1. 解

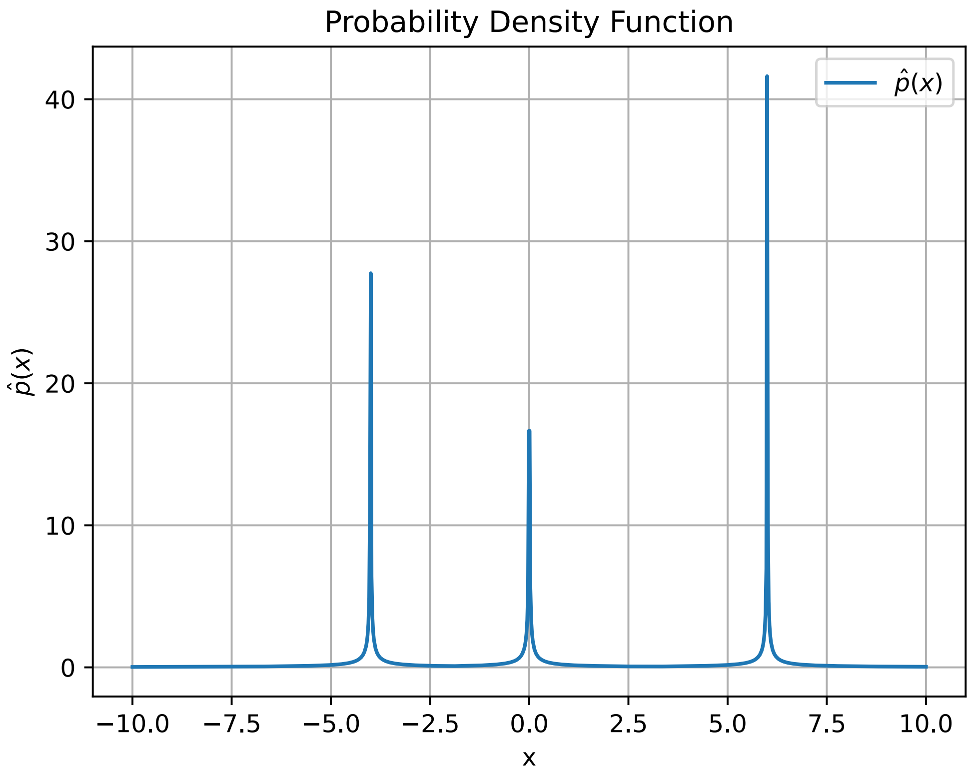
$$
\begin{aligned}
p_n(x) &= \frac{1}{nh_n^d}\sum_{i=1}^{n}\varphi(\frac{x-x_i}{h_n}) \\
&= \frac{1}{nh_n^d}\sum_{i=1}^{n}\frac{1}{\sqrt{2\pi}}\exp(-\frac{(x-x_i)^2}{2h_n^2})
\end{aligned}
$$

---

2. 解

$$
\hat{p}(x) = \begin{cases} \frac{1}{6|x+4|}, & x < -2; \\ \frac{1}{6|x|}, & -2 \le x < 3; \\ \frac{1}{6|x-6|}, & x > 3; \end{cases}
$$

其概率密度曲线图为



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3. 解

EM算法步骤

——初始化参数$\boldsymbol{\theta}^{\text{old}}$;

——循环

E步: 估计隐变量后验分布$p(\mathbf{z}|\mathbf{x},\boldsymbol{\theta}^{\text{old}})$

M步: 更新得到$\boldsymbol{\theta}^{\text{old}}$

$$
\begin{aligned}
\boldsymbol{\theta}^{\text{new}} & =\arg \max _{\boldsymbol{\theta}} \sum_{i} E_{p\left(\mathbf{z}_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right)}\left[\ln \left(p\left(\mathbf{x}_{i}, \mathbf{z}_{i} \mid \boldsymbol{\theta}\right)\right)\right] \\
& =\arg \max _{\boldsymbol{\theta}} \sum_{i} \sum_{z_{i}} p\left(z_{i} \mid x_{i}, \boldsymbol{\theta}^{\text{old}}\right) \ln \left(p\left(x_{i}, z_{i} \mid \boldsymbol{\theta}\right)\right)
\end{aligned}
$$

---

4. 解

   其基本形式为

$$
\begin{aligned}
Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{\text{old}}\right) & =\sum_{i} \sum_{z_{i}} p\left(z_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right) \ln \left(p\left(\mathbf{x}_{i}, z_{i} \mid \boldsymbol{\theta}\right)\right) \\
& =\sum_{i} \sum_{z_{i}=1: k} p\left(z_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right) \ln \left(\pi_{z_{i}} p\left(\mathbf{x}_{i} \mid z_{i}, \boldsymbol{\theta}\right)\right) \\
& =\sum_{i} \sum_{z_{i}=1: k} p\left(z_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right) \ln \left(\pi_{z_{i}} \mathcal{N}\left(\mathbf{x}_{i} \mid \boldsymbol{\mu}_{z_{i}}, \boldsymbol{\Sigma}_{z_{i}}\right)\right) \\
& =\sum_{i} \sum_{z_{i}=1: k} p\left(z_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right)\left(\ln \left(\pi_{z_{i}}\right)+\ln \left(\mathcal{N}\left(\mathbf{x}_{i} \mid \boldsymbol{\mu}_{z_{i}}, \boldsymbol{\Sigma}_{z_{i}}\right)\right)\right) \\
& =\sum_{i} \sum_{z_{i}=1: k}\left(p\left(z_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right) \ln \left(\pi_{z_{i}}\right)+p\left(z_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}^{\text{old}}\right) \ln \left(\mathcal{N}\left(\mathbf{x}_{i} \mid \boldsymbol{\mu}_{z_{i}}, \boldsymbol{\Sigma}_{z_{i}}\right)\right)\right)
\end{aligned}
$$

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5. 解

   HMM有三个基本任务

6. 给定模型参数$[\mathbf{A,B,}\boldsymbol{\pi}]$，计算序列$\mathbf{x}=[\mathbf{x_1,x_2,...,x_n}]$的概率$P(\mathbf{x}|\mathbf{A},\mathbf{B},\boldsymbol{\pi})$
7. 参数估计问题。即，给定观测序列$\mathbf{x}=[\mathbf{x_1,x_2,...,x_n}]$，选择合适的模型参数$[\mathbf{A,B,}\boldsymbol{\pi}]$使得$P(\mathbf{x}|\mathbf{A},\mathbf{B},\boldsymbol{\pi})$概率最大。
8. 解码问题。给定模型参数$[\mathbf{A,B,}\boldsymbol{\pi}]$和观测序列$\mathbf{x}=[\mathbf{x_1,x_2,...,x_n}]$，如何找到与观测序列相匹配的状态序列给定模型参数$[\mathbf{A,B,}\boldsymbol{\pi}]$，计算序列$\mathbf{z}=[\mathbf{z_1,z_2,...,z_n}]$