

# Control and State Estimation for Compliant End-effectors as Constrained Optimization Problems

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**Abstract.** This project focuses on the execution of robotic manipulation tasks with a 6D compliant robot end-effector. When a robot end-effector has compliance, the forces are coupled with the configuration of the object-end-effector system, making the control and state estimation challenging. This project proposes to formulate the compliant end-effector control and state estimation as constrained optimization problems. Our method is verified with the block tilting task, where the close-loop control frequency can be as high as 50 Hz.

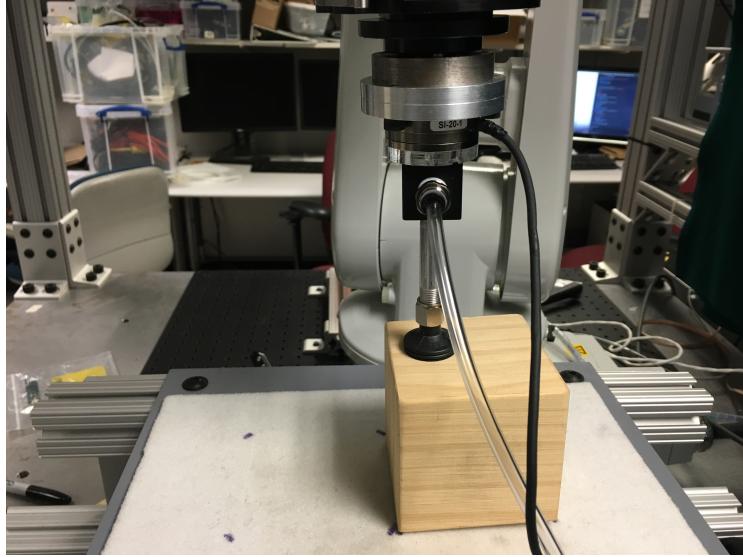
## 1 Introduction

Lots of robot end-effectors have some extent of compliance. Some examples are: all the soft robots, 6-axis compliant Stewart platform, vacuum suction cups, and rigid robot grippers with compliant mechanism. The passive compliance of these end-effectors increase the tolerance to positional errors, while compromises their abilities to control. The main problem is that the forces in the system and the system configuration are coupled. In other words, the forces and the positions cannot be controlled separately due to the compliance. This makes the execution of manipulation tasks with compliant end-effectors a challenging problem.

We propose a framework that enabling a 6-axis compliant end-effector to perform manipulation tasks through multiple external contacts, where the only sensor is a 6-axis force torque sensor. This framework consists of three parts: compliant system identification, state estimation and control. The focuses of this class project are the last two parts, using constrained optimization technique to solve for state estimation and control.

We verified our method with a block tilting example using a 6-axis compliant vacuum suction cup, as shown in 1. The computation time for both state estimation and control in one time-step is less than 20ms. This means that we can achieve the control frequency as high as 50 Hz.

The rest of this report is structured as follows. Section.2 briefly describes our work prior to this project on compliant system identification. Section.3 and Section.4 describes algorithms for the state estimation and control problem respectively. Section.5 we provide experiment setup and result of the example of block tilting using a vacuum suction cup.



**Fig. 1.** Our robot system for block titing with a vacuum suction cup.

## 2 System Identification

Given the force torque sensor reading, we first need to identify the compliant end-effector configuration. In this section, we develop a simplified 6D static model for compliant end-effectors and design a self-supervised data collection procedure to fit this model.

### 2.1 Static/Stiffness Model for Compliant End-effectors

For a short and non-actuated compliant end-effector, we can simply treat it as a 6-dof nonlinear spring. We define the compliant displace  $S$  and external wrench  $W$  at one end as

$$X = (x, y, z, \alpha, \gamma, \delta)$$

$$W = (F_x, F_y, F_z, T_x, T_y, T_z)$$

where  $(x, y, z, \alpha, \gamma, \delta)$  are the linear and angular displacement on one end of the compliant part;  $W$  is the respective wrench applied on this end.

