JOINT INSTITUTE
交大密西根学院

vv255SU2020_Assignment 2 **Due to June 5, 2020**

Problem 1

Consider the points $A_1(3,1,4)$, $A_2(-1,6,1)$, $A_3(-1,1,6)$, $A_4(0,4,-1)$.

A. Find the equations of the following objects:

a. the plane $A_1A_2A_3$, **b.** the line A_1A_2, **c.** the line A_4M perpendicular to the plane $A_1A_2A_3$, **d.** the line A_3N parallel to the line A_1A_2, **e.** the plane $\wp$: $A_4 \in \wp$, $\wp \perp (line\ A_1A_2)$.

B. Calculate:

a. $\sin\theta$, where θ is the angle between the line A_1A_4 and the plane $A_1A_2A_3$,

b. $\cos\varphi$, where φ is the angle between the coordinate plane $z=0$ and the plane $A_1A_2A_3$.

Problem 2

A. Find the equation of the plane that passes through the lines

$$\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{2}, \qquad \frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{2}.$$

B. Find the equation of the plane that passes through the origin and is perpendicular to the planes

$$2x-3y+z-1=0, \qquad x-y+5z+3=0.$$

C. Find the symmetric point of $M(4,3,10)$ about the line

$$\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{5}$$

Problem 3

1. Find the vector function $\bar r(t)$ if $\bar r'(t)=t\bar\imath+e^t\bar\jmath+te^t\bar k$ and $\bar r(0)=\bar\imath+\bar\jmath+\bar k$.
2. The curve C in $\mathbb{R}^2$ is defined by a vector function $\bar r(t)=t^2\bar\imath+(t^3-t)\bar\jmath$.

A. Find the point P at which the curve C intersects itself.

B. Does the curve C have more than one tangent line at P? If yes, find the angle between tangent lines.

Problem 4

1. A particle of unit mass moves under a constant force $\bar F$. If a particle was initially at the point $\bar r_0$ and passed through the point $\bar r_1$ after 2 units of time, find the initial velocity of the particle. What was the velocity of the particle when it passed through $\bar r_1$?
2. Consider the curve $\bar r(t)=\cos t\,\bar\imath+\sin t\,\bar\jmath+\frac{2}{3}t^{\frac{3}{2}}\bar k$ and find

A. the length of the curve from $t=0$ to $t=2\pi$.

B. the equation of the tangent line at the point $t=0$.

C. the speed of the point moving along the curve at the point $t=2\pi$.

Problem 5

Find an equation for the osculating and normal planes for the curve $\bar r(t)=(\ln t, 2t, t^2)$ at the point P_0 of its intersection with the plane $y-z=1$. A plane is normal to a curve at a point if the tangent to the curve at that point is normal to the plane.

Problem 6

A particle travels along a helix of radius R that rises h units of length per turn. Let the z axis be the symmetry axis of the helix. If a particle travels the distance $4\pi R$ from the point $(R,0,0)$, find the position vector of the particle.

Dr. Olga Danilkina 中国 上海闵行区东川路 800 号 邮编 200240

Tel: +86-21-34206765 Ext. 4372 http://umji.sjtu.edu.cn