

——从0开始实现OpenGL

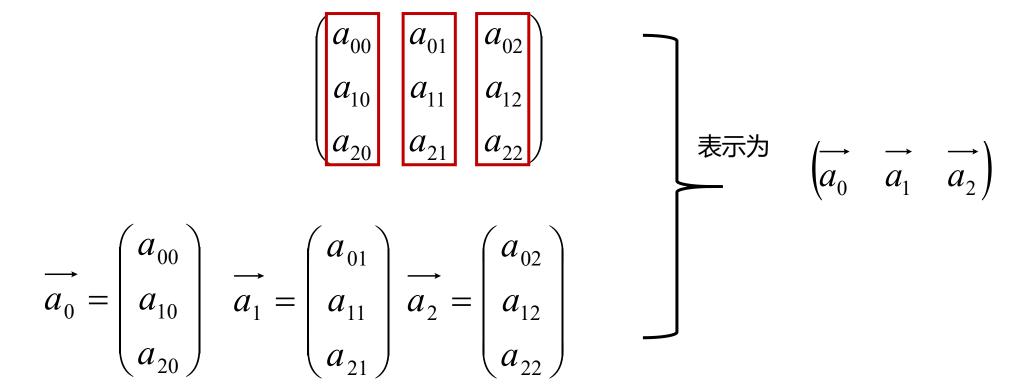
线性代数—矩阵行列视图



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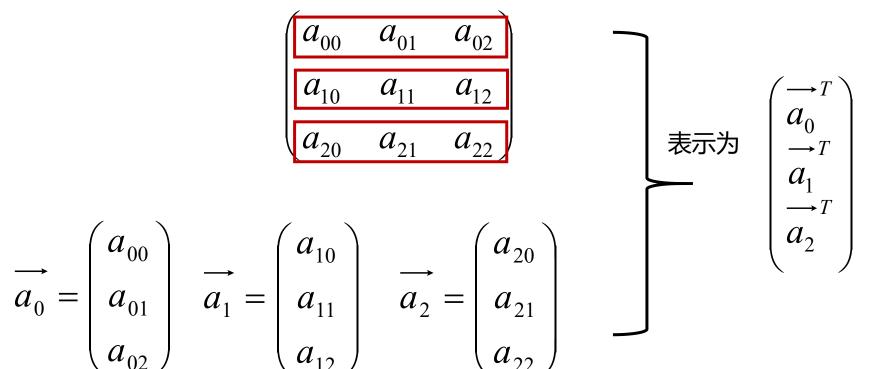
### 矩阵的列视图

• 矩阵可以看做由多个**列向量**排列构成,如下所示:



### 矩阵的行视图

• 矩阵可以看做由多个**行向量**排列构成,如下所示:



#### 己知矩阵乘法

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{00} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{pmatrix}$$

#### 举例:

$$\mathbf{c}_{21} = \begin{pmatrix} a_{20} \\ a_{21} \\ a_{22} \end{pmatrix} \cdot \begin{pmatrix} b_{01} \\ b_{11} \\ b_{21} \end{pmatrix} = a_{20} \cdot b_{01} + a_{21} \cdot b_{11} + a_{22} \cdot b_{21}$$

#### 矩阵与向量乘法理解

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{00} & x + a_{01} & y + a_{02} & z \\ a_{10} & x + a_{11} & y + a_{12} & z \\ a_{20} & x + a_{21} & y + a_{22} & z \end{pmatrix}$$

• 矩阵每一列,都理解为一个向量,乘法可以看作三个向量的加权和

$$\vec{a}_{0} = \begin{pmatrix} a_{00} \\ a_{10} \\ a_{20} \end{pmatrix} \vec{a}_{1} = \begin{pmatrix} a_{01} \\ a_{11} \\ a_{21} \end{pmatrix} \vec{a}_{2} = \begin{pmatrix} a_{02} \\ a_{12} \\ a_{22} \end{pmatrix}$$

$$result = x.a_0 + y.a_1 + z.a_2$$

#### 思考题

考虑成加权和,那么**xyz**表示的权重是什么?每个列的向量含义是什么?

