Ds to eta

** Xiao Guangyan **

Contents

1 Preliminary Investigation **2**

1.1 Types of Mesons 2

1.2 Decay process 3

2 LCSR for $D_s^+ \to \eta^{(\prime)}$ **3**

2.1 Correlation Function 3

2.2 Hadron Representation 5

2.3 Quark Representation 6

1 Preliminary Investigation

1.1 Types of Mesons

The meson can be categorized into five types based on its spin and parity. As shown in the table below:

Table 1: Types of Mesons

Type of Meson	**S**	**L**	**P**	**J**	$\mathbf{J^P}$
Pseudoscalar Meson	0	0	-	0	0-
Pseudovector Meson	0, 1	1	+	1	1+
Vector Meson	1	0, 2	-	1	1-
Scalar Meson	1	1	+	0	0+
Tensor Meson	1	1, 3	+	2	2+

The quark composition of the D_s^+ meson is $c\bar{s}$ while the D_s^- meson has a quark composition of $\bar{c}s$. The η and η' mesons are isosinglet mesons made of a mixture of up, down, and strange quarks and their antiquarks. The basic SU(3) symmetry theory of quarks for the three lightest quarks, which only takes into account the strong force, predicts corresponding particles

$$\eta_1 = \frac{1}{\sqrt{3}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}}) \tag{1}$$

and

$$\eta_8 = \frac{1}{\sqrt{6}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} - 2\ \mathrm{s\bar{s}}) \tag{2}$$

However, the electroweak interaction - which can transform one flavor of quark into another - causes a small but significant amount of ”mixing” of the eigenstates (with mixing angle

θ_{P}), so that the actual quark composition is a linear combination of these formulae. That is:

$$\begin{pmatrix} \cos\theta_{\mathrm{P}} & -\sin\theta_{\mathrm{P}} \\ \sin\theta_{\mathrm{P}} & \cos\theta_{\mathrm{P}} \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \tag{3}$$

The unsubscripted name η refers to the real particle that is actually observed and close to the η_8. The η' is the observed particle close to η_1. Both D_s^+ and $\eta^{(\prime)}$ are pseudoscalar mesons.

1.2 Decay process

The process from D_s^+ to $\eta^{(\prime)}\ell\nu$ involves both strong and weak interactions. The Feynman diagram is shown in the figure below.

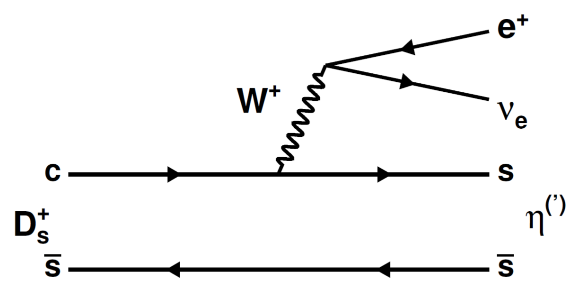

Figure 1: The dominant Feynman diagram.

2 LCSR for $D_s^+ \to \eta^{(\prime)}$

2.1 Correlation Function

LCSR starts with the correlation function Eq.4.

$$\Pi_\mu(p,q) = i\int d^4x e^{iqx} \left\langle \eta^{(\prime)}(p) \left| \mathrm{T}\left\{ j_{\mu,1}(x), j_2(0) \right\} \right| 0 \right\rangle \tag{4}$$

In reference [1], the currents entering the correlation function are defined as shown in Table 2.

Table 2: Currents entering the correlation function[1].

Decay	**Interpolation current**	**Weak current**
$D_s^+ \to \eta^{(\prime)} l^+ \nu_l$	$j_2 = j_{D_s^+}^\dagger = m_c \bar{c} i \gamma_5 s$	$j_{\mu,1} = V_\mu^{(\eta,\eta')} = \bar{s}\gamma_\mu c$ $\tilde{j}_{\mu,1} = \tilde{V}_\mu^{(\eta,\eta')} = \bar{s}\sigma_{\mu\nu} q^\nu c$

The Feynman diagram corresponding to the formula is shown in fig.2.

