

## SUPPLEMENTARY MATERIAL

### Supplementary Material for "Design with the Maximin Efficiency Robust Test for an Immunotherapy under the Generalized Delayed Treatment Effect Pattern"

Bosheng Li<sup>a</sup>, Jingyi Zhang<sup>a</sup> and Fangrong Yan<sup>a</sup>

<sup>a</sup> Research Center of Biostatistics and Computational Pharmacy, China Pharmaceutical University, Nanjing China.

#### ARTICLE HISTORY

Compiled March 10, 2022

#### 1. Detailed simulation results

In this section, we drew Tables S1-S3 to present the detailed simulation results, namely the accurate values of the sample size estimations and empirical power, which are obtained in the simulation studies in the section 3.1 of the main text. Tables S1-S3 correspond to the considered asymptotic variances  $\sigma_1^2, \sigma_2^2$  and  $\sigma_3^2$ , respectively.

**Table 1.** Accuracies of sample size estimations with  $\sigma_1^2$  under a variety of scenarios

| $n_1/n_0$ | $\lambda$ | $\tau$ | Sample size (Empirical power) |               |               |               |               |
|-----------|-----------|--------|-------------------------------|---------------|---------------|---------------|---------------|
|           |           |        | Scenario 1                    | Scenario 2    | Scenario 3    | Scenario 4    | Scenario 5    |
| 1         | 0.4       | 60     | 87(0.89693)                   | 97(0.89749)   | 92(0.89775)   | 90(0.89773)   | 94(0.90174)   |
| 1         | 0.5       | 60     | 143(0.89937)                  | 158(0.89887)  | 150(0.89794)  | 148(0.89797)  | 153(0.89858)  |
| 1         | 0.6       | 60     | 251(0.90004)                  | 277(0.89927)  | 263(0.89844)  | 259(0.90026)  | 268(0.89965)  |
| 1         | 0.7       | 60     | 495(0.89828)                  | 545(0.89985)  | 519(0.90038)  | 511(0.89842)  | 528(0.89982)  |
| 1         | 0.8       | 60     | 1229(0.90059)                 | 1350(0.89956) | 1287(0.89978) | 1267(0.90043) | 1307(0.90049) |
| 2         | 0.4       | 60     | 90(0.88551)                   | 101(0.88918)  | 96(0.89048)   | 94(0.89082)   | 97(0.8891)    |
| 2         | 0.5       | 60     | 152(0.89368)                  | 169(0.89345)  | 160(0.89204)  | 157(0.89294)  | 163(0.89317)  |
| 2         | 0.6       | 60     | 271(0.89564)                  | 300(0.89593)  | 285(0.89498)  | 280(0.89375)  | 290(0.8963)   |
| 2         | 0.7       | 60     | 543(0.89537)                  | 599(0.89672)  | 570(0.89553)  | 561(0.89493)  | 579(0.89517)  |
| 2         | 0.8       | 60     | 1362(0.89808)                 | 1496(0.90116) | 1426(0.89819) | 1404(0.89809) | 1449(0.89808) |
