SUPPLEMENTARY MATERIAL

Supplementary Material for "Design with the Maximin Efficiency Robust Test for an Immunotherapy under the Generalized Delayed Treatment Effect Pattern"

ARTICLE HISTORY

Compiled June 26, 2022

1. Detailed simulation results

In this section, we drew Tables S1-S3 to present the detailed simulation results, namely the accurate values of the sample size estimations and empirical power, which are obtained in the simulation studies in the section 3.1 of the main text. Tables S1-S3 correspond to the considered asymptotic variances σ_1^2, σ_2^2 and σ_3^2 , respectively.

Table 1. Accuracies of sample size estimations with σ_1^2 under a variety of scenarios

| | | | Sample size (Empirical power) | | | | | |
|-----------|-----------|----|-------------------------------|---------------|---------------|---------------|---------------|--|
| n_1/n_0 | λ | au | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 | |
| 1 | 0.4 | 60 | 87(0.89693) | 97(0.89749) | 92(0.89775) | 90(0.89773) | 94(0.90174) | |
| 1 | 0.5 | 60 | 143(0.89937) | 158(0.89887) | 150(0.89794) | 148(0.89797) | 153(0.89858) | |
| 1 | 0.6 | 60 | 251(0.90004) | 277(0.89927) | 263(0.89844) | 259(0.90026) | 268(0.89965) | |
| 1 | 0.7 | 60 | 495(0.89828) | 545(0.89985) | 519(0.90038) | 511(0.89842) | 528(0.89982) | |
| 1 | 0.8 | 60 | 1229(0.90059) | 1350(0.89956) | 1287(0.89978) | 1267(0.90043) | 1307(0.90049) | |
| 2 | 0.4 | 60 | 90(0.88551) | 101(0.88918) | 96(0.89048) | 94(0.89082) | 97(0.8891) | |
| 2 | 0.5 | 60 | 152(0.89368) | 169(0.89345) | 160(0.89204) | 157(0.89294) | 163(0.89317) | |
| 2 | 0.6 | 60 | 271(0.89564) | 300(0.89593) | 285(0.89498) | 280(0.89375) | 290(0.8963) | |
| 2 | 0.7 | 60 | 543(0.89537) | 599(0.89672) | 570(0.89553) | 561(0.89493) | 579(0.89517) | |
| 2 | 0.8 | 60 | 1362(0.89808) | 1496(0.90116) | 1426(0.89819) | 1404(0.89809) | 1449(0.89808) | |