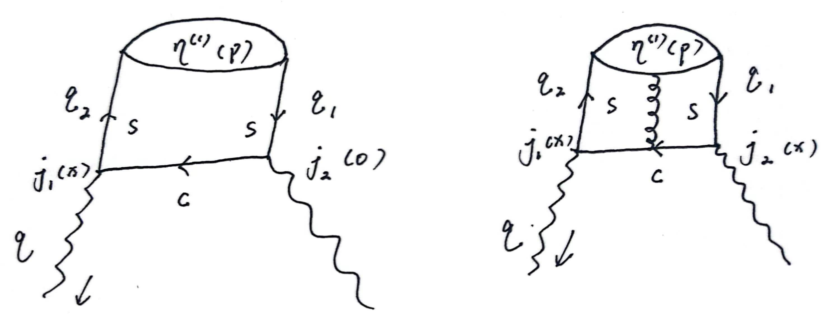

Figure 2: Diagrams corresponding to the leading-order terms in the hard-scattering amplitudes involving the two-particle (left) and three-particle (right)

It has been showen by Källén and Lehmann [2, 3] quite a long time ago that the two-point correlation functions obey dispersion relation:

$$\Pi_\mu(p,q) = \int_0^\infty dt\, \rho_\mu(t) \frac{1}{t-(p+q)^2 - i\epsilon} \tag{5}$$

For $(p+q)^2 > 0$, we have

$$\begin{aligned} \Pi_\mu(p,q) \quad &= \mathcal{P} \int_0^\infty dt\, \rho_\mu(t) \tfrac{1}{t-(p+q)^2} + i\pi\rho_\mu((p+q)^2) \\ &= \mathrm{Re}\Pi_\mu(p,q) + \mathrm{Im}\Pi_\mu(p,q) \end{aligned} \tag{6}$$

Clearly, $\mathrm{Im}\Pi_\mu(p,q)$ has a corresponding relationship with $\pi\rho_\mu((p+q)^2)$, the spectral function density $\rho_\mu((p+q)^2)$ is a scalar function of the Lorentz invariant $(p+q)^2$.

$$\rho_\mu(t) = \frac{1}{\pi}\mathrm{Im}\Pi_\mu(t) \equiv \sum_\Gamma \langle \eta^{(\prime)}|j_{\mu,1}|\Gamma\rangle\langle\Gamma|j_2|0\rangle(2\pi)^4\delta^{(4)}\left(q-p_\Gamma\right) \tag{7}$$

Given the definition of $\mathrm{Im}\Pi_\mu(t)$, naturally, we have

$$\Pi_\mu(p,q) = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\Pi_\mu(t)}{t-(p+q)^2-i\epsilon} \tag{8}$$

2.2 Hadron Representation

After inserting a complete set of hadronic states, specifically $|D_s^+(p+q)\rangle$, we have $(p+q)^2 \geq m_{Ds}$ in the physical region.

$$\begin{aligned}
\Pi_\mu(p,q)^{hadron} &= i\int d^4x e^{iqx}\left\langle \eta^{(\prime)}(p)\left|\mathrm{T}\left\{j_1(x),j_2(0)\right\}\right|0\right\rangle + \frac{1}{\pi}\int_{s_0}^\infty dt\,\frac{\mathrm{Im}\Pi_\mu^{hadron}(t)}{t-(p+q)^2-i\epsilon} \\
&= \frac{-im_{D_s}^2 f_{D_s}[2f^+_{D_{(s)}\eta^{(\prime)}}(q^2)\,p_\mu + \left(f^+_{D_{(s)}\eta^{(\prime)}}(q^2) + f^-_{D_{(s)}\eta^{(\prime)}}(q^2)\right)q_\mu]}{(m_c+m_s)[m_{D_s^+}^2-(p+q)^2]} \\
&\quad + \frac{1}{\pi}\int_{s_0}^\infty dt\,\frac{\mathrm{Im}\Pi_+^{hadron}(t,q^2)p_\mu + \mathrm{Im}\Pi_-^{hadron}(t,q^2)q_\mu}{t-(p+q)^2-i\epsilon}
\end{aligned} \tag{9}$$

where $f^+_{D_{(s)}\eta^{(\prime)}}$ and $f^-_{D_{(s)}\eta^{(\prime)}}$ are the form factors defined as Eq.10:

$$\left\langle \eta^{(\prime)}|j_{\mu,1}|D_s^+(p+q)\right\rangle = 2f^+_{D_{(s)}\eta^{(\prime)}}\left(q^2\right)p_\mu + \left(f^+_{D_{(s)}\eta^{(\prime)}}\left(q^2\right) + f^-_{D_{(s)}\eta^{(\prime)}}\left(q^2\right)\right)q_\mu, \tag{10}$$

