Acousto-Optic Tunable Filter*

S. E. Harris and R. W. Wallace

Department of Electrical Engineering, Stanford University, Stanford, California 94305

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This paper proposes a new type of electronically tunable optical filter. The basic idea is to utilize collinear acousto-optic diffraction in an optically anisotropic media. Changing the driving acoustic frequency changes the band of optical frequencies that the filter passes. A $LiNbO_3$ acousto-optic filter with a pass band approximately 1.3 cm^{-1} wide should be tunable from 4000 to 7000 Å by changing the acoustic frequency from 428 to 990 Mc/sec. For this case, the angular aperture will be about 1.5°, and, theoretically 100% transmittance should be attained at the filter center frequency by use of about 14 mW of acoustic power per mm^2 of filter aperture.

Index Headings: Filter; Polarization; Monochromators; Birefringence; Electro-optics.

In this paper, we propose a new type of electronically tunable optical filter. The basic idea is to utilize collinear acousto-optic diffraction in an optically anisotropic medium.[1] When an acoustic wave travels in a solid or liquid, the strain-induced change of the refractive index of the medium may diffract a light beam that is incident on the medium. In an isotropic medium, the polarization of the diffracted light is unchanged and the diffraction is particularly strong when the light is incident at the Bragg angle.[2] In an anisotropic medium, for certain crystal orientations, light may be diffracted from one polarization to another.[1] In this case, the condition for particularly strong interaction between the acoustic wave and the light wave is that the sum of the $\bar{k}$ vectors of the incident light and the acoustic wave equal the $\bar{k}$ vector of the orthogonally polarized diffracted wave.[3]

In the acousto-optic filter proposed here, a crystal orientation is chosen such that an incident optical signal of one polarization is diffracted into the orthogonal polarization by a collinearly propagating acoustic beam. For a given acoustic frequency, only a small range of optical frequencies will satisfy the $\bar{k}$ vector-matching condition, and only this small range of frequencies will be cumulatively diffracted into the orthogonal polarization. If the acoustic frequency is changed, the band of optical frequencies which the filter will pass is changed.

A $LiNbO_3$ acousto-optic filter will be shown to have a pass band approximately 1.3 cm wide, which is tunable from 4000 Å to 7000 Å by changing the acoustic frequency from 428 Mc/sec to 990 Mc/sec. For this case, the angular aperture will be about 1.5°, and theoretically, 100% transmittance at the filter center frequency should be obtained with 14 mW of propagating acoustic power per mm^2 of filter aperture.

ANALYSIS

The proposed acousto-optic filter consists of an input polarizer, a crystal with an appropriate acoustic transducer, and an output polarizer. A number of different crystal orientations, involving either longitudinal or shear waves, would allow collinear diffraction of light into the orthogonal polarization. One possible configuration for the filter using $LiNbO_3$ is shown in Fig. 1. In this case, the acoustic wave is brought in as a longitudinal wave, which is then converted to a shear wave upon reflection at the input face of the crystal.[4] The acoustic shear wave and the input optical beam then propagate collinearly down the y axis of the crystal, along which the acousto-optic interaction takes place.

We take the input optical beam to be an extraordinary wave polarized along the z or optic axis of the crystal. The output or diffracted optical beam will be an ordinary wave polarized along the x axis of the crystal. The acoustic wave that is necessary to accomplish the diffraction into the orthogonal polarization is an S_6 shear wave, and is set up in the configuration of Fig. 1. The three waves are then taken as plane waves and are given by

$$\hat{E}_z(y,t)=[E_z(y)/2]\exp j(\omega_e t-k_e y)+\text{c.c.}$$

(input optical wave)

$$\hat{E}_x(y,t)=[E_x(y)/2]\exp j(\omega_0 t-k_0 y)+\text{c.c.} \qquad (1)$$

(output optical wave)

$$\hat{S}_6(y,t)=[S_6(y)/2]\exp j(\omega_a t-k_a y)+\text{c.c.}$$

(acoustic shear wave)

The quantities ω_e, ω_0, ω_a, and k_e, k_0, k_a are the angular frequencies and $\bar{k}$ vectors of the input optical wave, output optical wave, and acoustic wave, respectively. The symbol $\wedge$ denotes variables which have the complete time and spacial dependence, as opposed to the envelope variables $E_z(y)$, etc. The acoustic wave mixes

* The work reported here was sponsored by the National Aeronautics and Space Administration under Grant NGR-05-020-103.

