Brief note[*][†]

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Based on Mathematica code calculating the impedance of infinite thickness with elliptical cross section, this note is showing the detailed process of calculation code, which will be extended using Python platform that will be edited by Dengjie Xiao

1 Gaussian Elimination

Step 1: parameters & Definitions & Elements

Parameters
$A = 0.003 \times t, B = 0.003, l = \sqrt{A^2 - B^2}, u = \text{arccosh}[u]$
$\sigma_0 = 4.228 \times 10^7, \tau = 8.0055 * 10^{-15}, \sigma = \frac{\sigma_0}{1 - i\omega \times \tau}$
$Z0 = 377, c = 2.997925 \times 10^8, \epsilon = 8.8 \times 10^{-12}, Truncation = m$
Definitions
$k = \frac{\omega}{c}, W = ikl^2, u1 = v1 = 0, \lambda = \sqrt{ik \times Z0 \times \sigma}, Ru0 = il\lambda \cosh[u]$
Elements
$n = 0$
$d[0] = -\frac{W}{4} \sinh[2u] - (\frac{ik}{\lambda^2} + \frac{i}{k}) \times Ru0$
$z[0] = \frac{W}{8} \sinh[2u], \quad t[0] = -\frac{1}{2\pi\epsilon c}, \quad s[0] = \frac{W}{4} \cosh[2u]$
$n > 0$
$s[n\_] := \frac{W}{8(2n+1)} (\frac{\sinh[2nu]}{\sinh[(2n+2)u]})$
$z[n\_] := \frac{W}{8(2n+1)} (\frac{\sinh[(2n+2)u]}{\sinh[2nu]} + \frac{\cosh[(2n+2)u]}{\cosh[2nu]})$
$t[n\_] := \frac{1}{\pi\epsilon c} \cosh[2nu1] \cos 2nv1 (\tanh[2nu] - \coth[2nu])$
$d[n\_] := -W(\frac{\sinh[(2n+2)u]}{(16n+8)\sinh[2nu]} + \frac{\cosh[(2n+2)u]}{(16n+8)\cosh[2nu]} + \frac{\sinh[(2n-2)u]}{(16n-8)\sinh[2nu]} + \frac{\cosh[(2n-2)u]}{(16n-8)\cosh[2nu]})$
$+ \frac{4ni}{k} - \frac{i}{k} Ru0 \coth[2nu] - \frac{ik}{\lambda^2} [Ru0(\tanh[2nu] + \coth[2nu]) - 4n]$

[*] Yutsing's Mathematica Code
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Step 2: Gaussian Elimination

$CC[0] = d[0] \qquad n = 0$
$CC[i\_] := d[i] - \frac{s[i-1]z[i-1]}{CC[i-1]} \qquad m \geq n > 0$

$DD[1] = \frac{s[0]}{CC[0]} \qquad n = 0$
$DD[i\_] := \frac{s[i-1]}{CC[i-1]} \qquad m \geq n > 0$

$T[0] = t[0] \qquad n = 0$
$T[i\_] := t[i] - T[i-1]DD[i] \qquad m \geq n > 0$

$X[m] = \frac{T[m]}{CC[m]} \qquad n = m$
$X[i\_] := \frac{T[i]-z[i]X[i+1]}{CC[i]} \qquad 0 \leq n < m$

Step 3: Coefficients

$Coe = \sum_{i=0}^{m} (-1)^i X[i]$

Step 4: Plot

Plot[Coe], axis: (ω, Coe), Range: $\omega \in (10^1, 10^{13})$

For first test, set $t = 1.01$ and $truncation = m = 10$. Then set $t = 2.8$ and increase truncation number until the final result converges.