| 1         | 0.4       | 48     | 102(0.89978)                  | 115(0.89977)  | 108(0.9003)   | 106(0.90012)  | 110(0.89846)  |
| 1         | 0.5       | 48     | 165(0.89799)                  | 185(0.89804)  | 174(0.89883)  | 171(0.89854)  | 178(0.90007)  |
| 1         | 0.6       | 48     | 287(0.89874)                  | 321(0.8983)   | 303(0.89967)  | 297(0.89919)  | 309(0.89865)  |
| 1         | 0.7       | 48     | 563(0.89864)                  | 627(0.89798)  | 594(0.89881)  | 583(0.89907)  | 605(0.89832)  |
| 1         | 0.8       | 48     | 1388(0.89933)                 | 1542(0.90016) | 1461(0.90008) | 1436(0.89977) | 1487(0.9002)  |
| 2         | 0.4       | 48     | 104(0.88563)                  | 118(0.88744)  | 110(0.88409)  | 108(0.88355)  | 113(0.88293)  |
| 2         | 0.5       | 48     | 173(0.88709)                  | 195(0.88888)  | 183(0.88574)  | 180(0.889)    | 187(0.88918)  |
| 2         | 0.6       | 48     | 308(0.89097)                  | 345(0.8944)   | 325(0.89314)  | 319(0.89236)  | 331(0.89164)  |
| 2         | 0.7       | 48     | 614(0.8959)                   | 684(0.89464)  | 647(0.89486)  | 636(0.89585)  | 659(0.89508)  |
| 2         | 0.8       | 48     | 1533(0.89761)                 | 1703(0.89766) | 1614(0.89531) | 1586(0.89669) | 1642(0.89644) |
| 1         | 0.4       | 36     | 136(0.90075)                  | 158(0.899)    | 146(0.89825)  | 143(0.90046)  | 150(0.90074)  |
| 1         | 0.5       | 36     | 218(0.89841)                  | 252(0.89911)  | 234(0.89977)  | 228(0.89845)  | 240(0.89857)  |
| 1         | 0.6       | 36     | 375(0.89914)                  | 432(0.89874)  | 402(0.89817)  | 393(0.89931)  | 412(0.89993)  |
| 1         | 0.7       | 36     | 730(0.89927)                  | 837(0.89989)  | 780(0.89785)  | 763(0.8994)   | 798(0.89838)  |
| 1         | 0.8       | 36     | 1784(0.89978)                 | 2039(0.89987) | 1904(0.90101) | 1862(0.89929) | 1947(0.90013) |
| 2         | 0.4       | 36     | 137(0.87992)                  | 160(0.88184)  | 148(0.88253)  | 144(0.87859)  | 152(0.8816)   |
| 2         | 0.5       | 36     | 226(0.88472)                  | 262(0.88715)  | 243(0.88534)  | 237(0.88514)  | 249(0.88448)  |
| 2         | 0.6       | 36     | 398(0.88812)                  | 460(0.8916)   | 427(0.89055)  | 417(0.8887)   | 437(0.89023)  |
| 2         | 0.7       | 36     | 789(0.89014)                  | 906(0.89276)  | 844(0.8937)   | 825(0.89234)  | 864(0.89375)  |
| 2         | 0.8       | 36     | 1960(0.89636)                 | 2241(0.8963)  | 2092(0.89337) | 2046(0.89711) | 2139(0.89639) |