2.3 Quark Representation

Computed in the perturbative theory with the help of OPE technique at the deep Euclidean region $(p^2, q^2 = -Q^2 \ll 0)$, we have:

$$\begin{aligned}
\Pi_\mu(p,q)^{OPE} &= i\int d^4x e^{iqx} \left\langle \eta^{(\prime)}(p) \left| \mathrm{T}\left\{ j_{\mu,1}(x), j_2(0) \right\} \right| 0 \right\rangle \\
&= i\int d^4x e^{iqx} \left\langle \eta^{(\prime)}(p) \left| \mathrm{T}\left\{ \bar{s}(x)\gamma_\mu c(x), \bar{c}(0) i\gamma_5 s(0) \right\} \right| 0 \right\rangle \\
&= i\int d^4x e^{iqx} \left\langle \eta^{(\prime)}(p) \left| \mathrm{T}\left\{ \bar{s}_\alpha(x) \left[\gamma_\mu\right]_{\alpha\alpha'} c_{\alpha'}(x), \bar{c}_\beta(0) \left[i\gamma_5\right]_{\beta\beta'} s(0)_{\beta'} \right\} \right| 0 \right\rangle \\
&= i\int d^4x e^{iqx} \left\langle \eta^{(\prime)}(p) \left| \mathrm{T}\left\{ \bar{s}_\alpha(x) s(0)_{\beta'} \right\} \right| 0 \right\rangle \left\langle 0 \left| \mathrm{T}\left\{ c_{\alpha'}(x)\bar{c}_\beta(0) \right\} \right| 0 \right\rangle \left[\gamma_\mu\right]_{\alpha\alpha'} \left[i\gamma_5\right]_{\beta\beta'}
\end{aligned} \tag{11}$$

Where $\left\langle 0 \left| \mathrm{T}\left\{ c_{\alpha'}(x)\bar{c}_\beta(0) \right\} \right| 0 \right\rangle \equiv S_c^{\alpha'\beta}(x, 0, m_c)$ is the full propagator of c quark. The light-cone expansion of the quark propagator in the external gluon field is made in ref [4]. The propagator receives contributions from higher Fock states proportional to the condensates of the operators $\bar{q}Gq$, $\bar{q}GGq$ and $\bar{q}q\bar{q}q$. We neglect contributions with two gluons as well as four quark operators due to the fact that their contributions are small [5]. In this approximation the $S_c(x, 0, m_c)$ is given as:

$$
\begin{aligned}
S_c\left(x,0,m_c\right) &= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\frac{\not{k}+m_c}{k^2-m_c^2} \\
&\quad - \mathrm{i}g_s \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}kx}\int_0^1 \mathrm{d}u\left[\frac{1}{2}\frac{\not{k}+m_c}{(m_c^2-k^2)^2}G_{\mu\nu}(ux)\sigma^{\mu\nu}\right. \\
&\quad \left. + \frac{1}{m_c^2-k^2}ux_\mu G^{\mu\nu}(ux)\gamma_\nu + \cdots\right] \\
&\qquad\qquad \cdots(\text{Fourier transform})\cdots \\
&= \frac{-im_c^2}{4\pi^2}\left[\frac{K_1\left(m\sqrt{|x^2|}\right)}{\sqrt{|x^2|}} + i\frac{\not{x}}{|x^2|}K_2\left(m_c\sqrt{|x^2|}\right)\right] \\
&\quad - \frac{ig_s}{16\pi^2}\int_0^1 du\left[m_cK_0\left(m_c\sqrt{|x^2|}\right)(G\cdot\sigma)\right. \\
&\quad \left. + \frac{im_c}{\sqrt{|x^2|}}K_1\left(m_c\sqrt{|x^2|}\right)\left(\not{x}(G\cdot\sigma) - 4iux_\mu G^{\mu\nu}r_\nu\right) + \cdots\right]
\end{aligned}
\tag{12}
$$

where $G\mu\nu$ is the gluonic field strength tensor and g_s is the strong coupling constant, $K_0\left(m_c\sqrt{|x^2|}\right)$, $K_1\left(m_c\sqrt{|x^2|}\right)$ and $K_2\left(m_c\sqrt{|x^2|}\right)$ are the Bessel functions. Simplify Eq.11:

$$
\begin{aligned}
\Pi_\mu(p,q)^{OPE} &= i\int d^4xe^{iqx}\left\langle\eta^{(\prime)}(p)\left|\mathrm{T}\left\{j_{\mu,1}(x),j_2(0)\right\}\right|0\right\rangle \\
&= i\int d^4xe^{iqx}\left\langle\eta^{(\prime)}(p)\left|\mathrm{T}\left\{\bar{s}_\alpha(x)s(0)_{\beta'}\right\}\right|0\right\rangle\left[\gamma_\mu\right]_{\alpha\alpha'}\left[i\gamma_5\right]_{\beta\beta'} \\
&\quad \times\left\{\frac{-im_c^2}{4\pi^2}\left[\frac{K_1\left(m\sqrt{|x^2|}\right)}{\sqrt{|x^2|}} + i\frac{\not{x}}{|x^2|}K_2\left(m_c\sqrt{|x^2|}\right)\right]\right. \\
&\quad \left. - \frac{ig_s}{16\pi^2}\int_0^1 du\left[m_cK_0\left(m_c\sqrt{|x^2|}\right)(G\cdot\sigma) + \frac{im_c}{\sqrt{|x^2|}}K_1\left(m_c\sqrt{|x^2|}\right)\not{x}(G\cdot\sigma) + \cdots\right]\right\}_{\alpha'\beta}
\end{aligned}
\tag{13}
$$