[1] R. W. Dixon, IEEE J. Quant. Elect. QE-3, 85 (1967).

[2] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, Inc., New York, 1964), p. 593–609.

[3] In fact, the Bragg condition is a special case of $\bar{k}$ vector matching and is strictly correct only when the acoustic frequency is negligibly small as compared with the optical frequency.

[4] E. G. H. Lean and H. J. Shaw, Appl. Phys. Letters 9, 372 (1966).

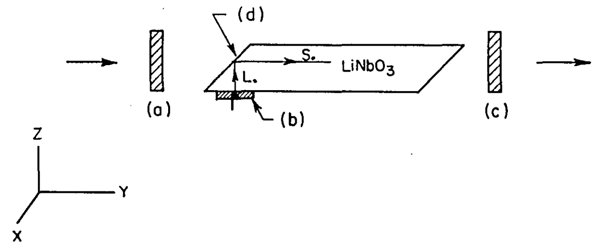


FIG. 1. $LiNbO_3$ acousto-optic filter. Longitudinal wave (L), shear wave (S).

with the input optical signal to produce forcing optical polarization waves at frequencies $\omega_e+\omega_a$, and $\omega_e-\omega_a$. These forcing waves propagate with $\bar{k}$ vectors whose magnitudes are k_e+k_a and k_e-k_a, respectively. Only if the $\bar{k}$ vector of this forcing wave is equal or nearly equal to that of the freely propagating electromagnetic wave, will a cumulative interaction over many wavelengths take place. In $LiNbO_3$, the ordinary refractive index is greater than the extraordinary index, which for forward-propagating waves requires phase matching such that $k_e+k_a=k_0$; this, in turn, results in the frequency of the ordinary wave (the output frequency in our example) being greater than that of the extraordinary wave by ω_a.

The interaction between the acoustic and optical waves takes place as a result of the photoelastic effect. This effect is described as a perturbation of the elements of the impermeability tensor b_{ij} such that $\Delta b_{ij}=p_{ijkl}S_{kl}$, where p_{ijkl} are the components of the photoelastic tensor and S_{kl} is the propagating strain wave. This perturbation of the impermeability tensor is equivalent to the creation of a driving polarization, which for our example may be shown to be given by

$$\hat{P}_x=-e_0n_0^2n_e^2p_{41}\hat{S}_6\hat{E}_z$$
$$\hat{P}_z=-e_0n_0^2n_e^2p_{41}\hat{S}_6\hat{E}_x, \tag{2}$$

where e_0 is the dielectric constant of free space and n_0 and n_e are the refractive indices for the ordinary and extraordinary waves respectively. If we substitute Eqs. (1) and (2) into the one-dimensional driven-wave equation for lossless media, i.e.,

$$\frac{\partial^2\hat{E}}{\partial y^2}-\frac{1}{c^2}\frac{\partial^2\hat{E}}{\partial t^2}=\mu_0\frac{\partial^2\hat{P}}{\partial t^2} \tag{3}$$

and make use of the fact that $E_z(y)$ and $E_x(y)$ are slowly varying functions of y, then the following coupled complex equations may be obtained

$$\frac{dE_x}{dy}=j\frac{n_0n_e^2p_{41}\omega_0}{4c}S_6E_z\exp(j\Delta ky)$$
$$\frac{dE_z}{dy}=j\frac{n_en_0^2p_{41}\omega_e}{4c}S_6{}^*E_x\exp(-j\Delta ky), \tag{4}$$

where we have defined a $\bar{k}$ vector mis-match $\Delta k=k_0-k_e-k_a$. In these equations, the acoustic wave is assumed to propagate losslessly and thus the acoustic strain S_6 is assumed to be independent of position in the crystal.

