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1. 解

因为线性系统输入为零均值白高斯噪声,所以输出 $\xi(t)$ 为高斯过程,因此 $\eta(t)$ 也为高斯过程。

$$\mathbb{E}(\eta(t)) = 0$$

$$S_{\xi\xi}(w) = ||H(jw)||^2 S_{nn}(w) = rac{N_0}{2(a^2+w^2)}$$

因此,有

$$R_{\xi}(au) = rac{1}{2\pi} \int_{-\infty}^{\infty} S_{\xi\xi}(w) e^{jw au} dw = rac{N_0}{4lpha} e^{-lpha au}$$

所以 $\eta(t)$ 的方差为

$$egin{aligned} D(\eta(t)) &= \mathbb{E}(\eta(t)\eta(t)) \ &= R_{\xi}(0) - 2R_{\xi}(T) - R_{\xi}(0) \ &= rac{N_0}{2lpha}(1 - e^{-lpha au}) \end{aligned}$$

所以有

$$f_{\xi}(y) = rac{1}{\sqrt{2\pi}\sigma_n^2} \mathrm{exp}(-rac{y^2}{2\sigma_\eta^2})$$

其中,定义 $\sigma_{\eta}^2 = rac{N_0}{2lpha}(1-e^{-lpha au})$

2. 解

(1)

由题可知 η_1, η_2 皆为高斯分布;

因此,有

$$\mathbb{E}(\eta_1(t)) = E_s + \int_0^T s(t) \mathbb{E}(n(t)) dt = E_s$$

$$egin{aligned} D(\eta_1(t)) &= \mathbb{E}(\eta_1^2(t)) - \mathbb{E}^2(\eta_1(t)) \ &= \mathbb{E}\left[(\int_0^T s(t)(s(t) + n(t))dt)^2
ight] - E_s^2 \ &= \mathbb{E}\left[(\int_0^T s(t)n(t)dt)^2
ight] \ &= \mathbb{E}\left[(\int_0^T s(u)n(u)du) (\int_0^T s(v)n(v)dv)
ight] \ &= \left[\int_0^T \int_0^T s(u)s(v)\mathbb{E}(n(u)n(v))dudv
ight] \ &= rac{N_0}{2} \int_0^T \int_0^T s(u)s(v)\delta(u-v)dudv \ &= rac{N_0}{2} E_s = \sigma_{\eta_1}^2 \end{aligned}$$

所以有

$$egin{align} f_{\eta_1}(u) &= rac{1}{\sqrt{2\pi}\sigma_{\eta_1}^2} ext{exp}(-rac{(u-E_s)^2}{2\sigma_{\eta_1}^2}) \ P(\eta_1 > \gamma) &= \int_{\gamma}^{\infty} f_{\eta_1}(u) du \ \end{cases}$$

(2)

同理,有

$$egin{aligned} \mathbb{E}(\eta_2(t)) &= \int_0^T s(t) \mathbb{E}(n(t)) dt = 0 \ D(\eta_2(t)) &= \mathbb{E}(\eta_2^2(t)) - \mathbb{E}^2(\eta_2(t)) \ &= \left[\int_0^T \int_0^T s(u) s(v) \mathbb{E}(n(u) n(v)) du dv
ight] \ &= rac{N_0}{2} E_s = \sigma_{\eta_2}^2 \end{aligned}$$

所以,有

$$egin{align} f_{\eta_2}(v) &= rac{1}{\sqrt{2\pi}\sigma_{\eta_2}^2} ext{exp}(-rac{u^2}{2\sigma_{\eta_2}^2}) \ P(\eta_2 > \gamma) &= \int_{\gamma}^{\infty} f_{\eta_2}(u) du \ \end{array}$$