NCAR/TN–???+STR
NCAR TECHNICAL NOTE
February 2007

The WRF Variational Data Assimilation System (WRF-Var) Version 2

Dale M. Barker
John R. Bray
Meral Demirtas
Yong-Run Guo
Hans Huang
Hui-Chuan Lin
Zhiquan Liu
Syed R. H. Rizvi
Hui Shao
Qingnong Xiao

Mesoscale and Microscale Meteorology Division
National Center for Atmospheric Research
Boulder, Colorado, USA

NCAR TECHNICAL NOTES

The Technical Note series provides an outlet for a variety of NCAR manuscripts that contribute in specialized ways to the body of scientific knowledge but which are not suitable for journal, monograph, or book publication. Reports in this series are issued by the NCAR Scientific Divisions; copies may be obtained on request from the Publications Office of NCAR. Designation symbols for the series include:

EDD: *Engineering, Design, or Development Reports*
Equipment descriptions, test results, instrumentation, and operating and maintenance manuals.

IA: *Instructional Aids*
Instruction manuals, bibliographies, film supplements, and other research or instructional aids.

PPR: *Program Progress Reports*
Field program reports, interim and working reports, survey reports, and plans for experiments.

PROC: *Proceedings*
Documentation of symposia, colloquia, conferences, workshops, and lectures. (Distribution may be limited to attendees.)

STR: *Scientific and Technical Reports*
Data compilations, theoretical and numerical investigations, and experimental results.

The National Center for Atmospheric Research (NCAR) is operated by the University Corporation for Atmospheric Research (UCAR) and is sponsored by the National Science Foundation. Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Contents

Appendices

List of Figures

List of Tables

Acknowledgments

Many people beyond this document's author list have contributed to the development of the WRF-Var system.

NCAR scientists (Dale)

SE achnowledgements (Bray)

The development of WRF-Var represents an international team effort. We would like to acknowledge the following people for their contributions to the WRF-Var system: Mike McAtee, Roy Peck, Steve Rugg, Jerry Wegiel, Wan-Shu Wu, Dezso Devenyi, Mi-Seon Lee, Ki-Han Youn, Eunha Lim, Hyun-Cheol Shin, Shu-Hua Chen, Ananda Das, Ashish Routray, etc.....

Tech note reviewers (Dale)

The WRF-Var effort is supported by the National Science Foundation (ATM and Office of Polar Programs), the US Air Force Weather Agency, NASA, the Korean Meteorological Administration, the Japanese Central Research Institute for the Power Industry, the Taiwanese Civil Aeronautics Administration and Central Weather Bureau, and the Beijing Meteorological Bureau.

Chapter 1

Introduction

Barker, Huang

1.1 Overview of Variational Data Assimilation

1.2 Overview of WRF-Var

Chapter 2

Minimization Algorithm

Rizvi, Barker

2.1 Inner/Outer Loop Structure

2.2 Conjugate Gradient Minimization

etc....

Rizvi, Barker

Chapter 3

Background Error Covariance Estimation

Barker, Lin

3.1 Overview

Forecast (first guess or background) error covariances are a vital input to any modern data assimilation system. They influence the analysis fit to observations and also completely define the analysis response away from observations. The latter impact is particularly important in data-sparse areas of the globe. Unlike ensemble filter data assimilation techniques (e.g. the Ensemble Adjustment Kalman Filter, the Ensemble Transform Kalman Filter), 3/4D-Var systems do not implicitly evolve forecast error covariances in real-time. Instead, climatologic statistics are typically estimated offline, and tuned to represent forecast errors for particular applications. This document describes the generation and tuning of forecast error covariances for the WRF-Var system.

WRF-Var is a freely available community variational data assimilation system. It is used in a number of applications spanning convective to synoptic scales, tropical to polar domains, and variable observation distributions. In addition, the WRF-Var system is used to provide initial conditions for a number of forecast models in addition to WRF, e.g. MM5, the Korean Meteorological Administration's (KMA's) global spectral model, and the Taiwanese Nonhydrostatic Forecast Model (NFS). Perhaps understandably, the issue of forecast error generation for WRF-Var applications is the number 1 user question to date. For all these reasons, it is therefore vital that an accurate, portable, flexible, and efficient utility to generate and tune forecast error statistics be made available for community use.

