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12.解

(a)

$$\begin{aligned}\mathbb{E}(X(t)) &= \int_{-\pi}^{\pi} \frac{1}{2\pi} A\cos(\omega_c t+\phi)d\phi \\ &= 0\end{aligned}$$

$$\begin{aligned}D(X(t)) &= \mathbb{E}(X^2(t)) - \mathbb{E}(X(t))^2 \\ &= \int_{-\pi}^{\pi} \frac{1}{2\pi} A^2\cos^2(\omega_c t+\phi)d\phi \\ &= \frac{A^2}{2}\end{aligned}$$

$$\begin{aligned}R_{XX}(t_1,t_2) &= \mathbb{E}(X(t_1)X(t_2)) \\ &= \int_{-\pi}^{\pi} \frac{1}{2\pi} A^2\cos(\omega_c t_1+\phi)\cos(\omega_c t_2+\phi)d\phi \\ &= \frac{A^2}{2}\cos(\omega_c(t_1-t_2))\end{aligned}$$

(b)

$$\begin{aligned}R_{XY}(t_1,t_2) &= \mathbb{E}(X(t_1)Y(t_2)) \\ &= \int_{-\infty}^{\infty} f(B)dB \int_{-\pi}^{\pi} \frac{1}{2\pi} AB\cos(\omega_c t_1+\phi)\cos(\omega_c t_2)d\phi \\ &= 0\end{aligned}$$

---

13. 解

$$\begin{aligned}\mathbb{E}(\xi(t)) &= \iint_{0\le x\le 1, 0\le y\le 1} x\sin(yt)dxdy \\ &= \frac{1-\cos(t)}{2t}\end{aligned}$$

$$\begin{aligned}R_{\xi}(t_1,t_2) &= \mathbb{E}(\xi(t_1)\xi(t_2)) \\ &= \iint_{0\le x\le 1, 0\le y\le 1} x^2\sin(yt_1)\sin(yt_2)dxdy \\ &= \frac{1}{6}(\frac{\sin(t_1-t_2)}{t_1-t_2} - \frac{\sin(t_1+t_2)}{t_1+t_2})\end{aligned}$$

---

15. 解

(1)

$$
\begin{aligned}
F_{X(t)}(z) &= P(X+e^{-t}Y \le z) \\
&= \sum_{k=1}^{\infty} P(X+e^{-t}Y \le z | Y=k) P(Y=k) \\
&= \sum_{k=1}^{\infty} \Phi(z-ke^{-t})(1-p)^{k-1}p
\end{aligned}
$$

因此有

$$
f_{X(t)}(z) = \sum_{k=1}^{\infty} \frac{(1-p)^{k-1}p}{\sqrt{2\pi}\sigma} \exp(-\frac{(z-ke^{-t}-\mu)^2}{2\sigma^2})
$$

(2)

$$
\begin{aligned}
\mathbb{E}(X(t)) &= \mathbb{E}(X+e^{-t}Y) \\
&= \mu + e^{-t}p^{-1}
\end{aligned}
$$

$$
\begin{aligned}
Cov(X(s),X(t)) &= R_X(s,t) - \mathbb{E}(X(t))\mathbb{E}(X(s)) \\
&= R_X(s,t) - (\mu+e^{-t}p^{-1})(\mu+e^{-s}p^{-1}) \\
&= \mathbb{E}[(X+e^{-t}Y)(X+e^{-s}Y)] - (\mu+e^{-t}p^{-1})(\mu+e^{-s}p^{-1}) \\
&= \mathbb{E}(X^2) + (e^{-t}+e^{-s})\mathbb{E}(XY) + e^{-(t+s)}\mathbb{E}(Y^2) - (\mu+e^{-t}p^{-1})(\mu+e^{-s}p^{-1}) \\
&= e^{-(t+s)}\frac{1-p}{p^2}
\end{aligned}
$$

---

17. 解

(1)

$$
\begin{aligned}
\mathbb{E}(\xi(t)) &= \mathbb{E}(X\cos 2t + Y\sin 2t) \\
&= \frac{1}{2}(\cos 2t + \sin 2t)
\end{aligned}
$$

$$
\begin{aligned}
R_\xi(t_1,t_2) &= \mathbb{E}(\xi(t_1)\xi(t_2)) \\
&= \cos 2t\cos 2s\mathbb{E}(X^2) + (\cos 2t\sin 2s + \sin 2t\cos 2s)\mathbb{E}(XY) + \sin 2t\sin 2s\mathbb{E}(Y^2) \\
&= \frac{1}{3}\cos(2(t-s)) + \frac{1}{4}\sin(2(t+s))
\end{aligned}
$$

(2)

$$
\begin{aligned}
\mathbb{E}(\xi(t)) &= \mathbb{E}(X\cos 2t + Y\sin 2t) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
R_\xi(t_1,t_2) &= \mathbb{E}(\xi(t_1)\xi(t_2)) \\
&= \cos 2t\cos 2s\mathbb{E}(X^2) + (\cos 2t\sin 2s + \sin 2t\cos 2s)\mathbb{E}(XY) + \sin 2t\sin 2s\mathbb{E}(Y^2) \\
&= \cos(2(t-s))
\end{aligned}
$$

---

20. 解

可知

$$
\Theta = \frac{\ln Z - \ln X}{\ln 2}
$$

则

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_\Theta(\frac{\ln z - \ln x}{\ln 2}) \frac{1}{z \ln 2} dx$$

因此有

$$f_Z(z) = \begin{cases} \frac{1}{8\ln 2} - \frac{1}{2\ln 2} z^{-1},\ 4 \le z < 8 \\ \frac{1}{2\ln 2} z^{-1},\ 8 \le z < 16 \\ \frac{1}{\ln 2} z^{-1} - \frac{1}{32\ln 2},\ 16 \le z < 32 \\ 0,\ 其它 \end{cases}$$

当给定随机过程$Z(t)$时，相应即有

$$f_{Z(t)}(z) = \begin{cases} \frac{1}{8\ln t} - \frac{1}{2\ln t} z^{-1},\ 4 \le z < 8 \\ \frac{1}{2\ln t} z^{-1},\ 8 \le z < 16 \\ \frac{1}{\ln t} z^{-1} - \frac{1}{32\ln t},\ 16 \le z < 32 \\ 0,\ 其它 \end{cases}$$