1. 解

(a)

$$\alpha_0 =< \varphi_0, v >= [1/\sqrt{2}, 1/\sqrt{2}][3,2]^T = \frac{5\sqrt{2}}{2}$$

$$\alpha_1 =< \varphi_1, v >= \frac{\sqrt{2}}{2}$$

$$v = \frac{5\sqrt{2}}{2}\varphi_0 + \frac{\sqrt{2}}{2}\varphi_1$$

(b)

由双正交可知

$$\alpha_0 =< \widetilde{\varphi}_0, v >= 1$$

$$\alpha_1 =< \widetilde{\varphi}_1, v >= 2$$

$$v = \varphi_0 + 2\varphi_1$$

(c)

$$\alpha_0 =< \widetilde{\varphi}_0, v >=< \frac{2}{3}\varphi_0, v >= 2$$

$$\alpha_1 =< \widetilde{\varphi}_1, v >=< \frac{2}{3}\varphi_1, v >= \frac{2\sqrt{3}}{3} - 1$$

$$\alpha_2 =< \widetilde{\varphi}_2, v >=< \frac{2}{3}\varphi_2, v >= -\frac{2\sqrt{3}}{3} - 1$$

$$v = 2\varphi_0 + (\frac{2\sqrt{3}}{3} - 1)\varphi_1 + (-\frac{2\sqrt{3}}{3} - 1)\varphi_2$$

2. 解

(a)

$$
\begin{aligned}
W_\varphi(1,0) &= \frac{1}{2}[f(0)\varphi_{1,0}(0) + f(1)\varphi_{1,0}(1) + f(2)\varphi_{1,0}(2) + f(3)\varphi_{1,0}(3)] \\
&= \frac{1}{2}[(1)(\sqrt{2}) + (4)(\sqrt{2}) + (-3)(0) + (0)(0)] = \frac{5\sqrt{2}}{2} \\
W_\varphi(1,1) &= \frac{1}{2}[f(0)\varphi_{1,1}(0) + f(1)\varphi_{1,1}(1) + f(2)\varphi_{1,1}(2) + f(3)\varphi_{1,1}(3)] \\
&= \frac{1}{2}[(1)(0) + (4)(0) + (-3)(\sqrt{2}) + (0)(\sqrt{2})] = \frac{-3\sqrt{2}}{2} \\
W_\psi(1,0) &= \frac{1}{2}[f(0)\psi_{1,0}(0) + f(1)\psi_{1,0}(1) + f(2)\psi_{1,0}(2) + f(3)\psi_{1,0}(3)] \\
&= \frac{1}{2}[(1)(\sqrt{2}) + (4)(-\sqrt{2}) + (-3)(0) + (0)(0)] = \frac{-3\sqrt{2}}{2} \\
W_\psi(1,1) &= \frac{1}{2}[f(0)\psi_{1,1}(0) + f(1)\psi_{1,1}(1) + f(2)\psi_{1,1}(2) + f(3)\psi_{1,1}(3)] \\
&= \frac{1}{2}[(1)(0) + (4)(0) + (-3)(\sqrt{2}) + (0)(-\sqrt{2})] = \frac{-3\sqrt{2}}{2}
\end{aligned}
$$

因此，相应的DWT为$\{\frac{5\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\}$

(b)

$$
f(1) = \frac{\sqrt{2}}{4}[5\varphi_{1,0}(1) - 3\varphi_{1,1}(1) - 3\psi_{1,0}(1) - 3\psi_{1,1}(1)] = 1
$$

3. 解

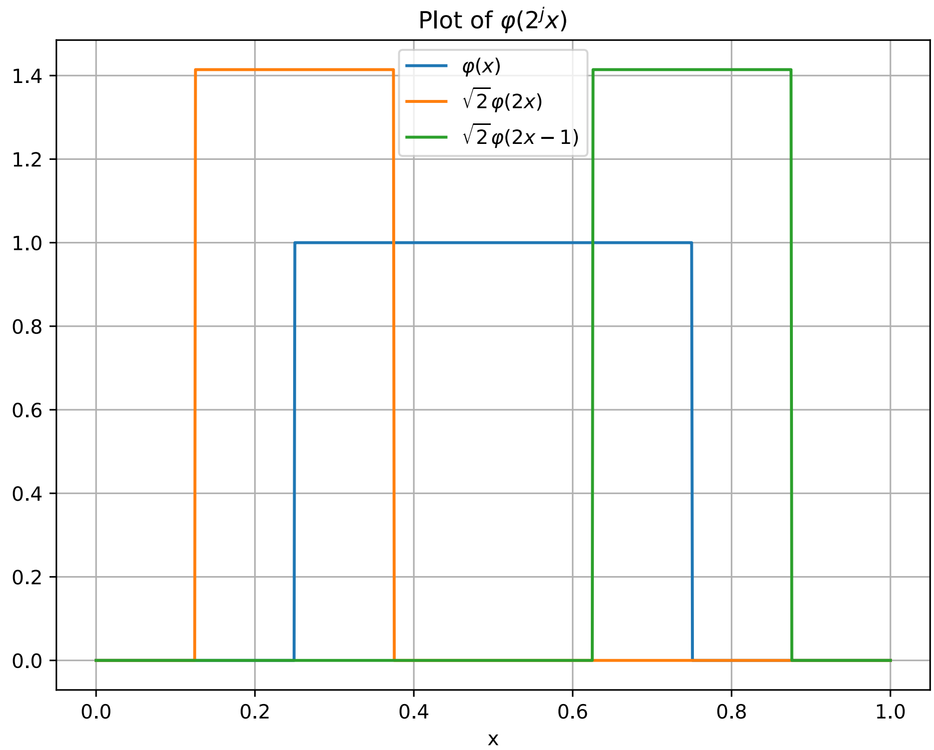
四个条件：

1. 尺度函数与其整数平移对应的函数正交；
2. 低尺度函数张成的子空间包含于高尺度函数张成的子空间
   $V_{-\infty} \subset \ldots \subset V_{-1} \subset V_1 \subset \ldots \subset V_\infty$
3. 唯一包含在所有V_j中的函数是$f(x)=0$

$$V_{-\infty} = \{f|f(x)=0\}$$

4. 任何函数都可以用任意精度表示

$$V_\infty = L^2(\mathbb{R})$$



由上图可知，$\varphi_{0,0}(x)$无法被$\varphi_{1,0}(x), \varphi_{1,1}(x)$线性表示。