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1. 解

$$\begin{aligned}
\mathbb{E}(X_n) &= \mathbb{E}\left(\sum_{k=1}^{N}\sigma_k\sqrt{2}\cos(\alpha_k n - U_k)\right) \\
&= \sum_{k=1}^{N}\sigma_k\sqrt{2}\mathbb{E}\left(\cos(\alpha_k n - U_k)\right) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
R_{X_n}(s,t) &= \mathbb{E}(X_s X_t) \\
&= \mathbb{E}\left[(\sum_{i=1}^{N}\sigma_i\sqrt{2}\cos(\alpha_i s - U_i))(\sum_{j=1}^{N}\sigma_j\sqrt{2}\cos(\alpha_j t - U_j))\right] \\
&= 2\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\cos(\alpha_i s - U_i)\cos(\alpha_j t - U_j)\right] \\
&= 2\sum_{i=1}^{N}\sigma_i^2\mathbb{E}\left[\cos(\alpha_i s - U_i)\cos(\alpha_i t - U_i)\right] \\
&= \sum_{i=1}^{N}\sigma_i^2\mathbb{E}\left[\cos(\alpha_i(s-t)) + \cos(\alpha_i(s+t) - 2U_i)\right] \\
&= \sum_{i=1}^{N}\sigma_i^2\cos(\alpha_i(s-t))
\end{aligned}$$

因此，该随机过程是平稳随机过程。

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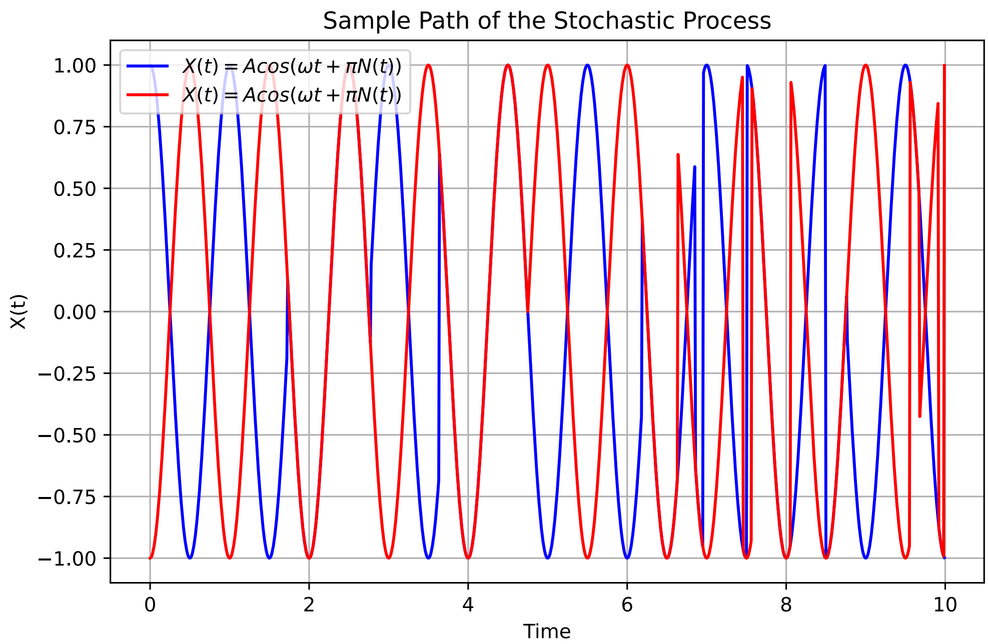
2. 解

(1)

如图所示



其中$\omega = 2\pi$，样本轨道不连续；（因为$\eta(t)$存在跳变）

(2)

不妨设$s > t$

则相应的相关函数为

$$
\begin{aligned}
R_{X_n}(s,t) &= \mathbb{E}(X(s)X(t)) \\
&= \mathbb{E}\left[A^2\cos(\omega s+\pi\eta(s))\cos(\omega t+\pi\eta(t))\right] \\
&= \mathbb{E}(A^2)\mathbb{E}\left[\cos(\omega s+\pi\eta(s))\cos(\omega t+\pi\eta(t))\right] \\
&= \frac{1}{2}\mathbb{E}\left[\cos(\omega(s+t)+\pi(\eta(s)+\eta(t))+\cos(\omega(s-t)+\pi(\eta(s)-\eta(t))\right] \\
&= \frac{1}{2}\mathbb{E}\left[\cos(\omega(s+t)+\pi(\eta(s)-\eta(t))+\cos(\omega(s-t)+\pi(\eta(s)-\eta(t))\right] \\
&= \frac{1}{2}\sum_k(-1)^k\left[\cos(\omega(s+t))+\cos(\omega(s-t))\right]\frac{(\lambda(s-t))^k}{k!}\exp(-\lambda(s-t)) \\
&= \frac{1}{2}\left[\cos(\omega(s+t))+\cos(\omega(s-t))\right]\exp(-2\lambda(s-t)) \\
&= \cos(\omega s)\cos(\omega t)\exp(-2\lambda(s-t))
\end{aligned}
$$

因为$R_X(t,t) = \cos^2(\omega t)$在$(t,t)$上均方连续

所以该过程均方连续。