Report on QED graphene

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1 Introduction

We consider the case when a graphene is coupled to a Right circular-polarized cavity, which breaks the time reversal symmetry, and calculate its influence on the dynamical properties of graphene, using the perturbation theory in the lowest order. The degeneracy at the Dirac point of ordinary graphene is entailed by both TMR and Inversion symmetry, first of which is broken by the polarized cavity in our model, and we therefore expect there will be gap-opening due to the coupling with photon field at previous Dirac point. Contrary to previous method to control the conductivity of materials such as the laser-driven system, this process break the TRS without the energy injection from the driven mechanism outside, thus will induce the insulating properties of graphene without heating up the system. The ARPES spectra from the analytical result demonstrate the gap-open feature and sub-band induced from the coupling, coinciding with the numerical calculation of perturbation method, and requires the further illustration in neDMFT simmulations.

2 Model Hamiltonian and parameter conditions

We assume that the system is initialized to thermal equilibrium, so that we can use the trick to derive the retarded self-energy from Mastubara components. Similar to the electron-phonon coupling, the thermal-initialization in our electron-photon model also eliminates the contribution from first-order perturbation. We also assume the weak coupling between photon and electron system to warrant the validity of the lowest order perturbation. Besides, we also assume the zero-momentum coupling for the simplification of calculation and the closure of the set of exact equations of motion.

$H = H_{e0} + H_{cav0} + V_{e-c}$

where

$$H_{e0} = \Sigma_k (c_{Ak}^\dagger \; c_{Bk}^\dagger) \begin{pmatrix} 0 & v_F(kx+iky) \\ v_F(kx-iky) & 0 \end{pmatrix} \begin{pmatrix} c_{Ak} \\ c_{Bk} \end{pmatrix}$$

$H_{cav0} = \omega a^\dagger a$

$$V_{e-c} = \Sigma_k V_k = \Sigma_k (c_{Ak}^\dagger \; c_{Bk}^\dagger) \begin{pmatrix} 0 & -ga^\dagger \\ -ga & 0 \end{pmatrix} \begin{pmatrix} c_{Ak} \\ c_{Bk} \end{pmatrix}$$

if we define the operator which pump the electron from state $|B,k>$ to $|A,k>$, that is $d_k = c_{Ak}^\dagger c_{Bk}$. Taking into account that a and d_k with different wave-vector k commutes with each other in Schrödinger picture, we can then derive the equations of motion for operators d_k and a in Heisenberg picture

$$\begin{cases} i\frac{\partial a}{\partial t} = \omega a - g\Sigma_k d_k \\ i\frac{\partial d_k}{\partial t} = (v_F(kx - iky) - ga)\left[d_k, d_k^\dagger\right]_- \end{cases} \tag{1}$$

Note that without the approximation of zero-momentum-coupling, H_{e0} and H_{e-c} will not have the same quadrature structure of electron operators, then we cannot derive the closed set of equations of motion.

To solve equation 1 , we first write down the formal solution of a

$$a_{(t)} = a_{(t0)} e^{-i\omega(t-t0)} + ige^{-i\omega t}\Sigma_k \int_{t0}^{t} \mathrm{d}t_1 e^{i\omega t_1} d_{k(t_1)}$$

Where $t0$ corresponds to the initial time we turn on the coupling between electron and photon field, thus their operators commute with each other initially in Heisenberg picture, so that $[d_{k(t0)}, a_{(t0)}]_- = 0$. Then we plug the formal solution back to derive equations for operators d_k.

$$i\frac{\partial d_{k(t)}}{\partial t} = \begin{cases} (v_F(kx - iky) - ga_{(t0)} \\ -ig^2 e^{-i\omega t}\Sigma_{k'} \int_{t0}^{t} \mathrm{d}t_1 e^{i\omega t_1} d_{k'(t_1)} \end{cases} [d_{k(t)}, d_{k(t)}^\dagger]_- \tag{2}$$

From the defnition of d_k it is clear that $[d_{k(t)}, d_{k(t)}^\dagger]_- = c_{Ak}^\dagger c_{Ak} - c_{Bk}^\dagger c_{Bk}$ is the occupation discrepancy between states of same wave-vector $|A,k>$ and $|B,k>$. From the closed equation of motion we know that the motion of d_k at t depends on initial photon field. However, it also depends on the electron operator at each time before t, revealing the appearance of electronic system's memory, orienting from the coupling to a cavity photon field.