**Table 2.** Accuracies of sample size estimations with  $\sigma_2^2$  under a variety of scenarios

| $n_1/n_0$ | $\lambda$ | $\tau$ | Sample size (Empirical power) |               |               |               |               |
|-----------|-----------|--------|-------------------------------|---------------|---------------|---------------|---------------|
|           |           |        | Scenario 1                    | Scenario 2    | Scenario 3    | Scenario 4    | Scenario 5    |
| 1         | 0.4       | 60     | 87(0.89872)                   | 96(0.8943)    | 92(0.89888)   | 90(0.89809)   | 93(0.89562)   |
| 1         | 0.5       | 60     | 143(0.89912)                  | 158(0.89888)  | 150(0.89882)  | 148(0.89988)  | 153(0.89733)  |
| 1         | 0.6       | 60     | 251(0.89861)                  | 277(0.89913)  | 264(0.89951)  | 260(0.90171)  | 268(0.89876)  |
| 1         | 0.7       | 60     | 497(0.90036)                  | 546(0.89999)  | 520(0.89878)  | 512(0.90017)  | 529(0.89896)  |
| 1         | 0.8       | 60     | 1231(0.9019)                  | 1351(0.89969) | 1288(0.89952) | 1268(0.89848) | 1308(0.89877) |
| 2         | 0.4       | 60     | 102(0.91878)                  | 113(0.91996)  | 107(0.92096)  | 105(0.91801)  | 109(0.92138)  |
| 2         | 0.5       | 60     | 167(0.91707)                  | 184(0.91714)  | 175(0.91791)  | 172(0.91808)  | 178(0.91816)  |
| 2         | 0.6       | 60     | 291(0.91231)                  | 321(0.91389)  | 305(0.91483)  | 300(0.91287)  | 310(0.91407)  |
| 2         | 0.7       | 60     | 571(0.91053)                  | 628(0.91167)  | 598(0.91171)  | 589(0.91034)  | 608(0.90874)  |
| 2         | 0.8       | 60     | 1405(0.90616)                 | 1541(0.90778) | 1470(0.90667) | 1448(0.90865) | 1493(0.90712) |
| 1         | 0.4       | 48     | 100(0.89433)                  | 112(0.89176)  | 106(0.89373)  | 104(0.89536)  | 108(0.89508)  |
| 1         | 0.5       | 48     | 164(0.89653)                  | 183(0.89614)  | 173(0.89699)  | 170(0.89674)  | 176(0.8951)   |
| 1         | 0.6       | 48     | 286(0.89811)                  | 320(0.89965)  | 302(0.89814)  | 297(0.89863)  | 308(0.9)      |
| 1         | 0.7       | 48     | 563(0.89957)                  | 627(0.89912)  | 593(0.898)    | 583(0.89943)  | 604(0.89998)  |
| 1         | 0.8       | 48     | 1389(0.89812)                 | 1542(0.90069) | 1461(0.90023) | 1436(0.89922) | 1487(0.901)   |
| 2         | 0.4       | 48     | 115(0.91295)                  | 130(0.9142)   | 122(0.91063)  | 120(0.9113)   | 125(0.91311)  |
| 2         | 0.5       | 48     | 189(0.91101)                  | 211(0.91268)  | 199(0.91087)  | 196(0.91315)  | 203(0.91244)  |
| 2         | 0.6       | 48     | 329(0.91175)                  | 367(0.91026)  | 347(0.91054)  | 341(0.91003)  | 353(0.91068)  |
| 2         | 0.7       | 48     | 644(0.90767)                  | 716(0.90791)  | 678(0.90763)  | 666(0.90735)  | 690(0.90663)  |
| 2         | 0.8       | 48     | 1579(0.90632)                 | 1752(0.90412) | 1662(0.90498) | 1633(0.90492) | 1691(0.90575) |
| 1         | 0.4       | 36     | 131(0.88911)                  | 153(0.88884)  | 141(0.88623)  | 138(0.88974)  | 145(0.89006)  |
| 1         | 0.5       | 36     | 214(0.89473)                  | 247(0.89206)  | 230(0.89464)  | 224(0.89509)  | 235(0.89286)  |
| 1         | 0.6       | 36     | 372(0.89798)                  | 428(0.89803)  | 399(0.8959)   | 389(0.89778)  | 408(0.89438)  |
| 1         | 0.7       | 36     | 727(0.89662)                  | 833(0.8988)   | 777(0.89807)  | 760(0.89682)  | 795(0.89725)  |
| 1         | 0.8       | 36     | 1782(0.9003)                  | 2036(0.9006)  | 1901(0.89837) | 1860(0.89903) | 1944(0.90056) |
| 2         | 0.4       | 36     | 149(0.90536)                  | 173(0.90366)  | 161(0.90472)  | 157(0.9058)   | 165(0.90556)  |
| 2         | 0.5       | 36     | 244(0.90619)                  | 281(0.90621)  | 261(0.90425)  | 255(0.90511)  | 268(0.90735)  |
| 2         | 0.6       | 36     | 423(0.9055)                   | 486(0.9043)   | 453(0.90502)  | 443(0.9071)   | 464(0.90851)  |
| 2         | 0.7       | 36     | 825(0.90568)                  | 945(0.90484)  | 882(0.90434)  | 862(0.90454)  | 902(0.90506)  |
| 2         | 0.8       | 36     | 2017(0.9031)                  | 2303(0.90308) | 2151(0.90385) | 2105(0.90385) | 2200(0.90328) |