Equations (4) are now solved subject to the boundary condition that $E_x=0$ and $E_z=E_z(0)$ at $y=0$. The ratio of the output power at $y=L$, $P_x(L)$, to the input power at $y=0$, $P_z(0)$ is found to be given by

$$\frac{P_x(L)}{P_z(0)}=\left(\frac{\omega_0}{\omega_e}\right)\Gamma^2L^2\frac{\sin^2\left[\left(\Gamma^2+\frac{\Delta k^2}{4}\right)^{\frac{1}{2}}L\right]}{\left(\Gamma^2+\frac{\Delta k^2}{4}\right)L^2}, \tag{5}$$

where

$$\Gamma^2=\frac{n_0^3n_e^3p_{41}^2\omega_0\omega_e}{16c^2}|S_6|^2.$$

We note that the frequency of the transmitted optical signal differs from that of the input signal by the acoustic frequency ω_a. There is also an insignificant Manley–Rowe-type power gain of magnitude ω_0/ω_e, which we neglect hereafter.

TRANSMITTANCE, TUNING RATE, BANDWIDTH, AND APERTURE

From Eq. (5) it is clear that the maximum transmittance of the filter will be attained when the input optical frequency is such that the momentum mis-match $\Delta k=0$. For this condition, we have

$$P_x(L)/P_z(0)=\sin^2\Gamma L \tag{6}$$

and thus for theoretical 100% peak transmittance we require $\Gamma L=\pi/2$. Expressing $|S_6|^2$ in terms of the acoustic power density P_A/A, we obtain

$$\Gamma^2=\frac{n_0^3n_e^3p_{41}^2\pi^2}{2\lambda_0^2}\frac{1}{\rho V^3}\frac{P_A}{A}, \tag{7}$$

where λ_0 is the optical wavelength, ρ is the density of the medium, V is the acoustic velocity, P_A is the total acoustic power, and A is the area of the acoustic and optical beams.

For a 5-cm-long crystal of $LiNbO_3$ at a central transmission frequency of $\lambda_0=5000$ Å, we have $p_{41}=0.155$,[5] $n_0=2.3$, $n_e=2.2$, $\rho=4.64$ g/cm³, $V=4.0\times10^5$ cm/sec; we therefore require an acoustic power density of 14 mW/mm² of filter aperture for 100% peak transmittance.

With the acoustic power adjusted to provide peak transmittance at the center frequency ($\Gamma L=\pi/2$), the frequency response of the filter is determined by the variation of Δk as the optical frequency is changed.

[5] R. W. Dixon, J. Appl. Phys. 38, 5149 (1967).

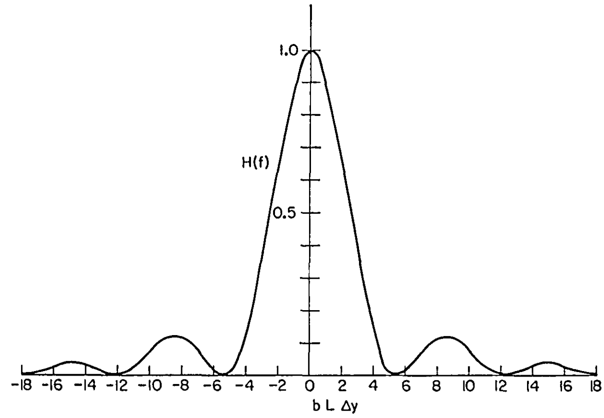


FIG. 2. Filter transmittance vs normalized frequency.

We let

$$\Delta k=\left(\frac{\partial k_0}{\partial y}-\frac{\partial k_e}{\partial y}\right)\Delta y$$

$$\equiv b\Delta y, \tag{8}$$

where Δy is the change in wave numbers of the optical frequency from the center frequency of the filter. From Eq. (5), the optical-frequency-response function of the filter $H(f)$ may then be written

$$H(f)=\pi^2\frac{\sin^2\frac{1}{2}(\pi^2+b^2L^2\Delta y^2)^{\frac{1}{2}}}{\pi^2+b^2L^2\Delta y^2}. \tag{9}$$