| 1 | 0.4 | 48 | 102(0.89978) | 115(0.89977) | 108(0.9003) | 106(0.90012) | 110(0.89846) | |
| 1 | 0.5 | 48 | 165(0.89799) | 185(0.89804) | 174(0.89883) | 171(0.89854) | 178(0.90007) | |
| 1 | 0.6 | 48 | 287(0.89874) | 321(0.8983) | 303(0.89967) | 297(0.89919) | 309(0.89865) | |
| 1 | 0.7 | 48 | 563(0.89864) | 627 (0.89798) | 594(0.89881) | 583(0.89907) | 605(0.89832) | |
| 1 | 0.8 | 48 | 1388(0.89933) | 1542(0.90016) | 1461(0.90008) | 1436(0.89977) | 1487(0.9002) | |
| 2 | 0.4 | 48 | 104(0.88563) | 118(0.88744) | 110(0.88409) | 108(0.88355) | 113(0.88293) | |
| 2 | 0.5 | 48 | 173(0.88709) | 195(0.88888) | 183(0.88574) | 180(0.889) | 187(0.88918) | |
| 2 | 0.6 | 48 | 308(0.89097) | 345(0.8944) | 325(0.89314) | 319(0.89236) | 331(0.89164) | |
| 2 | 0.7 | 48 | 614(0.8959) | 684(0.89464) | 647(0.89486) | 636(0.89585) | 659(0.89508) | |
| 2 | 0.8 | 48 | 1533(0.89761) | 1703(0.89766) | 1614(0.89531) | 1586(0.89669) | 1642(0.89644) | |
| 1 | 0.4 | 36 | 136(0.90075) | 158(0.899) | 146(0.89825) | 143(0.90046) | 150(0.90074) | |
| 1 | 0.5 | 36 | 218(0.89841) | 252(0.89911) | 234(0.89977) | 228(0.89845) | 240(0.89857) | |
| 1 | 0.6 | 36 | 375(0.89914) | 432(0.89874) | 402(0.89817) | 393(0.89931) | 412(0.89993) | |
| 1 | 0.7 | 36 | 730(0.89927) | 837 (0.89989) | 780(0.89785) | 763(0.8994) | 798(0.89838) | |
| 1 | 0.8 | 36 | 1784(0.89978) | 2039(0.89987) | 1904(0.90101) | 1862(0.89929) | 1947(0.90013) | |
| 2 | 0.4 | 36 | 137(0.87992) | 160(0.88184) | 148(0.88253) | 144(0.87859) | 152(0.8816) | |
| 2 | 0.5 | 36 | 226(0.88472) | 262(0.88715) | 243(0.88534) | 237(0.88514) | 249(0.88448) | |
| 2 | 0.6 | 36 | 398(0.88812) | 460(0.8916) | 427(0.89055) | 417(0.8887) | 437(0.89023) | |
| 2 | 0.7 | 36 | 789(0.89014) | 906(0.89276) | 844(0.8937) | 825(0.89234) | 864(0.89375) | |
| 2 | 0.8 | 36 | 1960(0.89636) | 2241(0.8963) | 2092(0.89337) | 2046(0.89711) | 2139(0.89639) | |

