序号：56

姓名：相贤泰

学号：202328019427026

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7. 解

$$
\begin{aligned}
\mathbb{E}(\xi(t)) &= \mathbb{E}(X)\mathbb{E}(\sin(Yt)) \\
&= \frac{1}{2}\int_0^1 \sin(yt)\, dy \\
&= \frac{1-\cos(yt)}{2t}
\end{aligned}
$$

$$
\begin{aligned}
R_\xi(s,t) &= \mathbb{E}(\xi(t)\xi(s)) \\
&= \mathbb{E}(X^2)\mathbb{E}(\sin(Yt)\sin(Ys)) \\
&= \frac{1}{6}\mathbb{E}\left(\cos(Y(t-s)) - \cos(Y(t+s))\right) \\
&= \frac{1}{6}(\frac{\sin(t-s)}{t-s} - \frac{\sin(t+s)}{t+s})
\end{aligned}
$$

因此，该过程不是**平稳过程**；**连续且可导**

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8. 解

$$
\begin{aligned}
\mathbb{E}(Y) &= \mathbb{E}(T^{-1}\int_0^T X(s)ds) \\
&= T^{-1}\int_0^T \mathbb{E}(X(s))ds \\
&= m
\end{aligned}
$$

$$
\begin{aligned}
&\operatorname{Var}(Y) = E\left\{Y^2\right\} - [E\{Y\}]^2 = E\left\{Y^2\right\} - m^2 \\
&= E\left\{\left[T^{-1}\int_0^T X(s)ds\right]\cdot\left[T^{-1}\int_0^T X(u)du\right]\right\} - m^2 \\
&= T^{-2}\int_0^T\int_0^T E\{X(s)X(u)\}dsdu - m^2 \\
&= T^{-2}\int_0^T\int_0^T R_X(s-u)dsdu - m^2 \\
&= T^{-2}\int_0^T\int_0^T \left[C_X(s-u)+m^2\right]dsdu - m^2 \\
&= T^{-2}\int_0^T\int_0^T ae^{-b|s-u|}dsdu \\
&= 2a\left[(bT)^{-1} - (bT)^{-2}\left(1-e^{-bT}\right)\right]
\end{aligned}
$$

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9. 解

(1)

$$\begin{aligned}\mathbb{E}(X(t)) &= \mathbb{E}(X) + t\mathbb{E}(Y) \\ &= 0\end{aligned}$$

$$\begin{aligned}\mathbb{E}(Y(t)) &= \int_0^t \mathbb{E}(X(u))du \\ &= 0\end{aligned}$$

$$\begin{aligned}Cov(X(s), X(t)) &= \mathbb{E}(X(s)X(t)) \\ &= \mathbb{E}(X^2) + (s+t)\mathbb{E}(XY) + st\mathbb{E}(Y^2) \\ &= \sigma_1^2 + \rho(s+t)\sigma_1\sigma_2 + st\sigma_2^2\end{aligned}$$

$$\begin{aligned}\mathbb{E}(Z(t)) &= \int_0^t \mathbb{E}(X^2(u))du \\ &= \int_0^t (\sigma_1^2 + 2\rho\sigma_1\sigma_2 u + \sigma_2^2 u^2)du \\ &= \sigma_1^2 t + \rho\sigma_1\sigma_2 t^2 + \frac{1}{3}\sigma_2^2 t^3\end{aligned}$$

$$\begin{aligned}Cov(Y(s), Y(t)) &= \mathbb{E}(Y(s)Y(t)) \\ &= \mathbb{E}(\int_0^s X(u)du \int_0^t X(v)dv) \\ &= \int_0^s du \int_0^t \mathbb{E}(X(u)X(v))dv \\ &= \sigma_1^2 st + \frac{1}{2}\rho\sigma_1\sigma_2 st(s+t) + \frac{1}{4}\sigma_2^2 s^2 t^2\end{aligned}$$

(2)

由

$$R_{X(t)}(s,t) = \sigma_1^2 + \rho(s+t)\sigma_1\sigma_2 + st\sigma_2^2$$

可知，$X(t)$均方连续且均方可导

(3)

$$\mathbb{E}(Y'(t)) = \frac{d}{dt}\mathbb{E}(Y(t)) = 0$$

$$R_{Y'}(s,t) = \frac{\partial R_Y(s,t)}{\partial s \partial t} = \sigma_1^2 + \rho\sigma_1\sigma_2(s+t) + \sigma_2^2 st$$

$$\mathbb{E}(Z'(t)) = \frac{d}{dt}\mathbb{E}(Z(t)) = \sigma_1^2 + 2\rho\sigma_1\sigma_2 t + \sigma_2^2 t^2$$

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10. 解

(1)

$$\begin{aligned}R_Y(s,t) &= \mathbb{E}(Y(t+\tau)Y(t)) \\ &= \mathbb{E}(|X(t+\tau)X(t)|) \\ &= \frac{2R_X(0)[\varphi\sin\varphi + \cos\varphi]}{\pi}\end{aligned}$$

其中$\sin\varphi = \frac{R_x(\tau)}{R_X(0)}$

因此，$Y(t)$是平稳过程

(2)

$$\mathbb{E}(Y(t)) = \int_R |x| f_X(x) dx = 2\int_0^{-\infty} x f_X(x) = \sqrt{\frac{2R_x(0)}{\pi}}$$

$$D(Y(t)) = \mathbb{E}(Y^2(t)) - \mathbb{E}^2(Y(t)) = (1-\frac{2}{\pi})R_X(0)$$

(3)

$$\begin{aligned} F_Y(y) &= P(|X(t)| \leq y) \\ &= \int_{-y}^{y} f_X(x) dx \end{aligned}$$

因此

$$f_Y(y) = f_X(y) + f_X(-y) = \frac{2}{\sqrt{2\pi R_X(0)}} \exp\left\{-\frac{y^2}{2R_X(0)}\right\}, \quad y > 0$$