Figure 2 shows the transmittance $H(f)$ plotted vs the normalized frequency variable $bL\Delta y$. The half-peak transmittances of the primary lobe of the filter occur when $bL\Delta y \cong \pm 2.5$. For $LiNbO_3$, the constant b may be obtained by differentiation of the Sellmeier equations of Hobden and Warner.[6] The result of this differentiation is given in Fig. 3, as a function of the optical wavelength, at a temperature of 200°C. This temperature was chosen

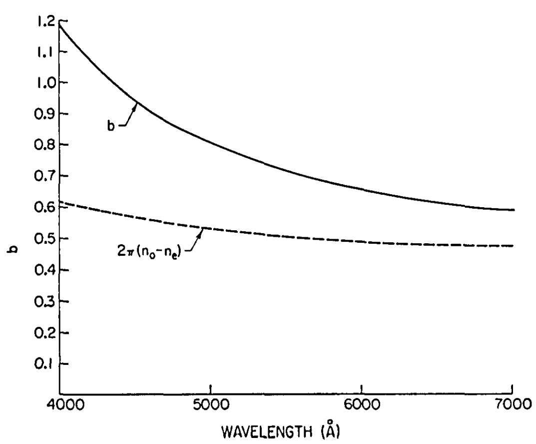


FIG. 3. b and $2\pi(n_0-n_e)$ vs λ.

[6] M. V. Hobden and J. Warner, Phys. Rev. Letters **22**, 243 (1966).

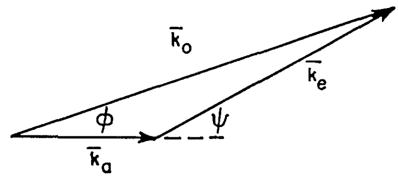


FIG. 4. $\bar{k}$ vector matching for nearly collinear propagation (not to scale).

because $LiNbO_3$ exhibits optical damage at temperatures lower than about 160°C.[7] Figure 3 shows that b is somewhat greater than the value $2\pi(n_0-n_e)$ that it would have in the absence of optical dispersion. The quantity $2\pi(n_0-n_e)$ is also shown in Fig. 3. For a 5-cm-long crystal of $LiNbO_3$ at 5000 Å, the total half-peak bandwidth of the first lobe of the filter ($\cong 5/bL$) is about 1.25 cm^{-1}=0.31 Å.

Electronic tuning of the filter may be accomplished by changing the frequency of the acoustic wave, thereby changing the length of its $\bar{k}$ vector. The acoustic frequency that will yield peak transmittance at an optical wavelength λ_0 is

$$f_a=(V/\lambda_0)(n_0-n_e), \tag{10}$$

where V is the acoustic velocity. For $LiNbO_3$ $V=4\times10^5$ cm/sec and n_0-n_e may be obtained from Fig. 3. The necessary acoustic frequency for peak transmittance at 5000 Å is 680 Mc/sec, and the region from 7000 Å to 4000 Å can be tuned by changing the acoustic frequency from about 428 Mc to about 990 Mc/sec. The rate of change of optical wave number per cycle change of the acoustic frequency is $\Delta y/\Delta f_a=2\pi/bV$, where b is defined in Eq. (8) and plotted in Fig. 3. We see that the average tuning rate for $LiNbO_3$ is about 20 wave numbers per megacycle change of the acoustic frequency. As the acoustic frequency is changed, the acoustic power should be varied inversely as the square of the acoustic frequency, if 100% peak filter transmittance is to be maintained [note Eqs. (6), (7), and (10)].

The optical angular aperture of the filter at the input frequency corresponding to peak collinear transmittance is determined by $\bar{k}$ vector mismatch. The half-peak, half-angle aperture occurs when $\Delta kL \cong \pi$. For nearly collinear propagation, we obtain from Fig. 4

$$\Delta k=k_0\cos\varphi-k_e\cos\psi-k_a$$

$$\cong k_0-k_e-k_a+\left(k_e-\frac{k_e^2}{k_0}\right)\frac{\psi^2}{2}$$

$$\cong(\pi/\lambda)\Delta n\psi^2. \tag{11}$$

The half-angle aperture taken inside the crystal is then about $\psi=(\lambda/L\Delta n)^{\frac{1}{2}}$. This is magnified by refraction at

[7] A. Ashkin, G. D. Boyd, J. M. Dziedzic, R. G. Smith, M. A. Ballman, J. J. Levinstein, and K. Nassau, Appl. Physics Letters **9**, 72 (1966).

the input of the crystal to yield a total aperture of about $2n_e(\lambda/L\Delta n)^{\frac{1}{2}}$. For a 5-cm crystal of $LiNbO_3$ at $\lambda = 5000$ Å, this yields a half-peak aperture, external to the crystal, of approximately 0.02 rad or 1.15°.