In the next subsection, a brief overview of the role of forecast error covariances in variational data assimilation is given. Default forecast error statistics are supplied with the WRF-Var release. These data files are made available primarily for training purposes, i.e. to permit the user to configure and test the WRF-Var/WRF system in their own application. It is important to note that the default statistics are not intended for extended testing or real-time applications of WRF-Var. For those applications, there is no substitute to creating one's own forecast error covariances with the *gen_be* utility, described in subsection (3.3). Having created domain-specific statistics, it may then be necessary to further tune both forecast and observation error statistics. A variety of algorithms are supplied with WRF-Var to perform this tuning, and documented in section 4. These tuning algorithms include both innovation vector-based approaches (Hollingsworth and Lonnberg, 1986) and variational tuning approaches (Desroziers and Ivanov, 2001). Clearly, the whole process of tuning error statistics for data assimilation is rather complex. However, it has been shown in many applications, that this work is vital if one is to produce the optimal analysis.

3.2 Scientific Background

As shown in Chapter (1), the basic goal of any variational data assimilation system is to produce an optimal estimate of the true atmospheric state at analysis time through iterative solution of a prescribed cost-function J. The particular variational data assimilation algorithm adopted in WRF-Var is a model-space, incremental formulation of the variational problem. In this approach, observations, previous forecasts, their errors, and physical laws are combined to produce analysis increments $\partial \mathbf{x^a}$, which are added to the first guess xb to provide an updated analysis.

The background (or "first guess") error covariance matrix is defined as

$$\mathbf{B} = \overline{\epsilon\epsilon^T} \simeq \overline{\mathbf{x'x'}^T}, \tag{3.1}$$

where the overbar denotes an average over time, and/or geographical area. The true background error ϵ is not known in reality, but is assumed to be statistically well represented by a model state perturbation $\mathbf{x}'$. In the standard NMC-method (Parrish and Derber, 1992), the perturbation $\mathbf{x}'$ is given by the difference between two forecasts (e.g. 24 hour minus 12 hour) verifying at the same time. Climatological estimates of background error may then be obtained by averaging such forecast differences over a period of time (e.g. one month). An alternative strategy proposed by (Fisher, 2003) makes use of ensemble forecast output, defining the $\mathbf{x}'$ vectors as ensemble perturbations (ensemble minus ensemble mean). In either approach, the end results is the same - an ensemble of model perturbation vectors from which estimates of background error may be derived. The new *gen_be* code has been designed to work with either forecast difference, or ensemble-based perturbations.

In model-space variational data assimilation systems, the background error covariances are specified not in model space $\mathbf{x}'$, but in a control variable space $\mathbf{v}$, related to the model variables (e.g. wind components, temperature, humidity, and surface pressure) via a control variable transform U defined by

$$\mathbf{x}' = \mathrm{U}\mathbf{v} = \mathrm{U}_p\mathrm{U}_v\mathrm{U}_h\mathbf{v}. \tag{3.2}$$

where the breakdown into components of U represents transformations of variable (p), and vertical (v) and horizontal (h) components of spatial error covariances. The transform (3.2), and its adjoint are required in the WRF-Var code, and will be dscribed in more detail in Chapter (4). In contrast, the gen_be code performs the inverse transform

$$\mathbf{v} = \mathrm{U}\mathbf{v} = \mathrm{U}_h^{-1}\mathrm{U}_v^{-1}\mathrm{U}_p^{-1}\mathbf{x}'. \tag{3.3}$$

In order to accumulate statistics for each component of the control vector $\mathbf{v}$. The detailed procedure for performing this calculation is described in the next section.

3.3 WRF's Forecast Error Covariance Estimator - gen_be

The background error covariance generation code gen_be is designed to take input from a variety of regional/global models (e.g. WRF, MM5, KMA global model, etc.), and process it in order to provide error covariance statistics. Note only the first stage (stage0) is model-specific - the output from stage0 is in a standard model-independent format. The gen_be output forecast error statistics may be used in a variety of ways, including: i) Forecast error covariances for multi-model applications of WRF-Var, ii) Multi-model error intercomparison studies, iii) Studying multivariate correlations within ensemble prediction systems, and iv) Defining optimal control variable/balance constraints for particular applications.

The gen_be utility comprises a number of fortran codes, as described in the following subsections.

3.3.1 gen_be_stage0_wrf - Calculate perturbations of standard fields

This stage is the only part of the process that has knowledge of the model supplying the forecast data and is specific to individual models. The purpose of this stage is to read model-specific, full field forecast data, to create model perturbations $\mathbf{x}'$, convert to standard perturbation fields (and metadata), and output them in a standard binary format for further processing as illustrated in Fig. 1.

