

****Ds to eta****

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1 Preliminary Investigation

1.1 Types of Mesons

The meson can be categorized into five types based on its spin and parity. As shown in the table below:

Table 1: Types of Mesons

Type of Meson	S	L	P	J	J ^P
Pseudoscalar Meson	0	0	-	0	0-
Pseudovector Meson	0, 1	1	+	1	1+
Vector Meson	1	0, 2	-	1	1-
Scalar Meson	1	1	+	0	0+
Tensor Meson	1	1, 3	+	2	2+

The quark composition of the D_s^+ meson is $c\bar{s}$ while the D_s^- meson has a quark composition of $\bar{c}s$. The η and η' mesons are isosinglet mesons made of a mixture of up, down, and strange quarks and their antiquarks. The basic SU(3) symmetry theory of quarks for the three lightest quarks, which only takes into account the strong force, predicts corresponding particles

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \quad (1)$$

and

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad (2)$$

However, the electroweak interaction - which can transform one flavor of quark into another - causes a small but significant amount of "mixing" of the eigenstates (with mixing angle

θ_P), so that the actual quark composition is a linear combination of these formulae. That is:

$$\begin{pmatrix} \cos \theta_P & -\sin \theta_P \\ \sin \theta_P & \cos \theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \quad (3)$$

The unsubscripted name η refers to the real particle that is actually observed and close to the η_8 . The η' is the observed particle close to η_1 . Both D_s^+ and $\eta^{(\prime)}$ are pseudoscalar mesons.

1.2 Decay process

The process from D_s^+ to $\eta^{(\prime)}\ell\nu$ involves both strong and weak interactions. The Feynman diagram is shown in the figure below.

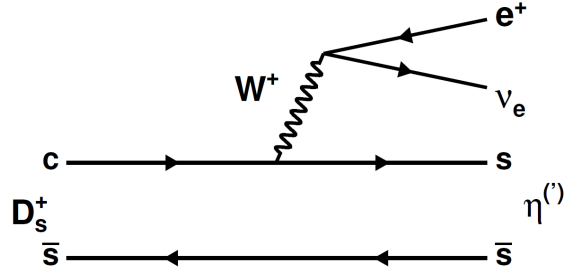


Figure 1: The dominant Feynman diagram.

2 LCSR for $D_s^+ \rightarrow \eta^{(\prime)}$

2.1 Correlation Function

LCSR starts with the correlation function Eq.4.

$$\Pi_\mu(p, q) = i \int d^4x e^{iqx} \langle \eta^{(\prime)}(p) | T \{ j_{\mu,1}(x), j_2(0) \} | 0 \rangle \quad (4)$$

In reference [1], the currents entering the correlation function are defined as shown in Table 2.

Table 2: Currents entering the correlation function[1].

Decay	Interpolation current	Weak current
$D_s^+ \rightarrow \eta^{(\prime)} l^+ \nu_l$	$j_2 = j_{D_s^+}^\dagger = m_c \bar{c} i \gamma_5 s$	$j_{\mu,1} = V_\mu^{(\eta,\eta')} = \bar{s} \gamma_\mu c$ $\tilde{j}_{\mu,1} = \tilde{V}_\mu^{(\eta,\eta')} = \bar{s} \sigma_{\mu\nu} q^\nu c$

The Feynman diagram corresponding to the formula is shown in fig.2.

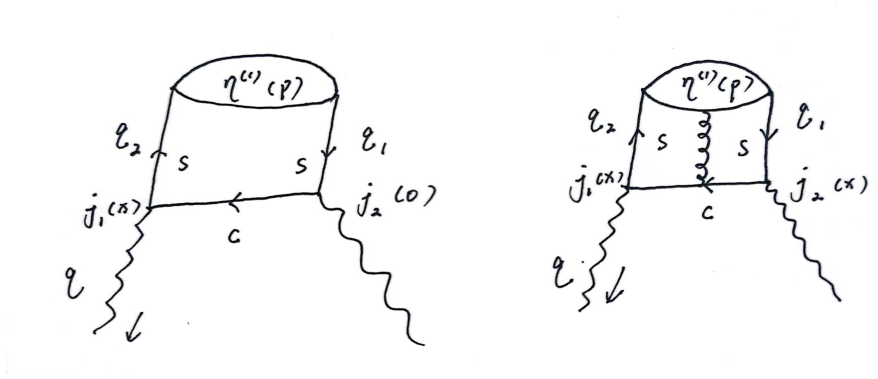


Figure 2: Diagrams corresponding to the leading-order terms in the hard-scattering amplitudes involving the two-particle (left) and three-particle (right)