The equation of motion is hard to solve numerically due to the commutation property of the photon field, and thus arbitrary order of a will be encountered in the evaluation of $d_{k(t)}$. However, the equation will be simplified greatly if we treat photon field as classically-driving. For the initially strong laser driven case, we can apply this classical field approximation by substitution of operator to a complex number $a_{(t0)} \to \alpha$. Then the equation of operators will become a set of matrix differential-integral equations with initial conditions relating to the basis we choose.

However, the initial condition we are interested in is the thermal equilibrium case, thus we must face the quantum properties of the cavity mode. Our motivation is that a cavity that only reflects one kind of circular polarization light while dissipates the other kind will break the time-reversal-symmetry of the system and led to the gap at the previous degenerate Dirac points in graphene, and this process will occur as long as the special cavity exist and require no driving from outside. Thus it will be possible to control the conductivity of the

inside graphene system only through changing the coupling coefficient g without heating the system through a driven laser that previously used for the control.

In thermal equilibrium, the Matsubara Hamiltonian that represents initial condition of system is $H^M = H - \mu(c^\dagger_{Ak}c_{Ak} + c^\dagger_{Bk}c_{Bk})$, it is clear that the coupling meliorates the distribution of the cavity photon, such as $< a^\dagger a >= \frac{Tr[e^{-\beta H^M}a^\dagger a]}{Tr[e^{-\beta H^M}]}$, due to the non-commutation relation $[V_k, a]_- \neq 0$. However, in the regime of weak coupling $g \to 0$, we recover the boson distribution $< a^\dagger a > \to \frac{1}{e^{\beta\omega}-1}$.

Besides, we can also deduce from the equation of motion that $\frac{\partial}{\partial t}(c^\dagger_{Ak}c_{Ak} + c^\dagger_{Bk}c_{Bk}) = 0$, asserting that the conservation of particle number at each specific wave-vector k, which stems from our zero momentum coupling approximation.

3 Lowest order perturbation theory of Matsubara Green's Function

From the perturbation theory we first write down the lowest-order correction of Matsubara Green's Function[1], then from the Dyson equation we find its correspondence with and thus deduce the lowest-order Matsubara self-energy. Finally we derive the Matsubara Green's Function from the Dyson Equation as well as the previously deduced self-energy.

In perturbation theory we separate total Hamiltonian H into bare Hamiltonian $H_0 = H_{e0} + H_{cav0}$ and perturbation operator $V = V_{e-c} = \Sigma_k V_k$. Then the Matsubara Green's Function

$$\begin{aligned} &G^M_{(k,Y,\tau\ ;k',X,\tau')} = \Sigma^\infty_{n=0} G^{M(n)}_{(k,Y,\tau\ ;k',X,\tau')} \\ &= -\Sigma^\infty_{n=0}(-1)^n \int_0^\beta \mathrm{d}\tau_1 \cdots \int_0^\beta \mathrm{d}\tau_\mathrm{n} < T_\tau c_{X,k(\tau)} V_{(\tau_1)} \cdots V_{(\tau_n)} c^\dagger_{Y,k'(\tau')} >_0 \end{aligned} \tag{3}$$

where β is inverse temperature, T_τ is contour-ordering operator along the vertical branch, and $< \theta_{(\tau)} >_0 = Tr[e^{-\beta H^M_0}\theta_{(\tau)}]$ for any operator $\theta_{(\tau)}$ with $H^M_0 = H^M - V$. We will see this separation of H_0 and V, together with our thermal equilibrium initial condition $H^M = H - \mu(c^\dagger_{Ak}c_{Ak} + c^\dagger_{Bk}c_{Bk})$, eliminates all perturbation terms with odd order. Note also that the second equality holds provide that we discard those disconnected terms of the operator correlators after applying Wick's theorem.