DISCUSSION

Probably the most severe limitation of the proposed filter is the difficulty of obtaining large apertures. Since we require 14 mW of propagating acoustic power per mm^2 of crystal aperture, a 1-cm-square aperture would require an acoustic power of 1.4 W. Broadband RF to acoustic transducers can now be constructed with about 10 dB conversion loss, thus requiring an RF power of 14 W. Also, at frequencies in the 400 to 1000 Mc/sec range, the construction of transducers having one cm^2 of area is somewhat ahead of the present state of the art.

The present analysis has neglected the acoustic attenuation that occurs as the acoustic wave propagates down the crystal. At room temperature, this attenuation should be about 0.75 dB/cm at 1000 Mc/sec[8]; and should vary approximately as the square of the acoustic frequency. Its effect will be equivalent to shortening the crystal and will thus lead to somewhat larger bandwidths and necessitate somewhat higher acoustic drive powers.

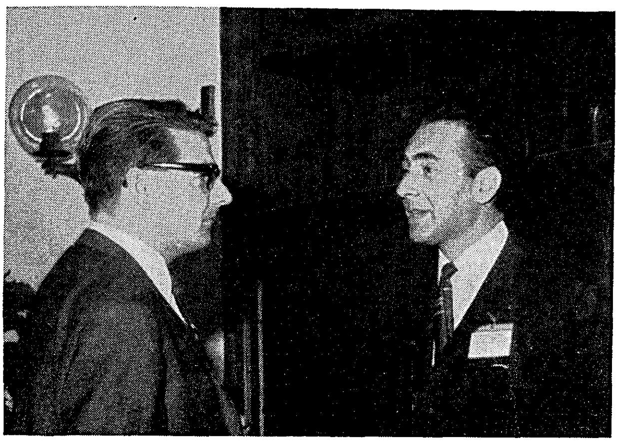
$LiNbO_3$ and the filter configuration of Fig. 1 are only one of a number of possible crystals and configurations that could be employed. The advantage of this configuration is that it allows the acoustic wave to be brought in at right angles to the light, and thus does not require the light to pass through an acoustic transducer. A disadvantage is that for shear-wave propagation down the y axis there is an approximately 7° divergence between the direction of the acoustic power flow and the acoustic $\bar{k}$ vector. This requires that the filter aperture be at least one part in ten of the crystal length. However, other crystal orientations allow diffraction into the orthogonal polarization and do not exhibit this divergence. For example, collinear propagation of a longitudinal acoustic wave and the optical signal down the x axis of a $LiNbO_3$ crystal accomplishes the desired result.[1]

Two other materials that may be useful for this type of filter are sapphire and quartz,[1] which have the same photoelastic tensor as does $LiNbO_3$. The birefringence of both of these materials is about 1/10 that of $LiNbO_3$. As a result, the necessary acoustic frequencies would be centered about 70 Mc/sec instead of 700 Mc/sec as in the $LiNbO_3$ filter. Both the tuning rate and also the bandwidth of these filters (for the same crystal length) would be about ten times as large as that of the $LiNbO_3$ filter. The angular aperture would be about three times as large as that of a $LiNbO_3$ filter of the same length. As a result of the lower refractive indices of these crystals, about ten to twenty times as much acoustic power would probably be required to obtain the theoretical 100% peak transmittance. However, this might be offset by use of longer crystals.

ACKNOWLEDGMENT

The authors gratefully acknowledge many helpful discussions with C. F. Quate.

[8] C. P. Wen and R. F. Mayo, Appl. Phys. Letters 9, 135 (1966).



At ICO-sponsored conference on Applications of Coherent Light, Florence, Italy, September 1968. Left: K. Schindl (Technische Hochschule) and C. Reichert (Opt. Werke AG, Vienna).