Table 2. Accuracies of sample size estimations with σ_2^2 under a variety of scenarios

| | | | Sample size (Empirical power) | | | | | |
|-----------|-----------|----|-------------------------------|---------------|---------------|---------------|---------------|--|
| n_1/n_0 | λ | au | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 | |
| 1 | 0.4 | 60 | 87(0.89872) | 96(0.8943) | 92(0.89888) | 90(0.89809) | 93(0.89562) | |
| 1 | 0.5 | 60 | 143(0.89912) | 158(0.89888) | 150(0.89882) | 148(0.89988) | 153(0.89733) | |
| 1 | 0.6 | 60 | 251(0.89861) | 277(0.89913) | 264(0.89951) | 260(0.90171) | 268(0.89876) | |
| 1 | 0.7 | 60 | 497(0.90036) | 546(0.89999) | 520(0.89878) | 512(0.90017) | 529(0.89896) | |
| 1 | 0.8 | 60 | 1231(0.9019) | 1351(0.89969) | 1288(0.89952) | 1268(0.89848) | 1308(0.89877) | |
| 2 | 0.4 | 60 | 102(0.91878) | 113(0.91996) | 107(0.92096) | 105(0.91801) | 109(0.92138) | |
| 2 | 0.5 | 60 | 167(0.91707) | 184(0.91714) | 175(0.91791) | 172(0.91808) | 178(0.91816) | |
| 2 | 0.6 | 60 | 291(0.91231) | 321(0.91389) | 305(0.91483) | 300(0.91287) | 310(0.91407) | |
| 2 | 0.7 | 60 | 571(0.91053) | 628 (0.91167) | 598(0.91171) | 589(0.91034) | 608(0.90874) | |
| 2 | 0.8 | 60 | 1405(0.90616) | 1541(0.90778) | 1470(0.90667) | 1448(0.90865) | 1493(0.90712) | |
| 1 | 0.4 | 48 | 100(0.89433) | 112(0.89176) | 106(0.89373) | 104(0.89536) | 108(0.89508) | |
| 1 | 0.5 | 48 | 164(0.89653) | 183(0.89614) | 173(0.89699) | 170(0.89674) | 176(0.8951) | |
| 1 | 0.6 | 48 | 286(0.89811) | 320(0.89965) | 302(0.89814) | 297 (0.89863) | 308(0.9) | |
| 1 | 0.7 | 48 | 563(0.89957) | 627 (0.89912) | 593(0.898) | 583(0.89943) | 604(0.89998) | |
| 1 | 0.8 | 48 | 1389(0.89812) | 1542(0.90069) | 1461(0.90023) | 1436(0.89922) | 1487(0.901) | |
| 2 | 0.4 | 48 | 115(0.91295) | 130(0.9142) | 122(0.91063) | 120(0.9113) | 125(0.91311) | |
| 2 | 0.5 | 48 | 189(0.91101) | 211(0.91268) | 199(0.91087) | 196(0.91315) | 203(0.91244) | |
| 2 | 0.6 | 48 | 329(0.91175) | 367(0.91026) | 347(0.91054) | 341(0.91003) | 353(0.91068) | |
| 2 | 0.7 | 48 | 644(0.90767) | 716(0.90791) | 678(0.90763) | 666(0.90735) | 690(0.90663) | |
| 2 | 0.8 | 48 | 1579(0.90632) | 1752(0.90412) | 1662(0.90498) | 1633(0.90492) | 1691(0.90575) | |
| 1 | 0.4 | 36 | 131(0.88911) | 153(0.88884) | 141(0.88623) | 138(0.88974) | 145(0.89006) | |
| 1 | 0.5 | 36 | 214(0.89473) | 247(0.89206) | 230(0.89464) | 224(0.89509) | 235(0.89286) | |
| 1 | 0.6 | 36 | 372(0.89798) | 428(0.89803) | 399(0.8959) | 389(0.89778) | 408(0.89438) | |
| 1 | 0.7 | 36 | 727 (0.89662) | 833(0.8988) | 777(0.89807) | 760(0.89682) | 795(0.89725) | |
| 1 | 0.8 | 36 | 1782(0.9003) | 2036(0.9006) | 1901(0.89837) | 1860(0.89903) | 1944(0.90056) | |
| 2 | 0.4 | 36 | 149(0.90536) | 173(0.90366) | 161(0.90472) | 157(0.9058) | 165(0.90556) | |
| 2 | 0.5 | 36 | 244(0.90619) | 281(0.90621) | 261(0.90425) | 255(0.90511) | 268(0.90735) | |
| 2 | 0.6 | 36 | 423(0.9055) | 486(0.9043) | 453(0.90502) | 443(0.9071) | 464(0.90851) | |
| 2 | 0.7 | 36 | 825 (0.90568) | 945(0.90484) | 882(0.90434) | 862(0.90454) | 902(0.90506) | |
| 2 | 0.8 | 36 | 2017(0.9031) | 2303(0.90308) | 2151(0.90385) | 2105(0.90385) | 2200(0.90328) | |

Table 3. Accuracies of sample size estimations with σ_3^2 under a variety of scenarios