At the same time, Matsubara Green's Function is also described by the Dyson equation

$$G^M_{(ip_n)} = G^M_{0(ip_n)} \Sigma^\infty_{n=0} (\Sigma^M_{(ip_n)} G^M_{0(ip_n)})^n \tag{4}$$

Where we use matrix form of

$$G^M_{(k,\tau\ ;k',0)} = -\begin{pmatrix} < T_\tau c_{A,k(\tau)} c^\dagger_{A,k'(0)} > & < T_\tau c_{A,k(\tau)} c^\dagger_{B,k'(0)} > \\ < T_\tau c_{B,k(\tau)} c^\dagger_{A,k'(0)} > & < T_\tau c_{B,k(\tau)} c^\dagger_{B,k'(0)} > \end{pmatrix} \tag{5}$$

$$G^M_{(k\ ;k')(ip_n)} = \int_0^\beta \mathrm{d}\tau e^{ip_n\tau} G^M_{(k,\tau\ ;k',0)} \tag{6}$$

$$G^M_{0(ip_n)} = Lim_{g\to 0} G^M_{(ip_n)} \tag{7}$$

where $< \theta(\tau) >= \frac{Tr[e^{-\beta H^M}\theta(\tau)]}{Tr[e^{-\beta H^M}]}$ is the quantum average of operator $\theta(\tau)$. And we try to deduce matrix $\Sigma^M_{(k\ ;k')(ip_n)}$ that suits this form of Dyson equation by expanding it to the order of coupling constant g. We just focus on the situation when wave-vector k is conserved, that is

$$\Sigma^M_{(k\ ;k)(ip_n)} = \Sigma^M_{(k\ ,ip_n)} = S^{M(0)}_{(k\ ,ip_n)} + gS^{M(1)}_{(k\ ,ip_n)} + g^2 S^{M(2)}_{(k\ ,ip_n)} + g^3 S^{M(3)}_{(k\ ,ip_n)} + g^4 S^{M(4)}_{(k\ ,ip_n)} \cdots \tag{8}$$

According to 7 we know that $\Sigma^M_{(k\ ,ip_n)} \to 0$ as $g \to 0$, thus

$$S^{M(0)}_{(k\ ,ip_n)} = 0 \tag{9}$$

In $G^{M(n)}_{(k,Y,\tau\ ;k',X,\tau')}$, we denote $Y = A/B$ the first/second row of matrix (5), and $X = A/B$ the first/second column of matrix (5). Applying Wick's theorem to equation 3, we find that every term of $G^{M(n)}_{(k,Y,\tau\ ;k',X,\tau')}$ contains photon field correlators to nth order, that is total number of $a_{(\tau_i)}$ and $a^\dagger_{(\tau_j)}$ in $< T_\tau \cdots a_{(\tau_i)} \cdots a^\dagger_{(\tau_j)} \cdots >_0$ is n. From the defnition of $< \theta_{(\tau)} >_0$ for $\theta_{(\tau)}$ as well as the equilibrium initial condition so that $H^M_0 = H_0 - \mu(c^\dagger_{Ak}c_{Ak} + c^\dagger_{Bk}c_{Bk})$, we know that any photon correlator with odd order of total number of photon operators will vanish. Thus

$$G^{M(n)}_{(k,Y,\tau\ ;k,X,\tau')} = G^{M(n)}_{kYX(\tau-\tau')} = 0\ ,\ \forall n \in odd \tag{10}$$

While from 3 we know that $G^{M(n)}_{(k,Y,\tau\ ;k,X,\tau')}$ only contains terms to the nth order of g. Thus from 4, we know that $S^{M(m)}_{(k\ ,ip_n)} = 0$ for all $m \in$ odd, thus the lowest perturbation correction to either self-energy or Green's Function is the second order.

Plugging the lowest order correction of self-energy, $\Sigma^M_{(k\ ,ip_n)} \to g^2 S^{M(2)}_{(k\ ,ip_n)}$, back to Dyson equation gives the lowest order of perturbation correction of $G^M_{(k\ ;k')(ip_n)}$, with the leading order of error equals to $g^4 G^M_{0(k\ ,ip_n)} S^{M(4)}_{(k\ ,ip_n)} G^M_{0(k\ ,ip_n)}$, which is small compared with second order correction $g^2 G^M_{0(k\ ,ip_n)} S^{M(2)}_{(k\ ,ip_n)} G^M_{0(k\ ,ip_n)}$ in weak coupling regime, and which thus can be neglected.