Using the NMC-method, $\mathbf{x}' = \mathbf{x_{T2}} - \mathbf{x_{T1}}$ where $T2$ and $T1$ are the forecast difference times (e.g. 48hr, 24hr for global, 24hr, 12hr for regional). Alternatively, for an ensemble-based approach, $\mathbf{x_k}' = \mathbf{x_k} - \bar{\mathbf{x}}$ where the overbar is an average over ensemble members $k = 1, n_e$. The total number of binary files produced by stage0 is $n_f \times n_e$ where n_f is the number of forecast times used (e.g. for 30 days with forecast every 12 hours, $n_f = 60$). Using the NMC-method, $n_e = 1$ (1 forecast difference per time). For ensemble-based statistics, n_e is the number of ensemble members.

Fig. 1: Sketch of gen_be's stage0 program. Model-specific (e.g. gen_be_stage0_wrf for WRF) processing of full field forecast fields is performed to convert to perturbation fields of "standard" variables and relevant metadata, e.g. latitude, height, land/sea, etc.

Input: Model-specific forecast files.

Processing: Read forecasts, calculate $\mathbf{x}'$ perturbations, transform to standard perturbation fields.

Output: File containing standard fields in specific binary format, with filename e.g. diff.2003-01-01_12:00:00 (perturbation valid at 12Z on 1 January 2003).

The standard gridpoint fields are:

- Perturbations: Streamfunction $\psi'(i,j,k)$, velocity potential $\chi'(i,j,k)$, temperature $T'(i,j,k)$, relative humidity $r'(i,j,k)$, surface pressure $p_s'(i,j)$.
- Full-fields: height $z(i,j,k)$, latitude $\phi(i,j)$. (These are required for the production of background error statistics stored in terms of physics variables, rather than tied to a specific grid. This flexibility is included in *gen_be* through a namelist option to define the bins over which data is averaged in a variety of ways (e.g., latitude height, grid points). Land-sea and orographic effects may also be represented in this way.

The calculation of streamfunction and velocity potential from input u and v fields is performed via a Fast Fourier Transform (FFT) decomposition of computed vorticity and divergence fields. Each model has it's own stage0 (only that for WRF - gen_be_stage0_wrf is supported at present) which outputs a common format so that the remaining stages below are the same for all model data.

3.3.2 gen_be_stage1 - Remove Mean

This stage simply reads the difference fields output by stage0 and removes the time/bin mean from each standard field and level. The zero-mean fields are output to separate directories/files for each variable, date and member (e.g. psi/2003010112.psi.NMC contains the 3D psi field from NMC-method data, with mean removed).

Input: Perturbation fields (output of stage 0)

Processing: Remove mean for each variable/level.

Output: Standard fields: $z(i,j,k)$, $\phi(i,j)$, $\psi'(i,j,k)$, $\chi'(i,j,k)$, $T'(i,j,k)$, $r'(i,j,k)$, $p'_s(i,j)$.

3.3.3 gen_be_stage2 - Calculate balance regressions coefficients

The standard control variables (i.e. those variable for which we assume cross-correlations are zero) in WRF-Var are streamfunction, "pseudo" relative humidity, and the unbalanced components of velocity potential, temperature, and surface pressure.

The unbalanced control variables are defined as the difference between full and balanced (or correlated) components of the field. In this stage of the calculation of background errors, the balanced component of particular fields is modeled via a regression analysis of the field using specified predictor fields (e.g. streamfunction). (see Wu et al. (2002) for further details). The resulting regression coefficients are output for use in WRF-Vars U_p transform, and are also used in gen_be_stage2a (see below). Currently, three regression analyses are performed resulting in three sets of regression coefficients (note: drop the perturbation notation from now on for clarity):

- Velocity potential/streamfunction regression: $\chi_b = c\psi$;
- Temperature/streamfunction regression: $T_{b,k1} = \sum_{k2} G_{k1,k2}\psi_{k2}$; and
- Surface pressure/streamfunction regression: $p_{sb} = \sum kW_k\psi_k$.

Data is read from all $n_f \times n_e$ files and sorted into bins defined via the namelist option *bin_type*. Regression coefficients $G(k1,k2)$ and $W(k)$ are computed individually for each bin (*bin_type*=1 is used here, representing latitudinal dependence) in order to allow representation of differences between e.g. polar, mid-latitude, and tropical dynamical and physical processes. In addition, the scalar coefficient c used to estimate velocity potential errors from those of streamfunction is calculated as a function of height to represent e.g. the impact of boundary-layer physics. Latitudinal /height smoothing of the resulting coefficients may be optionally performed to avoid artificial discontinuities at the edges of latitude/height boxes.

In summary, gen_be_stage2 proceeds as follows:

Input: Standard fields (output of stage 1).

Processing: Regression analysis to determine multivariate correlations between perturbation fields.

