Range Estimation based on OFDM signals

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Abstract

In this note, we introduce how to estimate the range information of the targets using OFDM signals. In Section (I), the system model and the OFDM signal model is first introduced. In Section, two range estimation methods, i.e., the periodogram based method and MUSIC method, are introudced.

I. System Model

Consider a sensing system consisting of one BS, one UE, and K targets. The BS transmits orthogonal frequency division multiplexing (OFDM) signal in the downlink. For convenience, denote N and Δf as the number of subcarriers and subcarrier spacing in the downlink, respectively. The transmitted symbol on the n-th subcarrier by the BS is denoted by $s_n, n = 0, \ldots, N-1$. Then, the transmitted signal in the m-th symbol block is expressed as [1], [2]

$$x_m(t) = \sum_{n=0}^{N-1} s_{m,n} e^{j2\pi n \Delta f t} \xi(t - mT),\ mT_d - T_{cp} \leq t \leq mT_d, \tag{1}$$

where $T = T_s + T_{cp}$ denote the duration of one OFDM symbol with T_d and T_{cp} denote the durations of the data and the CP, respectively, and

$$\xi(t) = \begin{cases} 1 & -T_{cp} \leq t \leq T_d, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The transmitted signal over M blocks can be expressed as

$$x(t) = \sum_{m=0}^{M-1} x_m(t),\ -T_{cp} \leq t \leq (M-1)T + T_d. \tag{3}$$

The emitted signals are reflected by the targets to the UE. The received baseband signals at the UE is expressed as

$$y(t) = \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k t} x(t - \tau_k) + z(t), \tag{4}$$

$$\simeq \sum_{m=0}^{M-1} \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k m T} x_m(t - \tau_k) + z(t), \tag{5}$$

where α_k is the attenuation coefficient including path loss reflection and processing gains, f_k denotes the Doppler frequency, and τ_k denotes the delay of the k-th path, and $z(t)\ \mathcal{CN}(0, \sigma_z^2)$ denotes the noise.

Moreover, the approximation in the last line is based on the assumption that the phase rotation within one OFDM block due to Doppler effect can be approximated as constant.

Then, after sampling at a frequency of $f_s = \frac{1}{T_s} = T_d/N$, we can obtain the following discrete signal in the p-th sample during the m-th block [3]

$$y_{m,p} = \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k mT} x_m(pT_s - \tau_k) + z_{m,p}, \tag{6}$$

$$= \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k mT} \sum_{n=0}^{N-1} s_{m,n} e^{j2\pi n\Delta f(pT_s - \tau_k)} + z_{m,p},\ p = 0.\dots, N-1. \tag{7}$$

After doing discrete Fourier transform (DFT), the received signal on the n-th subcarrier is expressed as

$$\tilde{y}_{m,n} = \sum_{p=0}^{N-1} y_p e^{-j2\pi \frac{np}{N}} \tag{8}$$

$$= \frac{1}{N} \sum_{p=0}^{N-1} e^{-j2\pi \frac{np}{N}} \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k mT} \sum_{n=0}^{N-1} s_{m,n} e^{j2\pi n\Delta f(pT_s - \tau_k)} + \sum_{p=0}^{N-1} z_p e^{-j2\pi \frac{np}{N}} \tag{9}$$

$$= \sum_{k=1}^{K} \alpha_k s_{m,n} e^{j2\pi f_k mT} e^{-j2\pi n\Delta f \tau_k} + \tilde{z}_n,\ \forall m, n. \tag{10}$$

$$\tag{11}$$

Divide the transmitted symbol $s_{m,n}$ from (10), we have

$$h_{m,n} = \frac{\tilde{y}_{m,n}}{s_{m,n}}, \tag{12}$$

$$= \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k mT} e^{-j2\pi n\Delta f \tau_k} + \frac{\tilde{z}_n}{s_{m,n}}, \tag{13}$$

$$= \sum_{k=1}^{K} \alpha_k e^{j2\pi f_k mT} e^{-j2\pi n\Delta f \tau_k} + \bar{z}_n,\ \forall m, n. \tag{14}$$

For convenience, denote $\mathbf{h}_m = [h_{m,0}, \dots, h_{m,N-1}]^T, \forall m = 0, \dots, M-1$ and $\mathbf{H} = [\mathbf{h}_0, \dots, \mathbf{h}_{M-1}]$. In the following, we show how to estimate the delays, i.e., τ_k's, based on (14).