Besides, note that the total Hamiltonian do not allow the momentum hopping due to the zero momentum coupling approximation, thus

$$G^M_{(k,\tau\ ;k',0)} = 0 \quad \forall k \neq k' \tag{11}$$

Finally, from the Martin-Schwinger hierarchy equations in matrix form we know that

$$G^M_{0(k,ip_n)} = (ip_n - \widehat{H_{0e(k)}})^{-1} = \begin{pmatrix} -\frac{ip_n}{(p_n)^2+(kV_F)^2} & -\frac{(k_x+ik_y)V_F}{(p_n)^2+(kV_F)^2} \\ -\frac{(k_x-ik_y)V_F}{(p_n)^2+(kV_F)^2} & -\frac{ip_n}{(p_n)^2+(kV_F)^2} \end{pmatrix} \tag{12}$$

From which we can calculate all terms of lowest order correction of Matsubara Green's Function.Note that for simplicity we assume the half-filled case so that chemical potential $\mu = 0$. Applying Wick's theorem to $G^{M(2)}_{kYX(\tau-\tau')}$ we get 12 terms belonging to 3 categories, corresponding to Fock, Hatree and disconnected terms

$$G^{M(2)}_{kXY(\tau\ ;\tau')} = G^{(2)F}_{kXY(\tau\ ;\tau')} + G^{(2)H}_{kXY(\tau\ ;\tau')} + G^{(2)D}_{kXY(\tau\ ;\tau')} \tag{13}$$

where after defining two propagators of polarized photon
$D^{(0)}_{1(\tau_1\ ;\tau_2)} =< T_\tau a^\dagger_{(\tau_1)} a_{(\tau_2)} >_0$ and $D^{(0)}_{2(\tau_1\ ;\tau_2)} =< T_\tau a_{(\tau_1)} a^\dagger_{(\tau_2)} >_0$
we get the explicit formula of these 3 categories above

$$\begin{array}{l} G^{(2)F}_{kXY(\tau\ ;0)} = \\ -g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{1(\tau_1\ ;\tau_2)} G^{(0)}_{kXA(\tau\ ;\tau_1)} G^{(0)}_{kBB(\tau_1\ ;\tau_2)} G^{(0)}_{kAY(\tau_2\ ;0)} \\ -g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{1(\tau_1\ ;\tau_2)} G^{(0)}_{kXB(\tau\ ;\tau_2)} G^{(0)}_{kAA(\tau_2\ ;\tau_1)} G^{(0)}_{kBY(\tau_1\ ;0)} \\ -g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{2(\tau_1\ ;\tau_2)} G^{(0)}_{kXA(\tau\ ;\tau_2)} G^{(0)}_{kBB(\tau_2\ ;\tau_1)} G^{(0)}_{kAY(\tau_1\ ;0)} \\ -g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{2(\tau_1\ ;\tau_2)} G^{(0)}_{kXB(\tau\ ;\tau_1)} G^{(0)}_{kAA(\tau_1\ ;\tau_2)} G^{(0)}_{kBY(\tau_2\ ;0)} \end{array} \tag{14}$$

and

$$\begin{array}{l} G^{(2)H}_{kXY(\tau\ ;0)} = \\ g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{1(\tau_1\ ;\tau_2)} G^{(0)}_{kXA(\tau\ ;\tau_1)} G^{(0)}_{kBY(\tau_1\ ;0)} \Sigma_{k'} G^{(0)}_{k'AB(\tau_2\ ;\tau_2)} \\ +g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{1(\tau_1\ ;\tau_2)} G^{(0)}_{kXB(\tau\ ;\tau_2)} G^{(0)}_{kAY(\tau_2\ ;0)} \Sigma_{k'} G^{(0)}_{k'BA(\tau_1\ ;\tau_1)} \\ +g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{2(\tau_1\ ;\tau_2)} G^{(0)}_{kXA(\tau\ ;\tau_2)} G^{(0)}_{kBY(\tau_2\ ;0)} \Sigma_{k'} G^{(0)}_{k'AB(\tau_1\ ;\tau_1)} \\ +g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{2(\tau_1\ ;\tau_2)} G^{(0)}_{kXB(\tau\ ;\tau_1)} G^{(0)}_{kAY(\tau_1\ ;0)} \Sigma_{k'} G^{(0)}_{k'BA(\tau_2\ ;\tau_2)} \end{array} \tag{15}$$