Output: Regression coefficients c, W, G.

3.3.4 gen_be_stage2a - Calculate unbalanced control variable fields

Having computed regression coefficients, the unbalanced components of the fields are calculated as $\chi_u = \chi - c\psi$, $T_{u,k1} = T_{k1} - \sum_{k2} G_{k1,k2}\psi_{k2}$, and $p_{su} = p_s - \sum_k W_k \psi_k$. These fields are output for the subsequent calculation of the spatial covariances as described below.

Input: Standard fields (output of stage 1) and regression coefficients (output of stage 2).

Processing: Compute unbalanced components of selected fields.

Output: Unbalanced fields χ_u, T_u, and p_{su}.

3.3.5 gen_be_stage3 - Eigenvectors/values of Vertical Error Covariances

The gen_be_stage3 program calculates the statistics required for the vertical component of the control variable transform of WRF-Var. This involves the projection of 3D fields on model-levels onto empirical orthogonal functions (EOFs) of the vertical component of background error covariances (Barker et al., 2004).

The *gen_be* code calculates both domain-averaged and local values of the vertical component of the background error covariance matrix. The definition of local again depends on the value of the namelist variable bin_type chosen. For example, for bin_type=1, a $kz \times kz$ (where kz is the number of vertical levels) vertical component of $\mathbf{B}$ is produced at every latitude (data is averaged over time and longitude) for each control variable. Eigendecomposition of the resulting climatological vertical error covariances $\mathbf{B} = \mathbf{E}\Lambda\mathbf{E}^T$ results in both domain-averaged and local eigenvectors $\mathbf{E}$ and eigenvalues Λ. Both sets of statistics are included in the dataset supplied to WRF-Var, allowing the choice between homogeneous (domain-averaged) or local (inhomogeneous) background error variances and vertical correlations to be chosen at run time (Barker et al., 2004).

Having calculated and stored eigenvectors and eigenvalues, the final part of *gen_be_stage3* is to project the entire sequence of 3D control variable fields into EOF space $\mathbf{v_v} = U_v^{-1}\mathbf{v_p} = \Lambda^{-1/2}\mathbf{E}^T\mathbf{v_p}$.

Input: 3D (i,j,k) control variable fields: ψ, χ_u, T_u, and r (output from stage 1 and 2a).

Processing: For each variable: compute vertical component of $\mathbf{B}$, perform eigenvector decomposition, and compute projections of fields onto eigenvectors.

Output: Eigenvectors $\mathbf{E}$, eigenvalues Λ, and projected 3D (i,j,m) control variable fields: ψ, χ_u, T_u, and r for calculation of horizontal correlations.

3.3.6 gen_be_stage4 - Horizontal Error Correlations

The last aspect of the climatological component of background error covariance data required for WRF-Var are the horizontal error correlations, the representation of which forms the largest difference between running WRF-Var in regional and global mode (it is however, still a fairly local change).

In global applications (gen_be_stage4_global). a spectral decomposition of the grid-point data is perfomed, and power spectra computed for each variable and vertical mode. Details of the spectral technique used to project gridpoint fields to spectral modes are given in Appendix B together with a description of the power spectra computed from the spectral modes.

In regional applications (gen_be_stage4_regional), a recursive filter is used to provide the horizontal correlations (Barker et al. (2004)). The error covariance inputs to the recursive filter is a correlation lengthscale (provided by gen_be_stage4_regional), and a WRF-Var namelist parameters to define the correlations shape (the number of passes through the recursive filter). The calculation of horizontal lengthscales in gen_be_stage4_regional is described in section 8c of (Barker et al. (2004)). Note: This is the most expensive part of the entire gen_be process.

Input: Projected 3D (i,j,m) control variable fields: ψ, χ_u, T_u, and r, and p_{su}(i,j).

Processing (regional): Perform linear regression of horizontal correlations to calculate recursive filter lengthscales (see Barker et al. (2004)).

Processing (global): Perform horizontal spectral decomposition, and compute power spectra for each field/mode (see Appendix B).

Output (regional): Recursive filter lengthscales for each control variable, vertical mode. Output (global): Power spectra for each control variable, vertical mode.

3.4 Example Results From An Application Of gen_be

Chapter 4

Background Error Covariance Modeling

Barker, Rizvi

Chapter 5

Conventional Observations

Guo, Rizvi

5.1 Observation Preprocessing

5.2 Observation Quality Control

5.3 Observation Data Formats

5.3.1 BUFR

5.3.2 ASCII format

5.4 First Guess at Appropriate Time

Chapter 6

Radiance Data Assimilation

Liu, Vukicevic

Chapter 7

Radar Data Assimilation

Xiao, Sun, Lim, etc?

