0.218717	0.522607

1.3.2 Near_psd Correlation

	x1	x2	x3	x4	x5
x1	1.000000	-0.483199	-0.241787	-0.067767	-0.714761
x2	-0.483199	1.000000	0.015446	0.405660	0.178286
x3	-0.241787	0.015446	1.000000	0.488250	0.336248
x4	-0.067767	0.405660	0.488250	1.000000	-0.322136
x5	-0.714761	0.178286	0.336248	-0.322136	1.000000

1.3.3 Higham covariance

	x1	x2	x3	x4	x5
x1	1.173986	-0.623870	-0.294335	-0.057677	-0.693888
x2	-0.623870	1.318197	0.016449	0.448579	0.143703
x3	-0.294335	0.016449	0.918102	0.354067	0.246866
x4	-0.057677	0.448579	0.354067	0.894764	-0.217062
x5	-0.693888	0.143703	0.246866	-0.217062	0.522607

1.3.4 Higham correlation

	x1	x2	x3	x4	x5
x1	1.000000	-0.483199	-0.241787	-0.067767	-0.714761
x2	-0.483199	1.000000	0.015446	0.405660	0.178286
x3	-0.241787	0.015446	1.000000	0.488250	0.336248
x4	-0.067767	0.405660	0.488250	1.000000	-0.322136
x5	-0.714761	0.178286	0.336248	-0.322136	1.000000

1.4 Cholesky factorization

(i) Column j , start on the diagonal element

(ii) Subtract the sum of the squares of the values on the root matrix for row j from the value on the input matrix on the diagonal.

(iii) Update the root matrix at position (j, j) with the square root of 2

(iv) Moving down the column, row i

(a) Calculate the dot product of sub matrix vector $[i, 1 : (j-1)]$ and $[j, 1 : (j-1)]$

(b) Subtract a. from the (i, j) element of the input matrix.

(c) Divide b. by the j diagonal element of the root matrix

(d) Store that value in element (i, j) of the root matrix.

(v) Repeat for the next column.

	x1	x2	x3	x4	x5
x1	1.083506	0.000000	0.000000	0.000000	0.000000e+00
x2	-0.570360	0.996437	0.000000	0.000000	0.000000e+00
x3	-0.262628	-0.133175	0.911807	0.000000	0.000000e+00
x4	-0.060130	0.412871	0.431384	0.731160	0.000000e+00
x5	-0.635240	-0.223938	0.054179	-0.256892	1.053671e-08

1.5 Normal simulation

$$x = F^{-1}(random\ uniform) \tag{11}$$

1.5.1 PD input

	x1	x2	x3	x4	x5
x1	0.085367	0.087933	0.042383	0.009032	0.003874
x2	0.087933	0.160844	0.058218	0.012410	0.005335
x3	0.042383	0.058218	0.037386	0.005975	0.002566
x4	0.009032	0.012410	0.005975	0.001695	0.000548
x5	0.003874	0.005335	0.002566	0.000548	0.000314

1.5.2 PSD input

	x1	x2	x3	x4	x5
x1	0.085474	0.117461	0.042377	0.008987	0.003869
x2	0.117461	0.161419	0.058236	0.012350	0.005317
x3	0.042377	0.058236	0.037285	0.005926	0.002564
x4	0.008987	0.012350	0.005926	0.001679	0.000543
x5	0.003869	0.005317	0.002564	0.000543	0.000314

1.5.3 NonPSD input, near_psd fix

	x1	x2	x3	x4	x5
x1	0.085318	0.008679	0.037962	0.008066	0.003476
x2	0.008679	0.160988	0.052052	0.011104	0.004768
x3	0.037962	0.052052	0.037545	0.006033	0.002593
x4	0.008066	0.011104	0.006033	0.001699	0.000553
x5	0.003476	0.004768	0.002593	0.000553	0.000315

1.5.4 NonPSD input Higham fix

	x1	x2	x3	x4	x5
x1	0.084845	0.013741	0.039073	0.008274	0.003577
x2	0.013741	0.160394	0.053686	0.011398	0.004918
x3	0.039073	0.053686	0.037571	0.006248	0.002700
x4	0.008274	0.011398	0.006248	0.001692	0.000572
x5	0.003577	0.004918	0.002700	0.000572	0.000315

1.5.5 PSD Input - PCA simulation

	x1	x2	x3	x4	x5
x1	0.085344	0.117282	0.042516	0.009038	0.003896
x2	0.117282	0.161173	0.058427	0.012420	0.005355
x3	0.042516	0.058427	0.037562	0.006046	0.002595
x4	0.009038	0.012420	0.006046	0.001103	0.000474
x5	0.003896	0.005355	0.002595	0.000474	0.000204