and

$$\begin{array}{l} G^{(2)D}_{kXY(\tau\ ;0)} = \\ -g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{1(\tau_1\ ;\tau_2)} G^{(0)}_{kXY(\tau\ ;0)} \Sigma_{k'} G^{(0)}_{k'BA(\tau_1\ ;\tau_1)} \Sigma_{k''} G^{(0)}_{k''AB(\tau_2\ ;\tau_2)} \\ +g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{1(\tau_1\ ;\tau_2)} G^{(0)}_{kXY(\tau\ ;0)} \Sigma_{k'} G^{(0)}_{k'BB(\tau_1\ ;\tau_2)} G^{(0)}_{k'AA(\tau_2\ ;\tau_1)} \\ -g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{2(\tau_1\ ;\tau_2)} G^{(0)}_{kXY(\tau\ ;0)} \Sigma_{k''} G^{(0)}_{k''BA(\tau_2\ ;\tau_2)} \Sigma_{k'} G^{(0)}_{k'AB(\tau_1\ ;\tau_1)} \\ +g^2 \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 D^{(0)}_{2(\tau_1\ ;\tau_2)} G^{(0)}_{kXY(\tau\ ;0)} \Sigma_{k'} G^{(0)}_{k'AA(\tau_1\ ;\tau_2)} G^{(0)}_{k'BB(\tau_2\ ;\tau_1)} \end{array} \tag{16}$$

Actually, each term above should contain two summations over wave-vectors k' and k'' at two internal vertexes, which stems from the summation over k of the perturbation operator $V = \Sigma_{k'} V_{k'}$. However, due to $G^{M(0)}_{(k,X,\tau\ ;k',Y,0)} = \delta_{k,k'} G^{(0)}_{kXY(\tau\ ;0)}$, summation $\Sigma_{k'}$ on vertexes that directly connected with any one of external line(either k, X, τ or $k, Y, 0$) reduces into the only term with $k' = k$.

Before evaluation of the formulae of Matsubara Green's Function above, first note that equation (3) requires us to discard the disconnected term $G^{(2)D}_{kXY(\tau\ ;0)}$.

Then we prove Hatree component $G^{(2)H}_{kXY(\tau\ ;0)}$ actually vanishes. Note that each term of Hatree component contains a factor that is the direct summation of off-diagnal non-interacting propagator over wave-vector, such as $\Sigma_{k'}G^{(0)}_{k'AB(\tau_2\ ;\tau_2)}$. But from formula (12) we find $G^{(0)}_{k'AB(\tau_2\ ;\tau_2)} + G^{(0)}_{-k'AB(\tau_2\ ;\tau_2)} = 0$, thus

$$G^{(2)H}_{kXY(\tau\ ;0)} = 0 \tag{17}$$

So all we need to evaluate is the Fock component of lowest order correction. Using relation in discrete Fourier Transformation $G^{(0)}_{k(\tau'\ ;\tau)} = \frac{1}{\beta}\Sigma_{p_n} e^{-ip_n(\tau'-\tau)}G^{(0)}_{(k,ip_n)}$ where $p_n = \frac{(2n+1)\pi}{\beta}$ is the Fermion frequency, $D^{(0)}_{(\tau'\ ;\tau)} = \frac{1}{\beta}\Sigma_{\omega_n} e^{-i\omega_n(\tau'-\tau)}D^{(0)}_{(i\omega_n)}$ where $\omega_n = \frac{2n\pi}{\beta}$ is the Boson frequency, as well as equation (6) and $\frac{1}{\beta}\int_0^\beta \mathrm{d}\tau e^{i(p_n-p_{n'})} = \delta_{p_n-p_{n'}}$, we rewrite equation 14 as