Chapter 8

Data Assimilation Diagnostics

Barker, Shao

Chapter 9

WRF Suite Scripts

Barker, Demirtas

Chapter 10

Software Engineering

Bray, Michalakes

Appendix A

Physical Constants

This is the WRF version. Needs to be updated for WRF-VAR!!!!!!!

The following is a list of physical constants used in the model.

π	$=$	3.1415926	Pi
k	$=$	0.4	Von Karman constant
r_e	$=$	$6.370 \times 10^6 \quad \mathrm{m}$	Radius of earth
g	$=$	$9.81 \quad \mathrm{m\,s^{-2}}$	Acceleration due to gravity
Ω_e	$=$	$7.2921 \times 10^{-5} \quad \mathrm{s^{-1}}$	Angular rotation rate of the earth
σ_B	$=$	$5.67051 \times 10^{-8} \quad \mathrm{W\,m^{-2}\,K^{-4}}$	$\mathrm{Stefan - Boltzmann\ constant}$
R_d	$=$	$287 \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Gas constant for dry air
R_v	$=$	$461.6 \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Gas constant for water vapor
c_p	$=$	$7 \times R_d/2 \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Specific heat of dry air at constant pressure
c_v	$=$	$c_p - R_d \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Specific heat of dry air at constant volume
c_{pv}	$=$	$4 \times R_v \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Specific heat of water vapor at constant pressure
c_{vv}	$=$	$c_{pv} - R_v \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Specific heat of water vapor at constant volume
c_{liq}	$=$	$4190 \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Specific heat capacity of water
c_{ice}	$=$	$2106 \quad \mathrm{J\,kg^{-1}\,K^{-1}}$	Specific heat capacity of ice
L_v	$=$	$2.5 \times 10^6 \quad \mathrm{J\,kg^{-1}}$	Latent heat of vaporization
L_s	$=$	$2.85 \times 10^6 \quad \mathrm{J\,kg^{-1}}$	Latent heat of sublimation
L_f	$=$	$3.50 \times 10^5 \quad \mathrm{J\,kg^{-1}}$	Latent heat of fusion
ρ_w	$=$	$1.0 \times 10^3 \quad \mathrm{kg\,m^{-3}}$	Density of liquid water

Appendix B

List of Symbols

This is the WRF version. Needs to be updated for WRF-VAR!!!!!!!

Symbols used in this document are listed in alphabatical order in this appendix.

Symbols	*Definition*
a	generic variable
A	coefficient (Chapter **??**), base-state lapse rate constant (Chapter **??**)
$\mathbf{B}$	background error covariance matrix
c	scalar coefficient
c_s	speed of sound
C_k	a constant used in TKE closure
Cr	Courant number
Cr_{max}	maximum Courant number
Cr_{theory}	Courant number from Table 3.1
Cr_β	activation Courant number in vertical velocity damping
C_s	a constant used in eddy viscosity calculation
D	deformation
D_{nm}	deformation tensor, where $n, m = 1$, 2 and 3
e	cosine component of the Coriolis term (Chapters **??**, **??**); turbulent kinetic energy (Chapter **??**)
$\mathbf{E}$	observation error covariance matrix
f	sine component of the Coriolis term
F	forcing terms for U, V, W, Θ and Q_m
$\mathbf{F}$	representivity error covariance matrix
$F_{X_{cor}}$	Coriolis forcing terms for $X = U$, V, and W
$F_{1,2}$	coefficients for weighting functions in specified boundary condition
g	acceleration due to gravity
G_k	regression coefficient
H	observation operator
J	cost function
$K_{dh,dv}$	horizontal and vertical eddy viscosity for gravity wave absorbing layer
$K_{h,v}$	horizontal and vertical eddy viscosities