We can write down the local linearization of this spring system as

$$K(X) : \mathbb{R}^6 \mapsto \mathbb{R}^6, \Delta W = K \Delta X$$

where  $K$  is the local stiffness matrix and is required to be positive semi-definite.

## 2.2 Data Collection

To collect the force-torque and the corresponding gripper configuration data for a vacuum suction cup, we design a self-supervised data collection procedure which doesn't need any extra sensor for displacement and doesn't require human supervision. The robotic system setup is shown in Figure ???. The only sensor used in our system is an ATI-mini force torque sensor mounted on the end of the gripper. To measure the gripper configuration, the open-end of the suction cup is fixed to a known point with vacuum on the table, thus the configuration is the difference of the fixed point and the robot end position. The data collection procedure is as follows:

1. go to a neutral position where there is only small configuration displacement in  $z$ , record force torque sensor data  $D_{f0}$ .
2. randomly sample a configuration within a roughly estimated range.
3. go to the robot position corresponding to the configuration sampled in 1, record the force torque sensor  $D_f$ .
4. go back to the neutral position, compare current force torque data with  $D_{f0}$ , if exceed a certain threshold, then the fix contact has slip; discard all data and reset the suction cup (go to a certain height that break the contact, then go back to the neutral position). Otherwise record the sample configuration and the force torque data  $D_f$ .

## 2.3 Model Identification

Using the data collection procedure, we collected 1637 samples for one suction cup. Given a new force torque data sample, we can fit a locally linear regression model  $W = K(X - X_0) + W_0$  with its nearest neighbors of weighted euclidean distance, and predict its corresponding configuration. The local linearity also contributes to the linearization of constraints in the optimization problems in Section 3 and Section 4.

## 3 State Estimation

This section addresses the problem of estimating object positions and compliant end-effector configuration using force torque data and known robot position. Although from Section 2 we obtain a mapping from force torque data to the compliant end-effector configuration directly, the mapping cannot be exactly accurate and could cause the inferred object position violating external contact constraints. Thus, we need to refine the rough estimation to let it satisfies the environment constraints and system stable condition. We formulate this as a constrained optimization problem, where we solve for the position of the object  $x_o$  and the configuration of compliant gripper  $x_c$ . In this problem, the following constraints should be satisfied:

1. robot position consistency: the robot position kinematically computed by  $x_o$  and  $x_c$  should be consistent with the actual robot position, which can be written as  $\Phi_{robot}(x_o, x_c) = x_{robot}$ ,
2. contact mode constraints: the object position should satisfy the natural constraints enforced by the task contact mode, which can be written as  $\Phi_{env}(x_o) = 0$ .

The cost function is the potential energy of the object-gripper system  $E(x_o, x_c) = E_{object}(x_o) + E_{elastic}(x_c)$ , since a stable configuration of a system is always at its local minima of energy under constraints. Thus, the state estimation problem can be written as:

$$\begin{aligned} \min_{x_o, x_c} \quad & E(x_o, x_c) \\ \text{s.t.} \quad & \Phi_{robot}(x_o, x_c) = x_{robot} \\ & \Phi_{env}(x_o, x_c) = 0 \end{aligned} \quad (1)$$

To speed up computation and ensure optimal solution, we can further turn this problem into quadratic programming using local linearization. If we solve for the change of  $x_o$  and  $x_c$  in the form of body twist represented as  $\Delta x_o$  and  $\Delta x_c$ , we can easily obtain a quadratic cost function as

$$\Delta E = mg\Delta x_{o_z} + \frac{1}{2}\Delta x_c^T K(x_c)\Delta x_c + W^T \Delta x_c$$

The robot position consistency constraints can be represented as

$$J_{\Phi_{robot}}(x_o, x_c)\Omega(x_o, x_c) \begin{pmatrix} \Delta x_o \\ \Delta x_c \end{pmatrix} = \Delta x_{robot}$$