It has been shown by Källén and Lehmann [2, 3] quite a long time ago that the two-point correlation functions obey dispersion relation:

$$\Pi_\mu(p, q) = \int_0^\infty dt \rho_\mu(t) \frac{1}{t - (p + q)^2 - i\epsilon} \quad (5)$$

For $(p + q)^2 > 0$, we have

$$\begin{aligned} \Pi_\mu(p, q) &= \mathcal{P} \int_0^\infty dt \rho_\mu(t) \frac{1}{t - (p + q)^2} + i\pi \rho_\mu((p + q)^2) \\ &= \text{Re}\Pi_\mu(p, q) + \text{Im}\Pi_\mu(p, q) \end{aligned} \quad (6)$$

Clearly, $\text{Im}\Pi_\mu(p, q)$ has a corresponding relationship with $\pi\rho_\mu((p+q)^2)$, the spectral function density $\rho_\mu((p+q)^2)$ is a scalar function of the Lorentz invariant $(p+q)^2$.

$$\rho_\mu(t) = \frac{1}{\pi} \text{Im}\Pi_\mu(t) \equiv \sum_{\Gamma} \langle \eta^{(\prime)} | j_{\mu,1} | \Gamma \rangle \langle \Gamma | j_2 | 0 \rangle (2\pi)^4 \delta^{(4)}(q - p_\Gamma) \quad (7)$$

Given the definition of $\text{Im}\Pi_\mu(t)$, naturally, we have

$$\Pi_\mu(p, q) = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im}\Pi_\mu(t)}{t - (p+q)^2 - i\epsilon} \quad (8)$$

2.2 Hadron Representation

After inserting a complete set of hadronic states, specifically $|D_s^+(p+q)\rangle$, we have $(p+q)^2 \geq m_{D_s}$ in the physical region.

$$\begin{aligned} \Pi_\mu(p, q)^{hadron} &= i \int d^4x e^{iqx} \langle \eta^{(\prime)}(p) | T \{ j_1(x), j_2(0) \} | 0 \rangle + \frac{1}{\pi} \int_{s_0}^\infty dt \frac{\text{Im}\Pi_\mu^{hadron}(t)}{t - (p+q)^2 - i\epsilon} \\ &= \frac{-im_{D_s}^2 f_{D_s} [2f_{D_{(s)}\eta^{(\prime)}}^+(q^2) p_\mu + (f_{D_{(s)}\eta^{(\prime)}}^+(q^2) + f_{D_{(s)}\eta^{(\prime)}}^-(q^2)) q_\mu]}{(m_c + m_s)[m_{D_s^+}^2 - (p+q)^2]} \\ &\quad + \frac{1}{\pi} \int_{s_0}^\infty dt \frac{\text{Im}\Pi_+^{hadron}(t, q^2) p_\mu + \text{Im}\Pi_-^{hadron}(t, q^2) q_\mu}{t - (p+q)^2 - i\epsilon} \end{aligned} \quad (9)$$

where $f_{D_{(s)}\eta^{(\prime)}}^+$ and $f_{D_{(s)}\eta^{(\prime)}}^-$ are the form factors defined as Eq.10:

$$\langle \eta^{(\prime)} | j_{\mu,1} | D_s^+(p+q) \rangle = 2f_{D_{(s)}\eta^{(\prime)}}^+(q^2) p_\mu + (f_{D_{(s)}\eta^{(\prime)}}^+(q^2) + f_{D_{(s)}\eta^{(\prime)}}^-(q^2)) q_\mu, \quad (10)$$

2.3 Quark Representation

Computed in the perturbative theory with the help of OPE technique at the deep Euclidean region $(p^2, q^2 = -Q^2 \ll 0)$, we have:

$$\begin{aligned}
\Pi_\mu(p, q)^{OPE} &= i \int d^4x e^{iqx} \langle \eta^{(\prime)}(p) | T \{ j_{\mu,1}(x), j_2(0) \} | 0 \rangle \\
&= i \int d^4x e^{iqx} \langle \eta^{(\prime)}(p) | T \{ \bar{s}(x) \gamma_\mu c(x), \bar{c}(0) i \gamma_5 s(0) \} | 0 \rangle
\end{aligned} \tag{11}$$

References

- [1] G. Duplancić and B. Melić. “J. High Energ. Phys. 2015, 138”. In: *Journal of High Energy Physics* 2015 (2015), p. 138. DOI: [https://doi.org/10.1007/JHEP11\(2015\)138](https://doi.org/10.1007/JHEP11(2015)138).
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