# Week04 Assignment

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March 2, 2024

## 1 Problem1

$$\begin{aligned} &P_t = P_{t-1} + r_t \\ &E(P_t) = E(P_{t-1}) + E(r_t) = P_{t-1} \\ &Var(P_t) = Var(P_{t-1}) + Var(r_t) = \sigma^2 \end{aligned} \tag{1}$$

$$\begin{aligned} &P_t = P_{t-1}(1 + r_t) \\ &E(P_t) = E(P_{t-1}(1 + r_t)) = P_{t-1}(1 + E(r_t)) = P_{t-1} \\ &Var(P_t) = Var(P_{t-1}(1 + r_t)) = P_{t-1}^2\sigma^2 \end{aligned} \tag{2}$$

$$\begin{aligned} &P_t = P_{t-1}e^{r_t} \\ &E(P_t) = E(P_{t-1}e^{r_t}) = P_{t-1}E(e^{r_t}) = P_{t-1}e^{\mu+\frac{1}{2}\sigma^2} \\ &Var(P_t) = Var(P_{t-1}e^{r_t}) = P_{t-1}^2 Var(e^{r_t}) = P_{t-1}^2(e^{\sigma^2} - 1)e^{2\mu+\sigma^2} \end{aligned} \tag{3}$$

Classical Brownian Motion:
mean: 10.00024525468145
std: 0.009852316885855281
Expectation:
mean: 10 difference: 0.0002452546814506462
std: 0.01 difference: -0.00014768311414471887
Arithmetic Return System:
mean: 9.998367145374376
std: 0.1024923953908439
Expectation:
mean: 10 difference: -0.001632854625624347
std: 0.1 difference: 0.0024923953908438934
Geometric Brownian Motion:
mean: 9.998319913345714
std: 0.09949480292518091
Expectation:
mean: 10.000500012500208 difference: -0.0021800991544935755
std: 0.10000750030211356 difference: -0.0005126973769326443

# 2 Problem2

Normal Distribution:
5% VaR: -5.42%
1% VaR: -7.66%
Normal Distribution with an Exponentially Weighted Variance:
5% VaR: -3.27%
1% VaR: -4.62%
MLE fitted T Distribution:
5% VaR: -2.9%
1% VaR: -4.11%
Fitte AR(1) Model:
5% VaR: -0.76%
1% VaR: -0.83%
Historic Simulation:
5% VaR: -22.55%
1% VaR: -24.16%

# 3 Problem3

——————————-EWMA——————————-
Symbol 5% VaR 1% VaR Arithmetic 5% VaR Geometric 5% VaR
0 SPY -1.05% -1.49% -4.053365 -4.032079
1 AAPL -1.69% -2.39% -2.621957 -2.599890
2 MSFT -1.83% -2.59% -4.644755 -4.602480
3 AMZN -2.37% -3.35% -3.017321 -2.981900
4 NVDA -3.28% -4.64% -4.471406 -4.398882
.. ... ... ... ... ...
96 LRCX -2.71% -3.83% -11.450139 -11.296544
97 ZTS -1.82% -2.58% -2.834646 -2.808982
98 C -1.71% -2.42% -0.798769 -0.791984
99 BSX -1.23% -1.74% -0.497205 -0.494164
100 AMT -1.8% -2.54% -4.374174 -4.335101
[101 rows x 5 columns]
Arithmetic Total VaR: -362.432911810523
Geometric Total VaR: -359.02848745056394
——————————Monte Carlo——————————-
Symbol 5% VaR 1% VaR Arithmetic 5% VaR Geometric 5% VaR
0 SPY -1.06% -1.49% -4.063846 -4.042451
1 AAPL -1.59% -2.25% -2.467194 -2.447649
2 MSFT -1.92% -2.72% -4.868885 -4.822445
3 AMZN -2.37% -3.35% -3.021181 -2.985671
4 NVDA -3.34% -4.73% -4.556723 -4.481421
.. ... ... ... ... ...
96 LRCX -2.63% -3.72% -11.138567 -10.993182

97 ZTS -1.86% -2.63% -2.891639 -2.864936
98 C -1.77% -2.5% -0.826620 -0.819354
99 BSX -1.24% -1.76% -0.502747 -0.499639
100 AMT -1.75% -2.48% -4.264427 -4.227285
[101 rows x 5 columns]
Arithmetic Total VaR: -358.28488443025043
Geometric Total VaR: -354.96695189354193
————————-Historical————————-
Symbol 5% VaR 1% VaR Arithmetic 5% VaR Geometric 5% VaR
0 SPY -1.01% -1.44% -3.903027 -3.883288
1 AAPL -1.7% -2.41% -2.639270 -2.616912
2 MSFT -1.72% -2.43% -4.353834 -4.316675
3 AMZN -2.23% -3.15% -2.841246 -2.809825
4 NVDA -3.56% -5.04% -4.858512 -4.772968
.. ... ... ... ... ...
96 LRCX -2.72% -3.85% -11.515370 -11.360029
97 ZTS -1.74% -2.46% -2.711432 -2.687945
98 C -1.75% -2.47% -0.817839 -0.810727
99 BSX -1.23% -1.74% -0.498718 -0.495659
100 AMT -1.67% -2.37% -4.070106 -4.036263
[101 rows x 5 columns]
Arithmetic Total VaR: -358.0565073942368
Geometric Total VaR: -354.70430245091774

EWMA's loss is highest, since it consider different weights for returns on different time. It is reasonable and take more tolerance of risk.