$$\begin{array}{l} G^{(2)F}_{XY(k,ip_n)} = \\ -\frac{g^2}{\beta}\Sigma_{\omega_m} D^{(0)}_{1(k,i\omega_m)} G^{(0)}_{XA(k,ip_n)} G^{(0)}_{BB(k,ip_n-i\omega_m)} G^{(0)}_{AY(k,ip_n)} \\ -\frac{g^2}{\beta}\Sigma_{\omega_m} D^{(0)}_{1(k,i\omega_m)} G^{(0)}_{XB(k,ip_n)} G^{(0)}_{AA(k,ip_n+i\omega_m)} G^{(0)}_{BY(k,ip_n)} \\ -\frac{g^2}{\beta}\Sigma_{\omega_m} D^{(0)}_{2(k,i\omega_m)} G^{(0)}_{XA(k,ip_n)} G^{(0)}_{BB(k,ip_n+i\omega_m)} G^{(0)}_{AY(k,ip_n)} \\ -\frac{g^2}{\beta}\Sigma_{\omega_m} D^{(0)}_{2(k,i\omega_m)} G^{(0)}_{XB(k,ip_n)} G^{(0)}_{AA(k,ip_n-i\omega_m)} G^{(0)}_{BY(k,ip_n)} \end{array} \tag{18}$$

From matrix equation $G^{M(2)}_{(k,ip_n)} = G^{(2)F}_{(k,ip_n)} = g^2 G^M_{0(k\ ,ip_n)} S^{M(2)}_{(k\ ,ip_n)} G^M_{0(k\ ,ip_n)}$ we know the relation of corresponding equations of matrix elements

$$G^{(2)F}_{XY(k,ip_n)} = g^2 \Sigma_{P=A,B}\Sigma_{Q=A,B} G^M_{0XP(k\ ,ip_n)} S^{M(2)}_{PQ(k\ ,ip_n)} G^M_{0QY(k\ ,ip_n)} \tag{19}$$

Comparing equation (18) and (19) we finally derived the expression of the Matsubara self-energy to the lowest order of correction in matrix form

$$\begin{array}{c} \Sigma_{(k,ip_n)} = \begin{pmatrix} \Sigma_{aa(k,ip_n)} & 0 \\ 0 & \Sigma_{bb(k,ip_n)} \end{pmatrix} \\ where \\ \Sigma_{aa(k,ip_n)} = -\frac{g^2}{\beta}\Sigma_{\omega_m}(D^{(0)}_{1(-i\omega_m)} + D^{(0)}_{2(i\omega_m)}) G^{(0)}_{BB(k,ip_n+i\omega_m)} \\ \Sigma_{bb} = -\Sigma_{aa} \ /.\ \{\omega \to -\omega\} \end{array} \tag{20}$$

From the definition of two photon propagators we know

$$D^{(0)}_{1(-i\omega_m)} = D^{(0)}_{2(i\omega_m)} = \frac{1}{i\omega_m - \omega} \tag{21}$$

Then

$$\begin{array}{c} \Sigma_{kaa(k,ip_n)} = 2\frac{g^2}{\beta}\Sigma_{\omega_m}\frac{1}{i\omega_m-\omega}\frac{ip_n+i\omega_m}{(p_n+\omega_m)^2+(kV_F)^2} \\ = g^2\big(\frac{2(ip_n+\omega)n^B_{(\omega)}}{(ip_n+\omega)^2-(kV_F)^2} + \frac{n^B_{(-ip_n+kV_F)}}{-ip_n+kV_F-\omega} + \frac{n^B_{(-ip_n-kV_F)}}{-ip_n-kV_F-\omega}\big) \\ = g^2\big(\frac{2(ip_n+\omega)n^B_{(\omega)}}{(ip_n+\omega)^2-(kV_F)^2} - \frac{n^F_{(kV_F)}}{-ip_n+kV_F-\omega} - \frac{n^F_{(-kV_F)}}{-ip_n-kV_F-\omega}\big) \end{array} \tag{22}$$

Where we define Boson distribution $n^B_{(x)} = \frac{1}{e^{\beta x}-1}$ and Fermion distribution $n^F_{(x)} = \frac{1}{e^{\beta x}+1}$. The first equation is derived from contour integral and the second equation holds because $e^{-ip_n\beta} = -1$ for Fermion frequencies $p_n = \frac{(2n+1)\pi}{\beta}$.

