序号：56

姓名：相贤泰

学号：202328019427026

---

1. 解

(1)

$$\begin{aligned}\mathbb{E}\{N(t)N(t+s)\} &= \sum_m \sum_n m(m+n)P(N(t)=m, N(t+s)=m+n) \\ &= \sum_m \sum_n m(m+n)\frac{\lambda^{m+n}t_1^m(t_2-t_1)^n}{m!n!}\exp(-\lambda t_2) \\ &= \lambda^2 t(t+s)+\lambda t\end{aligned}$$

$$\begin{aligned}\mathbb{E}\{N(t+s)|N(s)=n\} &= \sum_{k=n}^{\infty} kP(N(t+s)=k|N(s)=n) \\ &= \sum_{k=n}^{\infty} k\frac{P(N(t+s)=k, N(s)=n)}{P(N(s)=n)} \\ &= \sum_{k=n}^{\infty} k\frac{(\lambda t)^{k-n}}{(k-n)!}\exp(-\lambda t) \\ &= n+\lambda t\end{aligned}$$

(2)

由$N(t)$是独立增量过程，所以$X(t)$也是独立增量过程，因此是马氏过程；

$$\begin{aligned}p(s,i:t,j) &= \frac{P(X(s)=i, X(t)=j)}{P(X(s)=i)} \\ &= \frac{P(N(s)=2(i+1), N(t)=2(j+1))}{P(N(s)=2(i+1))} \\ &= \frac{(\lambda(t-s))^{2(j-i)}}{[2(j-i)]!}\exp(-\lambda(t-s))\end{aligned}$$

---

2. 解

(1)

$$\begin{aligned}\mathbb{E}(Z(t)) &= \mathbb{E}(X(t)-Y(t)) \\ &= (\lambda_1-\lambda_2)t\end{aligned}$$

$$\begin{aligned}\mathbb{E}(Z^2(t)) &= \mathbb{E}([X(t)-Y(t)]^2) \\ &= (\lambda_1+\lambda_2)t+(\lambda_1-\lambda_2)^2t^2\end{aligned}$$

(2)

由母函数可知

$$\begin{aligned}\Phi_{Z(t)}(u) &= \sum_{n=-\infty}^{+\infty} p_n(t)u^n = \Phi_{X(t)-Y(t)}(u) = \Phi_{X(t)+(-Y(t))}(u) = \Phi_{X(t)}(u)\Phi_{-Y(t)}(u) \\ &= \exp\left\{-(\lambda_1+\lambda_2)t\right\}\cdot\exp\left\{\lambda_1 ut+\lambda_2 u^{-1}t\right\}\end{aligned}$$

3. 解

(1)

$N_0(t)$状态空间为$\{\dots,-1,0,1,\dots\}$

(2)

$$\mathbb{E}(N_0(t))=\mathbb{E}(N_1(t)-N_2(t))=\lambda(t_1-t_2)$$