1. 答

   (a)

   当Q是正值时，$g(s,t)^{Q+1}$会因为正幂次被增大，使得暗点（如pepper噪声）值变大，当暗点像素值被增大后，椒盐噪声等则会被有效抑制。

   (b)

   同理，当Q为负值时，$g(s,t)^{Q+1}$会因为负幂次使得亮点像素的贡献变小，抑制领域内亮像素的影响。

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2. 证

   对于

$$\min \sigma^2(x,y) = \frac{1}{mn} \sum_{(r,c)\in S_{xy}} \{[g(r,c) - w(x,y)\eta(r,c)] - [\bar{g} - w(x,y)\bar{\eta}]\}^2$$

   计算

$$\frac{\partial}{\partial \omega(x,y)} \left(\frac{1}{mn} \sum_{(r,c)\in S_{xy}} \{[g(r,c) - w(x,y)\eta(r,c)] - [\bar{g} - w(x,y)\bar{\eta}]\}^2 \right) = 0$$

   化简有

$$\frac{1}{mn} \sum_{(r,c)\in S_{xy}} \{[g(r,c) - w(x,y)\eta(r,c)] - [\bar{g} - w(x,y)\bar{\eta}]\} \cdot \{-\eta(r,c) + \bar{\eta}\} = 0$$

   分离$w(x,y)$有

$$w(x,y) = \frac{\sum_{(r,c)\in S_{xy}} [g(r,c)\eta(r,c) - \bar{g}\eta(r,c) - g(r,c)\bar{\eta} + \bar{g}\bar{\eta}]}{\sum_{(r,c)\in S_{xy}} [\eta(r,c)^2 - \bar{\eta}^2]}$$

   化简则有

$$\omega(x,y) = \frac{\overline{\eta(x,y)g(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)}$$

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3. 解

$$
\begin{aligned}
H(u,v) &= \int_0^T e^{-j2\pi u x_0(t)} dt \\
&= \int_0^T e^{-j2\pi u\left[(1/2)at^2\right]} dt \\
&= \int_0^T e^{-j\pi u a t^2} dt \\
&= \int_0^T \left[\cos\left(\pi u a t^2\right) - j\sin\left(\pi u a t^2\right)\right] dt \\
&= \sqrt{\frac{T^2}{2\pi u a T^2}}\left[C(\sqrt{\pi u a}T) - jS(\sqrt{\pi u a}T)\right]
\end{aligned}
$$

其中

$$
C(z) = \sqrt{\frac{2\pi}{T}} \int_0^z \cos t^2 dt
$$

$$
S(z) = \sqrt{\frac{2}{\pi}} \int_0^z \sin t^2 dt
$$