4 achieve Retarded Green's Funciton from Matsubara component

When we start from the initial condition of thermal equilibrium, then from the Lehman representation, we know that the Retarded self-energy $\Sigma^R_{(k,\epsilon)}$ can be derived from Matusbara self-energy $\Sigma^M_{(k,ip_n)}$ using analytic continuation. Lehman representation of self-energy exists provided that the Hamiltonian is time-independent, and it states that

$$\begin{cases} \Sigma^R_{(k,\epsilon)} = \Sigma^M_{(k,ip_n \to \epsilon - \mu + i\eta)} \\ Lim_{|\epsilon| \to \infty} \Sigma^R_{(k,\epsilon)} \propto \frac{1}{\epsilon} \end{cases} \tag{23}$$

In which η is an infinitesimal positive number, and the second restriction guarantees the uniqueness of the possible analytic continuation, after which we get

$$\Sigma^R_{aa(k,\epsilon)} = g^2 \left(\frac{2(\epsilon + i\eta + \omega) n^B_{(\omega)}}{(\epsilon + i\eta + \omega)^2 - (kV_F)^2} + \frac{n^F_{(kV_F)}}{(\epsilon + i\eta) - kV_F + \omega} + \frac{n^F_{(-kV_F)}}{(\epsilon + i\eta) + kV_F + \omega} \right) \tag{24}$$

and

$$\Sigma^R_{bb(k,\epsilon)} = -g^2 \left(\frac{2(\epsilon + i\eta - \omega) n^B_{(-\omega)}}{(\epsilon + i\eta - \omega)^2 - (kV_F)^2} + \frac{n^F_{(kV_F)}}{(\epsilon + i\eta) - kV_F - \omega} + \frac{n^F_{(-kV_F)}}{(\epsilon + i\eta) + kV_F - \omega} \right) \tag{25}$$

Note that here we consider the case when $\mu = 0$ .At last we derive the Retarded Green's Function from the lowest order perturbation result $\Sigma^R_{(k,\epsilon)}$

$$G^R_{(k,\epsilon)} = ((G^R_{0(k,\epsilon)})^{-1} - \Sigma^R_{(k,\epsilon)})^{-1} \tag{26}$$

where

$$G^R_{0(k,\epsilon)} = \begin{pmatrix} \frac{\epsilon + i\eta}{(\epsilon + i\eta)^2 - (kV_F)^2} & \frac{(k_x + ik_y)V_F}{(\epsilon + i\eta)^2 - (kV_F)^2} \\ \frac{(k_x - ik_y)V_F}{(\epsilon + i\eta)^2 - (kV_F)^2} & \frac{\epsilon + i\eta}{(\epsilon + i\eta)^2 - (kV_F)^2} \end{pmatrix} \tag{27}$$

5 Spectral Properties of electronic structure

The spectra we first investigate is directly related to Retarded Green's Function

$$I_{(k,\epsilon)} = -\frac{1}{\pi} Im[TrG^R_{(k,\epsilon)}] \tag{28}$$

Below we plot the spectra under the condition of $\mu = 0,\ \beta = 2000,\ g = \sqrt{2} * 0.01 * 4.2,\ \omega = 0.1,\ \eta = 0.005$

