Adv Abstract Algebra: AAA #04

Due on March 2022 at 11:59PM

Prof. Peter Hermann Spr 2022

Xianzhi

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Problem 1

Let G be a finite group and p a prime. Suppose that $N_p \triangleleft G$ such that $|G : N_p|$ is some power of p and $|N_p|$ is not divisible by p. (Then N_p is called *a normal p-complement in G*.) Prove that (for the give p) N_p is unique, i.e., a group can have at most one normal p-complement.

Solution:

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Problem 2

Let G be a finite group, $L \triangleleft G$ and p a prime. Suppose that N_p is a normal p-complement in G. Show that $L \cap N_p$ is a normal p-complement in L and $L \cdot N_p / L$ is a normal p-complement in G/L.

Solution:

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Problem 3

Let $|G| = 2 \cdot 5 \cdot 7 \cdot 79^3$. Show that G is solvable.
Solution:
For the variance:

$$
\begin{aligned}
\mathrm{Var}[\hat{\beta}_1] &= \mathrm{Var}\left[\frac{\sum x_i Y_i}{\sum x_i^2}\right] \\
&= \frac{\sum x_i^2}{\sum x_i^2 \sum x_i^2} \mathrm{Var}[Y_i] \\
&= \frac{\sum x_i^2}{\sum x_i^2 \sum x_i^2} \mathrm{Var}[Y_i] \\
&= \frac{1}{\sum x_i^2} \mathrm{Var}[Y_i] \\
&= \frac{1}{\sum x_i^2} \sigma^2 \\
&= \frac{\sigma^2}{\sum x_i^2}
\end{aligned}
$$

Problem 4

Let a solvable group G act faithfully and transitively on the set Ω, where $|\Omega| = 35$.

1. Prove that this action is not primitive.

2. Show that if G is abelian then it must be cyclic.

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