The contact constraints can be represented as

$$J_{\Phi_{env}}(x_o)\Omega(x_o)\Delta x_o = 0$$

## 4 Control

The goal of control varies. It could be position/velocity control of the object, the force control on compliant contact, or even hybrid force velocity control of the whole system. But eventually we achieve these goals by commanding the robot position or velocity. Thus we can formulate the control problem as to find the control action (robot velocity) optimal to certain criteria under the task and environment constraints. The variables are robot body velocity  $v_{robot}^b$ , object body velocity  $v_{obj}^b$ , gripper contact force  $f_c$  and external contact force  $f_{env}$ . The following constraints should be satisfied:

1. task goal: enforce the controlled variable to satisfy the task goal, as  $S_v v_{obj}^b = v_{goal}$  and  $S_f f_c = f_{goal}$ .

2. contact constraints: maintain desired contact mode, written as  

$$J_{\Phi_{env}}(x_o)\Omega(x_o)v_{obj}^b = 0$$
3. quasi-static equilibrium (force balance on the object):  $G_o + f_c^o + f_{env}^o = 0$
4. guard conditions: satisfy the requirement of for each contact, like normal force greater a threshold and contact force within its friction cone, which can be written as  $f_{env}^n > f_{min}^n$  and  $\mu f_{env}^n - f_{env}^t > 0$ . The friction cone constraint can be turned into linear constraints by approximating the cone with octagonal polyhedron:  

$$\mu f_{env}^z = d_i^T \begin{pmatrix} f_{env}^x \\ f_{env}^y \\ f_{env}^z \end{pmatrix}, i = 1, 2, \dots, 8, \text{ where } d_i = [\sin(\frac{i\pi}{4}), \cos(\frac{i\pi}{4})].$$
5. compliant force relationship:

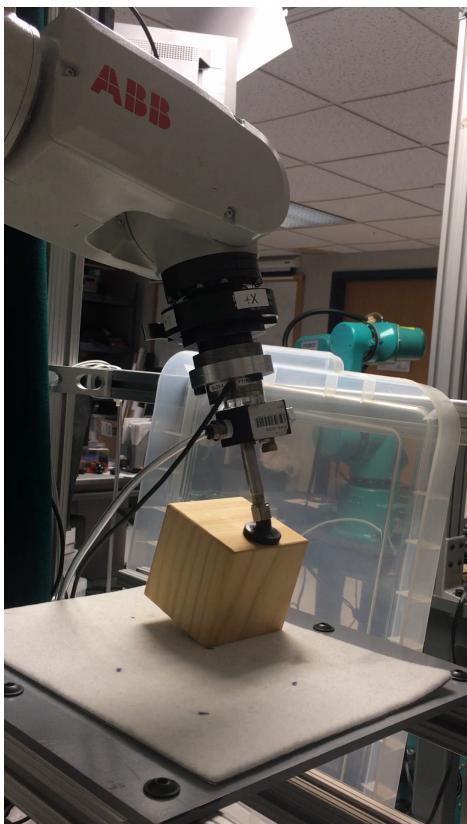
The cost function can be written as:

$$C(v_{robot}^b, v_{obj}^b, f_c, f_{env}) = v_{robot}^b {}^T v_{robot}^b + v_{obj}^b {}^T v_{obj}^b + f_c^T f_c + f_{env}^T f_{env}$$

This optimization problem can be easily solved with quadratic programming.

## 5 Experiments

We verified our method using a vacuum suction cup to perform a position controlled block tilting tasks, where only reference object position trajectory is given. The robotic system is shown in Figure.1. Figure.2 is taken during the block tilting process. We use an ABB 120 robot with Cartesian control. 1647 system identification samples were collected using the data collection procedure in Section.2. The computation time for one timestep is no more than 20ms. Videos for this experiments can be found in the folder `video` from this link [https://github.com/XianyiCheng/16745\\_Project](https://github.com/XianyiCheng/16745_Project).



**Fig. 2.** During the block tilting