Symbols	*Definition*
l_0	minimum length scale for dissipation
$l_{h,v}$	horizontal and vertical length scales for turbulence
l_{cr}	critical length scale for dissipation
L	latent heat of condensation
$L_{x,y}$	periodicity length in x and y
m	map scale factor
n_s	ratio of the RK3 time step to the acoustic time step
N	Brunt-Väisälä frequency
p	pressure
p'	perturbation pressure
p_0	reference sea-level pressure
p_h	hydrostatic pressure
$p_{ht,hs}$	hydrostatic pressure at the top and surface of the model
$p_{dht,dhs}$	dry hydrostatic pressure at the top and surface of the model
p_s	surface pressure
P_r	Prandtl number
q	generic scalar
$q_{c,i,r,s}$	mixing ratios for cloud water, ice, rain water and snow
q_m	generic mixing ratios for moisture
q_v	mixing ratio for water vapor
q_{vs}	saturation mixing ratio for water vapor
Q_m	generic coupled moisture variable
r	relative humidity
r_e	radius of earth
R	remaining terms in equations
R_d	gas constant for dry air
R_v	gas constant for water vapor
t	time
Δt	a full model time step
T	temperature
T_0	reference sea-level temperature
u	horizontal component of velocity in x-direction
U	coupled horizontal component of velocity in x-direction (Chapters **??**, **??**, **??**, **??**); control variable transform (Chapter **??**)
U_h	horizontal correlation
U_p	multivariate covariance
U_v	vertical covariance
v	horizontal component of velocity in y-direction
$\mathbf{v}$	three dimensional vector velocity
V	coupled horizontal component of velocity in y-direction
$\mathbf{V}$	three dimensional coupled vector velocity
w	vertical component of velocity
W	coupled vertical component of velocity

Symbols	*Definition*
W_k	regression coefficient
z	height
z_d	depth of damping layer
z_{top}	height of model top
α	inverse density of air
α'	perturbation inverse density of air
$\bar{\alpha}$	inverse density of air for the reference state
α_d	inverse density of dry air
α_r	local rotation angle between y-axis and the meridian
β	off-centering coefficient for semi-implicit acoustic step
γ	ratio of heat capacities for dry air at constant pressure and volume
γ_d	divergence damping coefficient
γ_e	external mode damping coefficient
γ_g	damping coefficient for upper boundary gravity wave absorbing layer
γ_r	Rayleigh damping coefficient
ϵ	molecular weight of water over the molecular weight of dry air (Chapter **??**); true background error (Chapter **??**)
η	terrain-following hydrostatic-pressure vertical coordinate
$\dot{\eta}$	contravariant 'vertical' velocity or coordinate velocity
θ	potential temperature
θ_e	equivalent potential temperature
θ_m	moist potential temperature
Θ	coupled potential temperature
μ	hydrostatic pressure difference between surface and top of the model
$\bar{\mu}$	reference state hydrostatic pressure difference between surface and top of the model
μ_d	dry hydrostatic pressure difference between surface and top of the model
τ	acoustic time (Chapter **??**), vertical structure function for Rayleigh damping (Chapter **??**)
τ_{nm}	stress tensor (Chapter **??**) where $n.m = 1$, 2 and 3
$\Delta\tau$	acoustic time step
ϕ	geopotential (Chapters **??**, **??**, **??**); latitude (Chapter **??**)
$\bar{\phi}$	geopotential for reference state
ϕ'	perturbation geopotential
Φ	generic prognostic variable (coupled)
ψ	generic variable (Chapter **??**)
ψ'	streamfunction increment
χ'	velocity potential increment
ω	same as $\dot{\eta}$
Ω	coupled coordinate velocity
Ω_e	angular rotation rate of the earth

Subscripts/Superscripts	*Definition*
$()_d$	dry
$()_h$	hydrostatic
$()_0$	base state sea-level constant
$\overline{()}$	reference state
$()'$	perturbation from reference state
$()^{t^*}$	full value at a Runge-Kutta step
$()''$	perturbation from Runge-Kutta step value in acoustic steps

Appendix C

Acronyms

This is the WRF version. Needs to be updated for WRF-VAR!!!!!!!

AFWA	Air Force Weather Agency
API	Application Program Interface
ARPS	Advanced Regional Prediction System
ARW	Advanced Research WRF
BUFR	Binary Universal Form for the Representation of Meteorological Data
CAPE	Convectively Available Potential Energy
CAPS	Center for the Analysis and Prediction of Storms
CGM	Conjugate Gradient Method
COAMPS	Coupled Ocean / Atmosphere Mesoscale Prediction System
COMET	Cooperative Program for Operational Meteorology, Education, and Training
DTC	Developmental Testbed Center
ECMWF	The European Centre for Medium-Range Weather Forecasts
EOF	Empirical Orthogonal Function
ESMF	Earth System Modeling Framework
FAA	Federal Aviation Administration
FGAT	First Guess at Appropriate Time
FSL	Forecast System Laboratory
GFDL	Geophysical Fluid Dynamics Laboratory
GFS	Global Forecast System
GRIB	Gridded Binary
KMA	Korean Meteorological Administration
LSM	Land Surface Model
MKS	Meter Kilogram Second
MM5	Pennsylvania State / NCAR Mesoscale Model Version 5
MMM	Mesoscale and Microscale Meteorology Division
MRF	Medium Range Forecast Model
NAM	North American Mesoscale Model
NCAR	National Center for Atmospheric Research
NCEP	National Centers for Environmental Prediction
NFS	Non-hydrostatic Forecast System (Central Weather Bureau of Taiwan)
NMM	Nonhydrostatic Mesoscale Model
NOAA	National Oceanographic and Atmospheric Administration
NRL	Navy Research Laboratory

