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Prior: $\beta \sim \text{Laplace}(Y)$.

$$P(\beta) = \left(\frac{\gamma}{2}\right)^p e^{(-\gamma \|\beta\|_1)}$$

$$\begin{cases} P(Y|\beta) \sim N(X\beta, \sigma^2) \\ P(Y|\beta) = \frac{P(Y, \beta)}{P(\beta)} \Rightarrow P(Y, \beta) = P(Y|\beta)P(\beta). \end{cases}$$

$$= \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left[-\frac{\|Y - X\beta\|_2^2}{2\sigma^2}\right] \left(\frac{\gamma}{2}\right)^p \exp[-\gamma \|\beta\|_1]$$

$$\log P(Y, \beta) = -\frac{\|Y - X\beta\|_2^2}{2\sigma^2} - \gamma \|\beta\|_1 + C.$$

$$\therefore \hat{\beta} = \underset{\beta}{\operatorname{argmin}} \left[-\frac{\|Y - X\beta\|_2^2}{2\sigma^2} - \gamma \|\beta\|_1 \right], \text{ which is related to Lasso Regression.}$$