II. Delay Estimation

In this section, we introduce two delay estimation methods. For ease of exposition, we assume the targets are static, i.e., $f_k = 0$;

A. Periodogram-based Method

1) Fundamentals: For the case of identifying sinusoids in a discrete-time signal, the periodogram is a well-understood tool and is in fact the optimal solution if the sinusoids are well resolved. Given a discret-time signal $s(k)$ of length N samples, the periodogram is defined as

$$\mathrm{Per}_{s(k)}(f) = \frac{1}{N} \left| \sum_{k=0}^{N-1} s(k) e^{-j2\pi f k} \right|^2 . \tag{15}$$

The common way to calculate this in digital systems is to quantize the frequency in regular intervals and use the Fast Fourier Transformation

$$\begin{aligned} \mathrm{Per}_{s(k)}(n) &= \frac{1}{N} \left| \sum_{k=0}^{N-1} s(k) e^{-j2\pi \frac{nk}{N_{\mathrm{Per}}}} \right|^2 \\ &= \frac{1}{N} \left| \mathrm{FFT}_{N_{\mathrm{Per}}}[s(k)] \right|^2 , \end{aligned} \tag{16}$$

where the notation $\mathrm{FFT}_{N_{\mathrm{Per}}}[s(k)]$ is used to denote an FFT of length N_{Per} on the input vector $s(k)$. Note that N_{Per} does not have to be equal to N. If $N_{\mathrm{Per}} > N$, zero-padding is used to increase the length of $s(k)$ to N_{Per}. This increases the number of supporting points of the discrete periodogram and hence the accuracy at which frequencies can be estimated.

2) Application: The periodogram for estimating the delay is calculated as

$$\mathrm{Per}_{\mathbf{H}}(l) = \frac{1}{NM} \sum_{m=0}^{M-1} \left| \sum_{n=0}^{N_{\mathrm{Per}}-1} h_{m,n} e^{-j2\pi \frac{nl}{N_{\mathrm{Per}}}} \right|^2 \tag{17}$$

Sinusoids in $\mathbf{H}$ will result in a peak in $\mathrm{Per}_{\mathbf{H}}(l)$. If $\mathrm{Per}_{\mathbf{H}}(\hat{l})$ corresponds to a peak value, then it means the estimated frequency is $\hat{\Omega} = 2\pi \hat{l}/N_{\mathrm{Per}}$. Furthermore, the corresponding estimated delay is $\hat{\tau} = \frac{\hat{l}}{\Delta f N_{\mathrm{Per}}}$ and the estimated range is

$$\hat{d} = \frac{c_0 \hat{l}}{2\Delta f N_{\mathrm{Per}}}, \tag{18}$$

where c_0 is the speed of light.

B. MUSIC based Method

1) Fundamentals: MUSIC is a subspace based method for spectral estimation . Specifically, consider the following signal model

$$\begin{aligned} \mathbf{y}(t) &= \sum_{k=1}^{K} \mathbf{a}(w_k) s_k(t) & (19) \\ &= [\mathbf{a}(w_1), \ldots, \mathbf{a}(w_K)] \mathbf{s}(t) + \mathbf{z}(t) & (20) \\ &= \mathbf{A}\mathbf{s}(t) + \mathbf{z}(t), & (21) \end{aligned}$$

where $\mathbf{a}(w_k) = [1, e^{-jw_k}, \ldots, e^{-j(N-1)w_k}]^T$ is genenrally called the steering vector at frequence $w_k, \forall k = 1, \ldots, K$ with K being the number of sources, $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$ denotes the transmit signal with the covariance matrix $\mathbf{R}_s = \mathbb{E}[\mathbf{s}(t)\mathbf{s}(t)^H]$ being a diagonal matrix (the sources are not correlated), and $\mathbf{z}(t) \sim \mathcal{CN}(0, \sigma_z^2 \mathbf{I})$ denote the noise. The objective of MUSIC method is to estimate w_k's given $\mathbf{y}(t)$. In the following, we introduce the specific steps for estimating the frequencies using MUSIC.

Step 1: Calculate the spatial covariance matrix.

$$\mathbf{R}_y = \mathbb{E}[\mathbf{y}(t)\mathbf{y}^H(t)]. \tag{22}$$

Step 2: Eigenvalue Decomposition (EVD)

Perform eigenvalue decomposition on $\mathbf{R}_y$:

$$\mathbf{R}_y = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^{\mathbf{H}}, \tag{23}$$

where $\mathbf{U} = [\mathbf{U}_s, \mathbf{U}_n]$ is the eigenvector matrix. The K eigenvectors corresponding to the K largest eigenvalues consists of the signal subspace $\mathbf{U}_s$ while the remaining eigenvector forms the noise subspace $\mathbf{U}_n$.