NWP	Numerical Weather Prediction
OSU	Oregon State University
PBL	Planetary Boundary Layer
PPM	Piecewise Parabolic Method
QNM	Quasi Newton Method
RHS	Right Hand Side
RRTM	Rapid Radiative Transfer Model
RUC	Rapid Update Cycle
SI	(WRF) Standard Initialization
TKE	Turbulent Kinetic Energy
UCAR	University Corporation for Atmospheric Research
YSU	Yonsei University (Korea)
VAR	Variational Assimilation
WRF	Weather Research and Forecasting Model
WSF	WRF Software Framework

Bibliography

Barker, D. M., W. Huang, Y.-R. Guo, and A. Bourgeois, 2003: A Three-Dimensional Variational (3DVAR) Data Assimilation System For Use With MM5. NCAR Tech Note, NCAR/TN-453+STR, 68 pp. [Available from UCAR Communications, P.O. Box 3000, Boulder, CO, 80307.].

Barker, D. M., W. Huang, Y.-R. Guo, A. Bourgeois, and X. N. Xiao, 2004: A Three-Dimensional Variational Data Assimilation System for MM5: Implementation and Initial Results *Mon. Wea. Rev.*, **132**, 897–914.

Barker, D. M., 2005: High Southern-Latitude Ensemble Data Assimilation in the Antarctic Mesoscale Prediction System. *Mon. Wea. Rev.*, **133**, 3431–3449.

Chen. S. -H., F. Vandenberghe. G. W. Petty, and J. Bresch, 2004: Application of SSM/I satellite data to a hurricane simulation. *Quart. J. Roy. Meteor. Soc.*, **130**, 801–825.

Cucurull, L., F. Vandenberghe, D. Barker, E. Vilaclara, and A. Rius, 2004: 3DVAR assimilation of GPS and meteorological observations in MM5 during the December 14th 2001 storm event over the western Mediterranean sea. *Mon. Wea. Rev.*, **132**, 749–763.

Cucurull, L., Y. H. Kuo, D. M. Barker, and S. R. H. Rizvi, 2006: Assessing the impact of COSMIC GPS radio occultation data on weather analysis and short-term forecasts over the Antarctic. *Mon. Wea. Rev.*, **134**, 3283–3296.

Daley, R., and E. Barker, 2001: NAVDAS: Formulation and Diagnostics. *Mon. Wea. Rev.*, **129**, 869–883.

Desroziers, G., 1997: A coordinate change for data assimilation in spherical geometry of frontal structures. *Mon. Wea. Rev.*, **125**, 3030–3039.

Desroziers, G., and S. Ivanov, 2001: Diagnosis and adaptive tuning of observation-error parameters in a variational assimilation. *Quart. J. Roy. Meteor. Soc.*, **127**, 1433–1452.

Faccani, C., R. Ferretti, R. Pacione, F. Vespe, L. Cucurull, and D. M. Barker, 2003: Near real-time data assimilation of GPS ZTD and PW into a non-hydrostatic model. *Atmospheric Remote Sensing using Satellite Navigation Systems.*

Faccani, C., and R. Ferretti, 2005: Data assimilation of high-density observations. I: Impact on initial conditions for the MAP/SOP IOP2b. *Quart. J. Roy. Meteor. Soc.*, **131**, 21–42.

Faccani, C., and R. Ferretti, 2005: Data assimilation of high-density observations. II: Impact on the forecast of the precipitation for the MAP/SOP IOP2b. *Quart. J. Roy. Meteor. Soc.*, **131**, 43–61.

Fisher, M., 2003: Background error covariance modeling. *Seminar on Recent Development in Data Assimilation for Atmosphere and Ocean*, 45–63, ECMWF.

Gu, J., Q. -N. Xiao, Y. -H. Kuo, D. M. Barker, J. Xue, and X. Ma, 2005: A Case Study Of Typhoon Rusa (2002) On Its Analysis And Simulation Using WRF 3DVAR And The WRF Modeling System. *Adv. In Atmos, Sci.*, **22**, 415–425.

Guo, Y. R., H. Kusaka, D. M. Barker, Y. H. Kuo, and A. Crook, 2005: Impact of Ground-Based GPS PW and MM5-3DVar Background Error Statistics on Forecast of a Convective Case. *SOLA*, **1**, 73–76.

