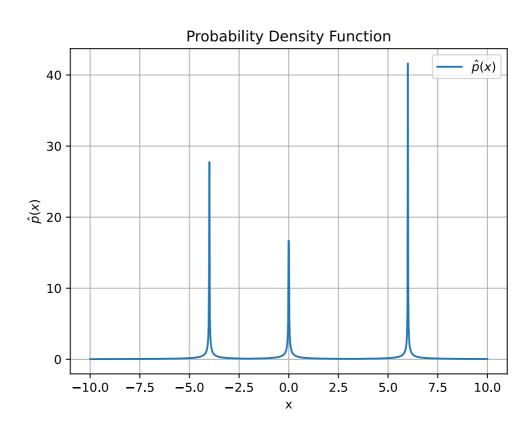
1.解

$$egin{aligned} p_n(x) &= rac{1}{nh_n^d} \sum_{i=1}^n arphi(rac{x-x_i}{h_n}) \ &= rac{1}{nh_n^d} \sum_{i=1}^n rac{1}{\sqrt{2\pi}} \mathrm{exp}(-rac{(x-x_i)^2}{2h_n^2}) \end{aligned}$$

2.解

$$\hat{p}(x) = egin{cases} rac{1}{6|x+4|}, & x < -2; \ rac{1}{6|x|}, & -2 \leq x < 3; \ rac{1}{6|x-6|}, & x > 3; \end{cases}$$

其概率密度曲线图为



3.解

EM算法步骤

——初始化参数 $oldsymbol{ heta}^{
m old}$;

——循环

E步: 估计隐变量后验分布 $p(\mathbf{z}|\mathbf{x}, oldsymbol{ heta}^{\mathrm{old}})$

M步: 更新得到 $oldsymbol{ heta}^{
m old}$

$$egin{aligned} oldsymbol{ heta}^{ ext{new}} &= rg\max_{oldsymbol{ heta}} \sum_{i} E_{p(\mathbf{z}_{i} \mid \mathbf{x}_{i}, oldsymbol{ heta}^{ ext{old}})} \left[\ln \left(p\left(\mathbf{x}_{i}, \mathbf{z}_{i} \mid oldsymbol{ heta}
ight)
ight)
ight] \ &= rg\max_{oldsymbol{ heta}} \sum_{i} \sum_{z_{i}} p\left(z_{i} \mid x_{i}, oldsymbol{ heta}^{ ext{old}}
ight) \ln \left(p\left(x_{i}, z_{i} \mid oldsymbol{ heta}
ight)
ight) \end{aligned}$$

4. 解

其基本形式为

$$egin{aligned} Q\left(oldsymbol{ heta}, oldsymbol{ heta}^{ ext{old}}
ight) &= \sum_{i} \sum_{z_{i}=1:k} p\left(z_{i} \mid \mathbf{x}_{i}, oldsymbol{ heta}^{ ext{old}}
ight) \ln\left(p\left(\mathbf{x}_{i}, z_{i} \mid oldsymbol{ heta}
ight)
ight) \ &= \sum_{i} \sum_{z_{i}=1:k} p\left(z_{i} \mid \mathbf{x}_{i}, oldsymbol{ heta}^{ ext{old}}
ight) \ln\left(\pi_{z_{i}} \mathcal{N}\left(\mathbf{x}_{i} \mid oldsymbol{\mu}_{z_{i}}, oldsymbol{\Sigma}_{z_{i}}
ight)
ight) \ &= \sum_{i} \sum_{z_{i}=1:k} p\left(z_{i} \mid \mathbf{x}_{i}, oldsymbol{ heta}^{ ext{old}}
ight) \left(\ln\left(\pi_{z_{i}}
ight) + \ln\left(\mathcal{N}\left(\mathbf{x}_{i} \mid oldsymbol{\mu}_{z_{i}}, oldsymbol{\Sigma}_{z_{i}}
ight)
ight) \ &= \sum_{i} \sum_{z_{i}=1:k} \left(p\left(z_{i} \mid \mathbf{x}_{i}, oldsymbol{ heta}^{ ext{old}}
ight) \ln\left(\pi_{z_{i}}
ight) + p\left(z_{i} \mid \mathbf{x}_{i}, oldsymbol{ heta}^{ ext{old}}
ight) \ln\left(\mathcal{N}\left(\mathbf{x}_{i} \mid oldsymbol{\mu}_{z_{i}}, oldsymbol{\Sigma}_{z_{i}}
ight)
ight) \end{aligned}$$

5. 解

HMM有三个基本任务

- 6. 给定模型参数 $[\mathbf{A}, \mathbf{B}, \pi]$, 计算序列 $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]$ 的概率 $P(\mathbf{x} | \mathbf{A}, \mathbf{B}, \pi)$
- 7. 参数估计问题。即,给定观测序列 $\mathbf{x}=[\mathbf{x_1,x_2,\dots,x_n}]$,选择合适的模型参数 $[\mathbf{A,B,\pi}]$ 使得 $P(\mathbf{x}|\mathbf{A,B,\pi})$ 概率最大。
- 8. 解码问题。给定模型参数 $[\mathbf{A}, \mathbf{B}, \pi]$ 和观测序列 $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]$,如何找到与观测序列相匹配的状态序列给定模型参数 $[\mathbf{A}, \mathbf{B}, \pi]$,计算序列 $\mathbf{z} = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n]$