Step 3: Compute the MUSIC Pseudo-Spectrum Since the noise subspace $\mathbf{U}_n$ is orthogonal to the signal's array manifold $\mathbf{a}(w_k)$, i.e., $\mathbf{U}_n^H\mathbf{a}(w_k) \simeq 0$, the MUSIC pseudo spectrum can be constructed as

$$P_{\text{MUSIC}}(w_k) = \frac{1}{\mathbf{a}^H(w_k)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(w_k)}. \tag{24}$$

Step 4: Peak Search for Estimating Frequency The MUSIC pseudo-spectrum exhibits peaks at the true frequency. Thus, we can determine $w_1, \ldots, w_K$ by searching for the peaks of $P_{\text{MUSIC}}(w_k)$.

2) Application: In this subsection, we show how to estimate the ranges based on $\mathbf{h}_m$. For ease of exposition, we define $\mathbf{h}_m = [\mathbf{h}_{m,0}, \ldots, \mathbf{h}_{m,N-1}]^T$ where $h_{m,n}$ is defined as

$$h_{m,n} = \sum_{k=1}^{K} \tilde{\alpha}_k e^{-j2\pi n\Delta f\tau_k} + \bar{z}_n, \ \forall m, n. \tag{25}$$

Define $\mathbf{a}(\tau_k) = [1, \ldots, e^{-j2\pi(N-1)\Delta f\tau_k}]^T$. Then, we have

$$\mathbf{h}_m = \mathbf{A}\bar{\mathbf{s}} + \mathbf{z}, \ \forall m, \tag{26}$$

where $\mathbf{A} = [\mathbf{a}(\tau_1), \ldots, \mathbf{a}(\tau_K)]$ and $\bar{\mathbf{s}} = [\alpha_{m,1}, \ldots, \alpha_{m,K}]^T$.

To use the MUSIC method, we should first find the covariance matrix. However, if we directly calculate the covariance matrix based on $\mathbf{h}_m$ as $\hat{\mathbf{R}}_h = \mathbb{E}\{\mathbf{h}_m * \mathbf{h}_m^H\}$, the rank of the covariance matrix is 1. In this case, MUSIC method does not work. To address this, we could apply the spatial smoothing method [4], which divides the array into multiple overlapping subarrays.

Specifically, denote the length of each subarray as N'. There are a total of $N - N' + 1$ subarrays. Define the receive signal at the i-th subarray as

$$\mathbf{h}_m^i = [h_{m,i}, \ldots, h_{m,i+N'-1}]^T. \tag{27}$$

Moreover, define the steering vector with a reduced dimension N' as

$$\bar{\mathbf{a}}(\tau_k) = [1, \ldots, e^{-j2\pi(N'-1)\Delta f\tau_k}]^T. \tag{28}$$

Then, we have

$$\mathbf{h}_m^i = \sum_{k=1}^{K} \bar{\alpha}_{m,k} \mathbf{D}^{i-1} \bar{\mathbf{a}}(\tau_k) + \mathbf{z}_i, \tag{29}$$

$$= \bar{\mathbf{A}} \mathbf{D}^{i-1} \bar{\mathbf{s}} + \mathbf{z}_i \; i = 1, \ldots, N - N' + 1, \tag{30}$$

where $\mathbf{A}' = [\mathbf{a}'(\tau_1), \ldots, \mathbf{a}'(\tau_K)]$ and $\mathbf{D} = \mathrm{diag}\{e^{-j2\pi\Delta f \tau_1}, \ldots, e^{-j2\pi\Delta f \tau_K}\}$.

Given (29), the covariance matrix for MUSIC is calculated as

$$\hat{\mathbf{R}} = \frac{1}{N - N' + 1} \sum_{i=1}^{N-N'+1} \mathbf{h}_m^i \mathbf{h}_m^{iH} \tag{31}$$

Denote the noise subspace derived from $\hat{\mathbf{R}}$ as $\hat{\mathbf{U}}_n$. The MUSIC pseudo-spectrum is expressed as

$$P_{\mathrm{MUSIC}} = \frac{1}{\bar{a}^H(\hat{\tau}_k) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \bar{a}(\hat{\tau}_k)}. \tag{32}$$

At last, given the target number K, we can easily obtain the range estimation by detecting the K peaks in the pseudo spectrum.

References

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