**Table 3.** Accuracies of sample size estimations with  $\sigma_3^2$  under a variety of scenarios

| $n_1/n_0$ | $\lambda$ | $\tau$ | Sample size (Empirical power) |               |               |               |               |
|-----------|-----------|--------|-------------------------------|---------------|---------------|---------------|---------------|
|           |           |        | Scenario 1                    | Scenario 2    | Scenario 3    | Scenario 4    | Scenario 5    |
| 1         | 0.4       | 60     | 87(0.8989)                    | 97(0.89706)   | 92(0.89796)   | 90(0.89703)   | 93(0.89508)   |
| 1         | 0.5       | 60     | 143(0.89904)                  | 158(0.89822)  | 150(0.89969)  | 148(0.89996)  | 153(0.8993)   |
| 1         | 0.6       | 60     | 251(0.89904)                  | 277(0.89918)  | 263(0.89842)  | 259(0.8979)   | 268(0.90015)  |
| 1         | 0.7       | 60     | 496(0.90062)                  | 546(0.89884)  | 520(0.90001)  | 512(0.8994)   | 528(0.90064)  |
| 1         | 0.8       | 60     | 1230(0.90073)                 | 1350(0.90109) | 1287(0.90019) | 1267(0.90034) | 1308(0.89972) |
| 2         | 0.4       | 60     | 96(0.90223)                   | 107(0.90588)  | 101(0.90553)  | 99(0.9017)    | 103(0.90819)  |
| 2         | 0.5       | 60     | 159(0.90395)                  | 176(0.90565)  | 167(0.90535)  | 164(0.90509)  | 170(0.90482)  |
| 2         | 0.6       | 60     | 281(0.90522)                  | 310(0.90587)  | 295(0.90613)  | 290(0.90474)  | 300(0.90473)  |
| 2         | 0.7       | 60     | 557(0.90248)                  | 613(0.90285)  | 584(0.90575)  | 574(0.90408)  | 593(0.90371)  |
| 2         | 0.8       | 60     | 1383(0.90167)                 | 1519(0.9028)  | 1448(0.9027)  | 1426(0.90239) | 1471(0.90459) |
| 1         | 0.4       | 48     | 101(0.89879)                  | 113(0.89618)  | 107(0.89533)  | 105(0.89648)  | 109(0.89673)  |
| 1         | 0.5       | 48     | 164(0.89713)                  | 184(0.90113)  | 174(0.89772)  | 170(0.89707)  | 177(0.89957)  |
| 1         | 0.6       | 48     | 287(0.89859)                  | 320(0.89996)  | 303(0.89952)  | 297(0.89856)  | 308(0.89974)  |
| 1         | 0.7       | 48     | 563(0.89778)                  | 627(0.89929)  | 594(0.89859)  | 583(0.90165)  | 604(0.90092)  |
| 1         | 0.8       | 48     | 1389(0.9)                     | 1542(0.89974) | 1461(0.9009)  | 1436(0.89935) | 1487(0.90079) |
| 2         | 0.4       | 48     | 109(0.89988)                  | 123(0.89814)  | 116(0.8987)   | 114(0.89796)  | 118(0.89957)  |
| 2         | 0.5       | 48     | 181(0.90205)                  | 203(0.89905)  | 191(0.90055)  | 188(0.90259)  | 195(0.90073)  |
| 2         | 0.6       | 48     | 318(0.90021)                  | 355(0.90085)  | 336(0.89927)  | 330(0.90027)  | 342(0.8995)   |
| 2         | 0.7       | 48     | 628(0.90038)                  | 700(0.90071)  | 662(0.9001)   | 651(0.90183)  | 674(0.90079)  |
| 2         | 0.8       | 48     | 1556(0.90079)                 | 1727(0.90094) | 1637(0.90164) | 1609(0.901)   | 1666(0.90076) |
| 1         | 0.4       | 36     | 134(0.89765)                  | 155(0.89287)  | 144(0.8956)   | 140(0.89449)  | 147(0.89291)  |
| 1         | 0.5       | 36     | 216(0.89701)                  | 250(0.89696)  | 232(0.89824)  | 226(0.8959)   | 238(0.89803)  |
| 1         | 0.6       | 36     | 374(0.89828)                  | 430(0.89757)  | 400(0.89845)  | 391(0.89775)  | 410(0.90023)  |
| 1         | 0.7       | 36     | 729(0.89888)                  | 835(0.89991)  | 779(0.89933)  | 761(0.8996)   | 797(0.89891)  |
| 1         | 0.8       | 36     | 1783(0.9025)                  | 2037(0.90062) | 1902(0.90119) | 1861(0.89829) | 1945(0.89887) |
| 2         | 0.4       | 36     | 143(0.89351)                  | 166(0.89273)  | 154(0.89419)  | 150(0.89158)  | 158(0.89325)  |
| 2         | 0.5       | 36     | 235(0.89654)                  | 271(0.89544)  | 252(0.89548)  | 246(0.89469)  | 258(0.89461)  |
| 2         | 0.6       | 36     | 411(0.89809)                  | 473(0.89868)  | 440(0.89906)  | 429(0.89791)  | 450(0.89829)  |
| 2         | 0.7       | 36     | 807(0.8999)                   | 925(0.89828)  | 863(0.90135)  | 843(0.89698)  | 883(0.89852)  |
| 2         | 0.8       | 36     | 1988(0.89884)                 | 2272(0.89976) | 2121(0.8971)  | 2075(0.90011) | 2169(0.89976) |