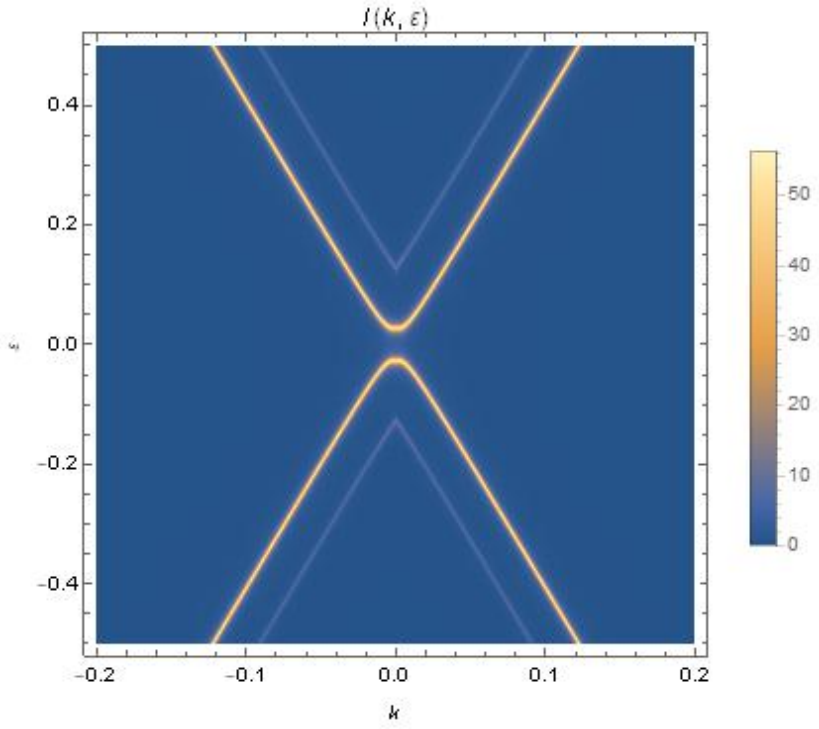

Figure 1: spectra

From this spectra we can find the opened gap Δ near the Dirac point

$$\Delta \to \sqrt{4g^2 + \omega^2} - \omega \quad when \quad \beta\omega \gg 1 \tag{29}$$

indicating the broken of time-reversal symmetry as well as the transition for graphene from conductor to insulator. There is also two sub-bands with lower intensity oriented from the coupling with the polarized cavity photon. The result above have been numerically checked in c++.

6 Issues in need of further study

The only 2 difference between my work and the original note on Graphene QED[2] are factor of two in self-energy and interchange of two photon propagators in the expression of $\Sigma_{aa(k,ip_n)}$ and$\Sigma_{bb(k,ip_n)}$.

More sub-bands may not occur even if higher order perturbation is applied, for the form of our Hamiltonian only allows hopping of electrons between two

internal energy eigenstates at same wave-vector k , this expectation requires further self-consistency numerical check.

Besides, we also need to deduce the topological property of the gap. And we also need to derived the case when $\mu \neq 0$.

7 Self-Consistency check and Pade's Method for analytical continuation

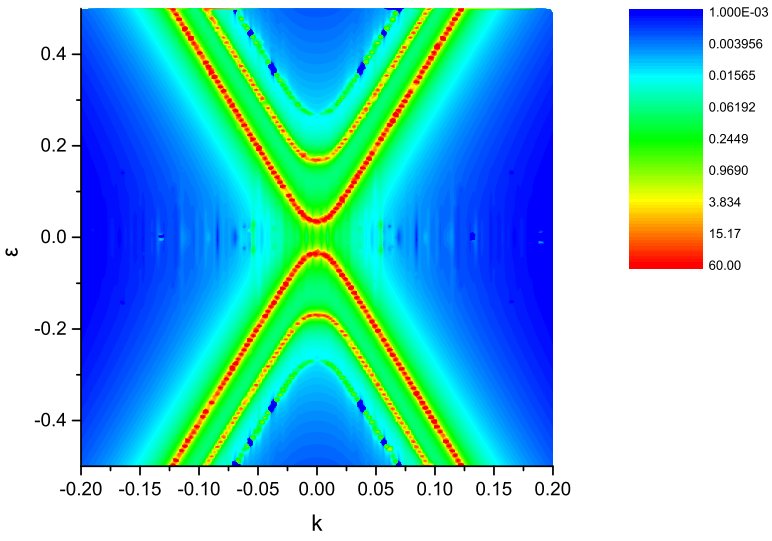

Figure 2: Numerical

References

[1] Gerald D. Manhan. *Many-Particle Physics*.

[2] Michael Sentef. *graphene QED*.