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1. 解

   因为线性系统输入为零均值白高斯噪声，所以输出$\xi(t)$为高斯过程，因此$\eta(t)$也为高斯过程。

$$\mathbb{E}(\eta(t)) = 0$$

$$S_{\xi\xi}(w) = ||H(jw)||^2 S_{nn}(w) = \frac{N_0}{2(a^2 + w^2)}$$

   因此，有

$$R_\xi(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\xi\xi}(w)e^{jw\tau}dw = \frac{N_0}{4\alpha}e^{-\alpha\tau}$$

   所以$\eta(t)$的方差为

$$\begin{aligned} D(\eta(t)) &= \mathbb{E}(\eta(t)\eta(t)) \\ &= R_\xi(0) - 2R_\xi(T) - R_\xi(0) \\ &= \frac{N_0}{2\alpha}(1 - e^{-\alpha\tau}) \end{aligned}$$

   所以有

$$f_\xi(y) = \frac{1}{\sqrt{2\pi}\sigma_\eta^2}\exp(-\frac{y^2}{2\sigma_\eta^2})$$

   其中，定义$\sigma_\eta^2 = \frac{N_0}{2\alpha}(1 - e^{-\alpha\tau})$

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2. 解

   (1)

   由题可知η_1, η_2皆为高斯分布；

   因此，有

$$\mathbb{E}(\eta_1(t)) = E_s + \int_0^T s(t)\mathbb{E}(n(t))dt = E_s$$

$$
\begin{aligned}
D(\eta_1(t)) &= \mathbb{E}(\eta_1^2(t)) - \mathbb{E}^2(\eta_1(t)) \\
&= \mathbb{E}\left[(\int_0^T s(t)(s(t)+n(t))dt)^2\right] - E_s^2 \\
&= \mathbb{E}\left[(\int_0^T s(t)n(t)dt)^2\right] \\
&= \mathbb{E}\left[(\int_0^T s(u)n(u)du)(\int_0^T s(v)n(v)dv)\right] \\
&= \left[\int_0^T \int_0^T s(u)s(v)\mathbb{E}(n(u)n(v))dudv\right] \\
&= \frac{N_0}{2}\int_0^T \int_0^T s(u)s(v)\delta(u-v)dudv \\
&= \frac{N_0}{2}E_s = \sigma_{\eta_1}^2
\end{aligned}
$$

所以有

$$f_{\eta_1}(u) = \frac{1}{\sqrt{2\pi}\sigma_{\eta_1}^2}\exp(-\frac{(u-E_s)^2}{2\sigma_{\eta_1}^2})$$

$$P(\eta_1 > \gamma) = \int_\gamma^\infty f_{\eta_1}(u)du$$

(2)

同理，有

$$\mathbb{E}(\eta_2(t)) = \int_0^T s(t)\mathbb{E}(n(t))dt = 0$$

$$
\begin{aligned}
D(\eta_2(t)) &= \mathbb{E}(\eta_2^2(t)) - \mathbb{E}^2(\eta_2(t)) \\
&= \left[\int_0^T \int_0^T s(u)s(v)\mathbb{E}(n(u)n(v))dudv\right] \\
&= \frac{N_0}{2}E_s = \sigma_{\eta_2}^2
\end{aligned}
$$

所以，有

$$f_{\eta_2}(v) = \frac{1}{\sqrt{2\pi}\sigma_{\eta_2}^2}\exp(-\frac{u^2}{2\sigma_{\eta_2}^2})$$

$$P(\eta_2 > \gamma) = \int_\gamma^\infty f_{\eta_2}(u)du$$