1.6 Returns

1.6.1 Arithmetic returns

$$P_t = P_{t-1}(1 + r_t) \tag{12}$$

1.7 Fit

1.7.1 Fit normal distribution

Date	SPY	AAPL	MSFT	AMZN	NVDA
2022-09-02	-0.010544	-0.013611	-0.016667	-0.002425	-0.020808
2022-09-06	-0.003773	-0.008215	-0.010974	-0.010980	-0.013336
2022-09-07	0.017965	0.009254	0.019111	0.026723	0.018795
2022-09-08	0.006536	-0.009618	0.001666	0.002626	0.020126
2022-09-09	0.015535	0.018840	0.022977	0.026575	0.028377
...	...	...	...	...	...
2023-09-18	0.000586	0.016913	-0.003513	-0.002920	0.001503
2023-09-19	-0.002074	0.006181	-0.001246	-0.016788	-0.010144
2023-09-20	-0.009193	-0.019992	-0.023977	-0.017002	-0.029435
2023-09-21	-0.016528	-0.008889	-0.003866	-0.044053	-0.028931
2023-09-22	-0.002249	0.004945	-0.007887	-0.001624	0.014457

1.7.2 Log returns

$$P_t = P_{t-1}e^{r_t} \tag{13}$$

1.8 Fit

1.8.1 Fit normal distribution

Date	SPY	AAPL	MSFT	AMZN	NVDA
2022-09-02	-0.010600	-0.013705	-0.016807	-0.002428	-0.021027
2022-09-06	-0.003780	-0.008249	-0.011035	-0.011040	-0.013426
2022-09-07	0.017806	0.009211	0.018931	0.026372	0.018621
2022-09-08	0.006515	-0.009664	0.001665	0.002623	0.019926
2022-09-09	0.015416	0.018664	0.022717	0.026228	0.027982
...	...	...	...	...	...
2023-09-18	0.000586	0.016772	-0.003519	-0.002925	0.001502
2023-09-19	-0.002076	0.006162	-0.001247	-0.016931	-0.010196
2023-09-20	-0.009236	-0.020195	-0.024269	-0.017148	-0.029877
2023-09-21	-0.016666	-0.008929	-0.003873	-0.045053	-0.029357
2023-09-22	-0.002251	0.004932	-0.007918	-0.001625	0.014354

1.9 Fit

1.9.1 Fit normal distribution

mu	sigma
0.046026	0.046545

1.9.2 Fit t distribution

$$ll = \frac{n}{2}ln(\sigma^2 2\pi) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{14}$$

mu	sigma	nu
0.04594	0.045443	6.336867

1.9.3 Fit t regression

mu	5.951481e-07
sigma	0.048548
nu	4.598303
Alpha	0.042633
B1	0.97501
B2	2.041187
B3	3.154751

1.10 VaR and ES

$$VaR_\alpha(x) = -F_x^{-1}(\alpha) \tag{15}$$

$$ES_\alpha(X) = -\frac{1}{\alpha}\int_{-\infty}^{-VaR(X)} xf(x)dx \tag{16}$$

1.10.1 VaR normal distribution

VaR Absolute	VaR Diff from Mean
0.030535	0.07656

1.10.2 VaR t distribution

VaR Absolute	VaR Diff from Mean
0.04153	0.08747

1.10.3 VaR simulation

VaR Absolute	VaR Diff from Mean
0.041848	0.087703

1.10.4 ES normal distribution

ES Absolute	ES Diff from Mean
0.049984	0.09601

1.10.5 ES t distribution

ES Absolute	ES Diff from Mean
0.075232	0.121172

1.10.6 VaR Simulation

ES Absolute	ES Diff from Mean
0.076033	0.122302

1.11 Risk with copula

$C_R(X) = \Phi_R(\Phi^{-1}(F_1(x_1)), \Phi^{-1}(F_2(x_2)), \ldots, \Phi^{-1}(F_n(x_n)))(17)$

Stock	VaR95	ES95	VaR95_Pct	ES95_Pct
A	93.986214	117.630036	0.046993	0.058815
B	108.399648	152.062133	0.036133	0.050687
Total	155.796148	193.914934	0.031159	0.038783

2 PROBLEM 2

2.1 a

Using a normal distribution with an exponentially weighted variance (lambda=0.97):

VaR Absolute	VaR Diff from Mean
0.091169	0.09029

ES Absolute	ES Diff from Mean
0.113349	0.113227

2.2 b

Using a MLE fitted T distribution:

VaR Absolute	VaR Diff from Mean
0.076476	0.076382

ES Absolute	ES Diff from Mean
0.113218	0.113124

2.3 c

Using a Historic Simulation:

VaR Absolute	VaR Diff from Mean
0.075981	0.075101

ES Absolute	ES Diff from Mean
0.116777	0.11372

3 PROBLEM 3

Stock	VaR95	ES95	VaR95_Pct	ES95_Pct
AAPL	317.465211	414.398701	0.036335	0.04743
ABBV	256.645275	322.998933	0.023323	0.029353
ABT	231.944693	290.561932	0.027168	0.034033
ACN	273.995667	363.900022	0.033133	0.044004
ADBE	319.080784	449.219532	0.042452	0.059767
...	... ...	...	...	
WFC	274.43128	389.418363	0.033347	0.047319
WMT	225.073975	359.760232	0.023049	0.036842
XOM	521.057161	697.143923	0.032548	0.043547
ZTS	253.575624	338.527727	0.032181	0.042962
Total	20319.687801	25526.91869	0.023508	0.029532