| | | | _ | | . /5 | ` | | |
|-----------|-----|--------|-------------------------------|---------------|---------------|---------------|---------------|--|
| , | | | Sample size (Empirical power) | | | | | |
| n_1/n_0 | λ | τ | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 | |
| 1 | 0.4 | 60 | 87(0.8989) | 97(0.89706) | 92(0.89796) | 90(0.89703) | 93(0.89508) | |
| 1 | 0.5 | 60 | 143(0.89904) | 158(0.89822) | 150(0.89969) | 148(0.89996) | 153(0.8993) | |
| 1 | 0.6 | 60 | 251(0.89904) | 277(0.89918) | 263(0.89842) | 259(0.8979) | 268(0.90015) | |
| 1 | 0.7 | 60 | 496(0.90062) | 546(0.89884) | 520(0.90001) | 512(0.8994) | 528 (0.90064) | |
| 1 | 0.8 | 60 | 1230(0.90073) | 1350(0.90109) | 1287(0.90019) | 1267(0.90034) | 1308(0.89972) | |
| 2 | 0.4 | 60 | 96(0.90223) | 107(0.90588) | 101(0.90553) | 99(0.9017) | 103(0.90819) | |
| 2 | 0.5 | 60 | 159(0.90395) | 176(0.90565) | 167(0.90535) | 164(0.90509) | 170(0.90482) | |
| 2 | 0.6 | 60 | 281(0.90522) | 310(0.90587) | 295(0.90613) | 290(0.90474) | 300(0.90473) | |
| 2 | 0.7 | 60 | 557(0.90248) | 613(0.90285) | 584(0.90575) | 574(0.90408) | 593(0.90371) | |
| 2 | 0.8 | 60 | 1383(0.90167) | 1519(0.9028) | 1448(0.9027) | 1426(0.90239) | 1471(0.90459) | |
| 1 | 0.4 | 48 | 101(0.89879) | 113(0.89618) | 107(0.89533) | 105(0.89648) | 109(0.89673) | |
| 1 | 0.5 | 48 | 164(0.89713) | 184(0.90113) | 174(0.89772) | 170(0.89707) | 177(0.89957) | |
| 1 | 0.6 | 48 | 287(0.89859) | 320(0.89996) | 303(0.89952) | 297(0.89856) | 308(0.89974) | |
| 1 | 0.7 | 48 | 563(0.89778) | 627 (0.89929) | 594(0.89859) | 583(0.90165) | 604(0.90092) | |
| 1 | 0.8 | 48 | 1389(0.9) | 1542(0.89974) | 1461(0.9009) | 1436(0.89935) | 1487(0.90079) | |
| 2 | 0.4 | 48 | 109(0.89988) | 123(0.89814) | 116(0.8987) | 114(0.89796) | 118(0.89957) | |
| 2 | 0.5 | 48 | 181(0.90205) | 203(0.89905) | 191(0.90055) | 188(0.90259) | 195(0.90073) | |
| 2 | 0.6 | 48 | 318(0.90021) | 355(0.90085) | 336(0.89927) | 330(0.90027) | 342(0.8995) | |
| 2 | 0.7 | 48 | 628(0.90038) | 700(0.90071) | 662(0.9001) | 651(0.90183) | 674(0.90079) | |
| 2 | 0.8 | 48 | 1556(0.90079) | 1727(0.90094) | 1637(0.90164) | 1609(0.901) | 1666(0.90076) | |
| 1 | 0.4 | 36 | 134(0.89765) | 155(0.89287) | 144(0.8956) | 140(0.89449) | 147(0.89291) | |
| 1 | 0.5 | 36 | 216(0.89701) | 250(0.89696) | 232(0.89824) | 226(0.8959) | 238(0.89803) | |
| 1 | 0.6 | 36 | 374(0.89828) | 430(0.89757) | 400(0.89845) | 391(0.89775) | 410(0.90023) | |
| 1 | 0.7 | 36 | 729(0.89888) | 835(0.89991) | 779(0.89933) | 761(0.8996) | 797(0.89891) | |
| 1 | 0.8 | 36 | 1783(0.9025) | 2037(0.90062) | 1902(0.90119) | 1861(0.89829) | 1945(0.89887) | |
| 2 | 0.4 | 36 | 143(0.89351) | 166(0.89273) | 154(0.89419) | 150(0.89158) | 158(0.89325) | |
| 2 | 0.5 | 36 | 235(0.89654) | 271(0.89544) | 252(0.89548) | 246(0.89469) | 258(0.89461) | |
| 2 | 0.6 | 36 | 411(0.89809) | 473(0.89868) | 440(0.89906) | 429(0.89791) | 450(0.89829) | |
| 2 | 0.7 | 36 | 807(0.8999) | 925 (0.89828) | 863(0.90135) | 843(0.89698) | 883(0.89852) | |
| 2 | 0.8 | 36 | 1988(0.89884) | 2272(0.89976) | 2121(0.8971) | 2075(0.90011) | 2169(0.89976) | |