Hollingsworth, A., and P. Lonnberg, 1986: The Statistical Structure Of Short-Range Forecast Errors As Determined From Radiosonde Data. Part I: The Wind Field. *Tellus*, **38A**, 111–136.

Ide, K., P. Courtier, M. Ghil, and A. C. Lorenc, 1997: Unified notation for data assimilation: Operational, sequential and variational. *J. Met. Soc. Japan*, **75**, 181–189.

Ingleby, N. B., 2001: The statistical structure of forecast errors and its representation in the Met. Office global 3-D variational data assimilation scheme. *Quart. J. Roy. Meteor. Soc.*, **127**, 209–232.

Lee, M. -S., and D. M. Barker, 2005: Preliminary Tests of First Guess at Appropriate Time (FGAT) with WRF 3DVAR and WRF Model. *Journ. Korean Met. Soc.*, 41, 495–505.
Lee, M. -S., Y. -H. Kuo, and D. M. Barker, 2006: Incremental Analysis Updates Initialization Technique Applied to 10-km MM5 and MM5 3DVAR. *Mon. Wea. Rev.*, **134**, 1389–1404.

Lee, M. -S., Y. -H. Kuo, and D. M. Barker, 2006: Incremental Analysis Updates Initialization Technique Applied to 10-km MM5 and MM5 3DVAR. *Mon. Wea. Rev.*, **134**, 1389–1404.

Lorenc, A. C., 1986: Analysis methods for numerical weather prediction. *Quart. J. Roy. Meteor. Soc.*, **112**, 1177–1194.

Lorenc, A.C., S.P. Ballard, R.S. Bell, N.B. Ingleby, P.L.F. Andrews, D. M. Barker, J.R. Bray, A.C. Clayton, T. Dalby, D. Li, T.J. Payne, and F.W. Saunders, 2000: The Met. Office Global 3- Dimensional Variational Data Assimilation Scheme. *Quart. J. Roy. Meteor. Soc.*, **126**, 2991–3012.

Parrish, D. F., and J. C. Derber, 1992: The National Meteorological Center's Spectral Statistical Interpolation analysis system. *Mon. Wea. Rev.*, **120**, 1747–1763.

Purser, R. J., W. -S. Wu, D. F. Parrish, and N. M. Roberts, 2003: Numerical aspects of the application of recursive filters to variational statistical analysis. Part I: Spatially homogeneous and isotropic Gaussian covariances. *Mon. Wea. Rev.*, **131**, 1524–1535.

Rabier, F., H., J. N. Thepaut, and P. Courtier, 1998: Extended assimilation and forecast experiments with a four-dimensional variational assimilation system. *Quart. J. Roy. Meteor. Soc.*, **124**, 1861–1887.

Skamarock, W. C., J. B. Klemp, J. Dudhia, D. O. Gill, D. M. Barker, W. Wang, and J. G. Powers, 2005: A Description Of The Advanced Research WRF Version. NCAR Tech Note, NCAR/TN-468+STR, 88 pp. [Available from UCAR Communications, P.O. Box 3000, Boulder, CO, 80307.].

Wang, W., D. Barker, C. Bruyère, J. Dudhia, D. Gill, and J. Michalakes, 2004: WRF Version 2 modeling system user's guide. *http://www.mmm.ucar.edu/wrf/users/docs/user_guide/*.

Wu, W. -S., R. J. Purser, and D. F. Parrish, 2002: Three-Dimensional Variational Analysis with Spatially Inhomogeneous Covariances. *Mon. Wea. Rev.*, **130**, 2905–2916.

Xiao, Q. N., Y. H. Kuo, J. Sun, W. C. Lee, E. Lim, Y. R. Guo, and D. M. Barker, 2004: Assimilation of Doppler Radar Observations with a Regional 3D-Var System: Impact of Doppler Velocities on Forecasts of a Heavy Rainfall Case. *J. Appl. Met.*, **44(6)**, 768–788.

Xiao, Q., Y.-H. Kuo, Y. Zhang, D. M. Barker, and D.-J. Won, 2006: A tropical cyclone bogus data assimilation scheme in the MM5 3D-Var system and numerical experiments with Typhoon Rusa (2002) near landfall. *J. Meteor. Soc. Japan*, **84(4)**, 671–689.

Xiao, Q., Y.-H. Kuo, J. Sun, W.-C. Lee, D. M. Barker, and E. Lim, 2006: An approach of Doppler reflectivity data assimilation and its assessment with the inland QPF of Typhoon Rusa (2002) at landfall. *J. Appl. Meteor.*, Revised.