计算机图形学小白入门

————从0开始实现OpenGL

线性代数—矩阵行列视图

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矩阵的列视图

- 矩阵可以看做由多个**列向量**排列构成，如下所示：

$$\begin{pmatrix} \boxed{\begin{matrix} a_{00} \\ a_{10} \\ a_{20} \end{matrix}} & \boxed{\begin{matrix} a_{01} \\ a_{11} \\ a_{21} \end{matrix}} & \boxed{\begin{matrix} a_{02} \\ a_{12} \\ a_{22} \end{matrix}} \end{pmatrix}$$

$$\vec{a}_0 = \begin{pmatrix} a_{00} \\ a_{10} \\ a_{20} \end{pmatrix} \quad \vec{a}_1 = \begin{pmatrix} a_{01} \\ a_{11} \\ a_{21} \end{pmatrix} \quad \vec{a}_2 = \begin{pmatrix} a_{02} \\ a_{12} \\ a_{22} \end{pmatrix}$$

表示为 $\begin{pmatrix} \vec{a}_0 & \vec{a}_1 & \vec{a}_2 \end{pmatrix}$

矩阵的行视图

- 矩阵可以看做由多个**行向量**排列构成，如下所示：

$$\begin{pmatrix} \boxed{a_{00} \quad a_{01} \quad a_{02}} \\ \boxed{a_{10} \quad a_{11} \quad a_{12}} \\ \boxed{a_{20} \quad a_{21} \quad a_{22}} \end{pmatrix}$$

$$\vec{a}_0 = \begin{pmatrix} a_{00} \\ a_{01} \\ a_{02} \end{pmatrix} \quad \vec{a}_1 = \begin{pmatrix} a_{10} \\ a_{11} \\ a_{12} \end{pmatrix} \quad \vec{a}_2 = \begin{pmatrix} a_{20} \\ a_{21} \\ a_{22} \end{pmatrix}$$

表示为

$$\begin{pmatrix} \vec{a}_0^{\,T} \\ \vec{a}_1^{\,T} \\ \vec{a}_2^{\,T} \end{pmatrix}$$

已知矩阵乘法

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{00} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{pmatrix}$$

举例：

$$\mathrm{c}_{21} = \begin{pmatrix} a_{20} \\ a_{21} \\ a_{22} \end{pmatrix} . \begin{pmatrix} b_{01} \\ b_{11} \\ b_{21} \end{pmatrix} = a_{20}.b_{01} + a_{21}.b_{11} + a_{22}.b_{21}$$

矩阵与向量乘法理解

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \boxed{a_{00}}.x + \boxed{a_{01}}.y + \boxed{a_{02}}.z \\ \boxed{a_{10}}.x + \boxed{a_{11}}.y + \boxed{a_{12}}.z \\ \boxed{a_{20}}.x + \boxed{a_{21}}.y + \boxed{a_{22}}.z \end{pmatrix}$$

- 矩阵每一列，都理解为一个向量，乘法可以看作三个向量的加权和

$$\overrightarrow{a_0} = \begin{pmatrix} a_{00} \\ a_{10} \\ a_{20} \end{pmatrix} \quad \overrightarrow{a_1} = \begin{pmatrix} a_{01} \\ a_{11} \\ a_{21} \end{pmatrix} \quad \overrightarrow{a_2} = \begin{pmatrix} a_{02} \\ a_{12} \\ a_{22} \end{pmatrix}$$

$$result = x.\overrightarrow{a_0} + y.\overrightarrow{a_1} + z.\overrightarrow{a_2}$$

思考题
考虑成加权和，那么**xyz**表示的权重是什么？每个列的向量含义是什么？

重新理解矩阵乘法（列视图）

$$\begin{pmatrix} \boxed{\begin{matrix} a_{00} \\ a_{10} \\ a_{20} \end{matrix}} & \boxed{\begin{matrix} a_{01} \\ a_{11} \\ a_{21} \end{matrix}} & \boxed{\begin{matrix} a_{02} \\ a_{12} \\ a_{22} \end{matrix}} \end{pmatrix} \times \begin{pmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} c_{00} \\ c_{10} \\ c_{20} \end{matrix}} & \boxed{\begin{matrix} c_{01} \\ c_{11} \\ c_{21} \end{matrix}} & \boxed{\begin{matrix} c_{02} \\ c_{12} \\ c_{22} \end{matrix}} \end{pmatrix}$$

- A矩阵看作**列向量**的组合，结果C矩阵也看作**列向量**组合

$$\overrightarrow{a_0} = \begin{pmatrix} a_{00} \\ a_{10} \\ a_{20} \end{pmatrix} \quad \overrightarrow{a_1} = \begin{pmatrix} a_{01} \\ a_{11} \\ a_{21} \end{pmatrix} \quad \overrightarrow{a_2} = \begin{pmatrix} a_{02} \\ a_{12} \\ a_{22} \end{pmatrix} \qquad \overrightarrow{c_0} = \begin{pmatrix} c_{00} \\ c_{10} \\ c_{20} \end{pmatrix} \quad \overrightarrow{c_1} = \begin{pmatrix} c_{01} \\ c_{11} \\ c_{21} \end{pmatrix} \quad \overrightarrow{c_2} = \begin{pmatrix} c_{02} \\ c_{12} \\ c_{22} \end{pmatrix}$$

