1. 证明

设三个通道的函数分别为$R(x,y),G(x,y),B(x,y)$

定义

$$\frac{\partial f}{\partial x}=\mathbf{u}=\frac{\partial R}{\partial x}\mathbf{r}+\frac{\partial G}{\partial x}\mathbf{g}+\frac{\partial B}{\partial x}\mathbf{b}$$
$$\frac{\partial f}{\partial y}=\mathbf{v}=\frac{\partial R}{\partial y}\mathbf{r}+\frac{\partial G}{\partial y}\mathbf{g}+\frac{\partial B}{\partial y}\mathbf{b}$$

由方向导数可知，在$\theta(x,y)$方向有

$$\begin{aligned}F_\theta=||\partial f_\theta||&=||\frac{\partial f}{\partial x}\cos\theta+\frac{\partial f}{\partial y}\sin\theta||\\&=||\mathbf{u}\cos\theta+\mathbf{v}\sin\theta||\\&=[(\mathbf{u}\cos\theta+\mathbf{v}\sin\theta)^T(\mathbf{u}\cos\theta+\mathbf{v}\sin\theta)]^{\frac{1}{2}}\\&=[g_{xx}\cos^2\theta+2g_{xy}\sin\theta\cos\theta+g_{yy}\sin^2\theta]^{\frac{1}{2}}\\&=\{[\frac{1}{2}[(g_{xx}+g_{yy})+(g_{xx}-g_{yy})\cos2\theta+2g_{xy}\sin2\theta]\}^{\frac{1}{2}}\end{aligned}$$

则，最大变换率的方向为

$$\theta=\arg\max_\theta F_\theta$$

即，求解

$$\frac{\partial F_\theta}{\partial\theta}=0$$

此时的θ为

$$\theta(x,y)=\frac{1}{2}\tan^{-1}[\frac{2g_{xy}}{g_{xx}-g_{yy}}]$$