### 重新理解矩阵乘法 (列视图)

• A矩阵看作**列向量**的组合,结果C矩阵也看作**列向量**组合

$$\overrightarrow{a_0} = \begin{pmatrix} a_{00} \\ a_{10} \\ a_{20} \end{pmatrix} \quad \overrightarrow{a_1} = \begin{pmatrix} a_{01} \\ a_{11} \\ a_{21} \end{pmatrix} \quad \overrightarrow{a_2} = \begin{pmatrix} a_{02} \\ a_{12} \\ a_{22} \end{pmatrix} \qquad \overrightarrow{c_0} = \begin{pmatrix} c_{00} \\ c_{10} \\ c_{20} \end{pmatrix} \quad \overrightarrow{c_1} = \begin{pmatrix} c_{01} \\ c_{11} \\ c_{21} \end{pmatrix} \quad \overrightarrow{c_2} = \begin{pmatrix} c_{02} \\ c_{12} \\ c_{22} \end{pmatrix}$$

#### 乘法可以表示为

$$(\overrightarrow{a_0} \quad \overrightarrow{a_1} \quad \overrightarrow{a_2}) \times \begin{pmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{pmatrix} = (\overrightarrow{c_0} \quad \overrightarrow{c_1} \quad \overrightarrow{c_2})$$

• 按照矩阵乘法, 我们可以得到如下等式

$$\overrightarrow{c_0} = b_{00}.\overrightarrow{a_0} + b_{10}.\overrightarrow{a_1} + b_{20}.\overrightarrow{a_2}$$

$$\overrightarrow{c_1} = b_{01}.\overrightarrow{a_0} + b_{11}.\overrightarrow{a_1} + b_{21}.\overrightarrow{a_2}$$

$$\overrightarrow{c_2} = b_{02}.\overrightarrow{a_0} + b_{12}.\overrightarrow{a_1} + b_{22}.\overrightarrow{a_2}$$

位于右边的矩阵B,每一列都可以理解为对A矩阵列向量的**调配比** 例 (或者说权重)

矩阵C的每一个列向量,都是A矩阵根据B提供的权重**加权相加**的结果

### 矩阵与向量(行) 乘法理解

$$(x \quad y \quad z) \times \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} = (a_{00}.x + a_{10}.y + a_{20}.z \quad a_{01}.x + a_{11}.y + a_{21}.z \quad a_{02}.x + a_{12}.y + a_{22}.z)$$

• 矩阵每一行,都理解为一个向量,乘法可以看作三个行向量的加权和

$$\overrightarrow{a_0} = \begin{pmatrix} a_{00} \\ a_{01} \\ a_{02} \end{pmatrix} \overrightarrow{a_1} = \begin{pmatrix} a_{10} \\ a_{11} \\ a_{12} \end{pmatrix} \quad \overrightarrow{a_2} = \begin{pmatrix} a_{20} \\ a_{21} \\ a_{22} \end{pmatrix}$$

$$result = \overrightarrow{x.a_0} + \overrightarrow{y.a_1} + \overrightarrow{z.a_2}$$

### 重新理解矩阵乘法 (行视图)

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{00} & c_{01} & c_{02} \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{pmatrix}$$

• B矩阵看作**行向量**的组合,结果C矩阵也看作**行向量**组合

$$\overrightarrow{b_0} = \begin{pmatrix} b_{00} \\ b_{01} \\ b_{02} \end{pmatrix} \overrightarrow{b_1} = \begin{pmatrix} b_{10} \\ b_{11} \\ b_{12} \end{pmatrix} \overrightarrow{b_2} = \begin{pmatrix} b_{20} \\ b_{21} \\ b_{22} \end{pmatrix} \qquad \overrightarrow{c_0} = \begin{pmatrix} c_{00} \\ c_{01} \\ c_{02} \end{pmatrix} \overrightarrow{c_1} = \begin{pmatrix} c_{10} \\ c_{11} \\ c_{12} \end{pmatrix} \overrightarrow{c_2} = \begin{pmatrix} c_{20} \\ c_{21} \\ c_{22} \end{pmatrix}$$

### 乘法可以表示为

$$\begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} \overrightarrow{b_0} \\ \overrightarrow{b_1} \\ \overrightarrow{b_1} \\ \overrightarrow{b_2} \end{pmatrix} = \begin{pmatrix} \overrightarrow{c_0} \\ \overrightarrow{c_1} \\ \overrightarrow{c_1} \\ \overrightarrow{c_2} \end{pmatrix}$$

• 按照矩阵乘法, 我们可以得到如下等式

$$\vec{c}_{0}^{T} = a_{00}.\vec{b}_{0}^{T} + a_{01}.\vec{b}_{1}^{T} + a_{02}.\vec{b}_{2}^{T} 
\vec{c}_{1}^{T} = a_{10}.\vec{b}_{0}^{T} + a_{11}.\vec{b}_{1}^{T} + a_{12}.\vec{b}_{2}^{T} 
\vec{c}_{2}^{T} = a_{20}.\vec{b}_{0}^{T} + a_{21}.\vec{b}_{1}^{T} + a_{22}.\vec{b}_{2}^{T}$$

位于左边的矩阵A,每一行都可以理解为对B矩阵行向量的**调配比例(或者说权重)** 

矩阵C的每一个行向量,都是B矩阵根据A提供的权重**加权相加**的结果