乘法可以表示为

$$\begin{pmatrix} \overrightarrow{a_0} & \overrightarrow{a_1} & \overrightarrow{a_2} \end{pmatrix} \times \begin{pmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} \overrightarrow{c_0} & \overrightarrow{c_1} & \overrightarrow{c_2} \end{pmatrix}$$

- 按照矩阵乘法，我们可以得到如下等式

$$\overrightarrow{c_0} = b_{00}.\overrightarrow{a_0} + b_{10}.\overrightarrow{a_1} + b_{20}.\overrightarrow{a_2}$$

$$\overrightarrow{c_1} = b_{01}.\overrightarrow{a_0} + b_{11}.\overrightarrow{a_1} + b_{21}.\overrightarrow{a_2}$$

$$\overrightarrow{c_2} = b_{02}.\overrightarrow{a_0} + b_{12}.\overrightarrow{a_1} + b_{22}.\overrightarrow{a_2}$$

位于右边的矩阵B，每一列都可以理解为对A矩阵列向量的**调配比例（或者说权重）**

矩阵C的每一个列向量，都是A矩阵根据B提供的权重**加权相加**的结果

矩阵与向量(行）乘法理解

$$(x \quad y \quad z)\times\begin{pmatrix} \boxed{a_{00} \quad a_{01} \quad a_{02}} \\ \boxed{a_{10} \quad a_{11} \quad a_{12}} \\ \boxed{a_{20} \quad a_{21} \quad a_{22}} \end{pmatrix}=(a_{00}.x+a_{10}.y+a_{20}.z \quad a_{01}.x+a_{11}.y+a_{21}.z \quad a_{02}.x+a_{12}.y+a_{22}.z)$$

- 矩阵每一**行**，都理解为一个向量，乘法可以看作三个**行**向量的加权和

$$\overrightarrow{a_0}=\begin{pmatrix} a_{00} \\ a_{01} \\ a_{02} \end{pmatrix} \quad \overrightarrow{a_1}=\begin{pmatrix} a_{10} \\ a_{11} \\ a_{12} \end{pmatrix} \quad \overrightarrow{a_2}=\begin{pmatrix} a_{20} \\ a_{21} \\ a_{22} \end{pmatrix}$$

$$result = x.\overrightarrow{a_0}^T + y.\overrightarrow{a_1}^T + z.\overrightarrow{a_2}^T$$

重新理解矩阵乘法（行视图）

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} \boxed{b_{00} \quad b_{01} \quad b_{02}} \\ \boxed{b_{10} \quad b_{11} \quad b_{12}} \\ \boxed{b_{20} \quad b_{21} \quad b_{22}} \end{pmatrix} = \begin{pmatrix} \boxed{c_{00} \quad c_{01} \quad c_{02}} \\ \boxed{c_{10} \quad c_{11} \quad c_{12}} \\ \boxed{c_{20} \quad c_{21} \quad c_{22}} \end{pmatrix}$$

- B矩阵看作**行向量**的组合，结果C矩阵也看作**行向量**组合

$$\vec{b_0} = \begin{pmatrix} b_{00} \\ b_{01} \\ b_{02} \end{pmatrix} \quad \vec{b_1} = \begin{pmatrix} b_{10} \\ b_{11} \\ b_{12} \end{pmatrix} \quad \vec{b_2} = \begin{pmatrix} b_{20} \\ b_{21} \\ b_{22} \end{pmatrix} \qquad \vec{c_0} = \begin{pmatrix} c_{00} \\ c_{01} \\ c_{02} \end{pmatrix} \quad \vec{c_1} = \begin{pmatrix} c_{10} \\ c_{11} \\ c_{12} \end{pmatrix} \quad \vec{c_2} = \begin{pmatrix} c_{20} \\ c_{21} \\ c_{22} \end{pmatrix}$$

乘法可以表示为

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} \vec{b_0}^T \\ \vec{b_1}^T \\ \vec{b_2}^T \end{pmatrix} = \begin{pmatrix} \vec{c_0}^T \\ \vec{c_1}^T \\ \vec{c_2}^T \end{pmatrix}$$

- 按照矩阵乘法，我们可以得到如下等式

$$\vec{c_0}^T = a_{00}.\vec{b_0}^T + a_{01}.\vec{b_1}^T + a_{02}.\vec{b_2}^T$$

$$\vec{c_1}^T = a_{10}.\vec{b_0}^T + a_{11}.\vec{b_1}^T + a_{12}.\vec{b_2}^T$$

$$\vec{c_2}^T = a_{20}.\vec{b_0}^T + a_{21}.\vec{b_1}^T + a_{22}.\vec{b_2}^T$$

位于左边的矩阵A，每一行都可以理解为对B矩阵行向量的**调配比例（或者说权重）**

矩阵C的每一个行向量，都是B矩阵根据A提供的权重**加权相加**的结果