from the above,

$$
\begin{aligned}
\Pi_\mu(p,q)^{OPE} =& \frac{1}{4\pi^2}\int d^4x e^{iqx}\left\langle \eta^{(\prime)}(p)\left|\mathrm{T}\left\{\bar{s}_\alpha(x)s(0)_{\beta'}\right\}\right|0\right\rangle [\gamma_\mu]_{\alpha\alpha'}[i\gamma_5]_{\beta\beta'} \\
& \times\left[\frac{m_c^2K_1\left(m\sqrt{|x^2|}\right)}{\sqrt{|x^2|}}+i\frac{\not{x}m_c^2K_2\left(m_c\sqrt{|x^2|}\right)}{|x^2|}\right]_{\alpha'\beta} \\
&+\frac{1}{4\pi^2}\int d^4x e^{iqx}\int_0^1 du\left\langle \eta^{(\prime)}(p)\left|\mathrm{T}\left\{\bar{s}_\alpha(x)g_sG_{\lambda\tau}s(0)_{\beta'}\right\}\right|0\right\rangle [\gamma_\mu]_{\alpha\alpha'}[i\gamma_5]_{\beta\beta'} \\
& \times\sigma^{\lambda\tau}\left[\frac{m_cK_0\left(m_c\sqrt{|x^2|}\right)}{4}+\frac{i\not{x}m_cK_1\left(m_c\sqrt{|x^2|}\right)}{4\sqrt{|x^2|}}+\cdots\right]_{\alpha'\beta}.
\end{aligned} \tag{14}
$$

In the Eq.14, the matrix element inside the Dirac brackets is provided by the LCDA[6]:

$$
\begin{aligned}
&\left\langle \eta(p)\left|\bar{q}^i_\omega(x_1)\,q^j_\xi(x_2)\right|0\right\rangle_{x^2\to 0} \\
&=\frac{i\delta^{ij}}{12}f_\eta\int_0^1 du e^{iup\cdot x_1+i\bar{u}p\cdot x_2}\Big([\not{p}\gamma_5]_{\xi\omega}\varphi_\eta(u)-[\gamma_5]_{\xi\omega}\mu_\eta\phi^p_{3\eta}(u) \\
&+\frac{1}{6}[\sigma_{\beta\tau}\gamma_5]_{\xi\omega}p_\beta(x_1-x_2)_\tau\mu_\eta\phi^\sigma_{3\eta}(u)+\frac{1}{16}[\not{p}\gamma_5]_{\xi\omega}(x_1-x_2)^2\phi_{4\eta}(u) \\
&-\frac{i}{2}[(\not{x}_1-\not{x}_2)\gamma_5]_{\xi\omega}\int_0^u\psi_{4\eta}(v)dv\Big),
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&\left\langle \eta(p)\left|\bar{q}^i_\omega(x_1)\,g_sG^a_{\mu\nu}(x_3)\,q^j_\xi(x_2)\right|0\right\rangle_{x^2\to 0} \\
&=\frac{\lambda^a_{ji}}{32}\int\mathcal{D}\alpha_i e^{ip(\alpha_1x_1+\alpha_2x_2+\alpha_3x_3)}\Big[if_{3\eta}(\sigma_{\lambda\rho}\gamma_5)_{\xi\omega}(p_\mu p_\lambda g_{\nu\rho}-p_\nu p_\lambda g_{\mu\rho})\Phi_{3\eta}(\alpha_i) \\
&-f_\eta(\gamma_\lambda\gamma_5)_{\xi\omega}\left\{(p_\nu g_{\mu\lambda}-p_\mu g_{\nu\lambda})\Psi_{4\eta}(\alpha_i)+\frac{p_\lambda(p_\mu x_\nu-p_\nu x_\mu)}{(p\cdot x)}(\Phi_{4\eta}(\alpha_i)+\Psi_{4\eta}(\alpha_i))\right\} \\
&-\frac{if_\eta}{2}\epsilon_{\mu\nu\delta\rho}(\gamma_\lambda)_{\xi\omega}\left\{\left(p^\rho g^{\delta\lambda}-p^\delta g^{\rho\lambda}\right)\tilde{\Psi}_{4\eta}(\alpha_i)+\frac{p_\lambda\left(p^\delta x^\rho-p^\rho x^\delta\right)}{(p\cdot x)}\left(\tilde{\Phi}_{4\eta}(\alpha_i)+\tilde{\Psi}_{4\eta}(\alpha_i)\right)\right\}\Big].
\end{aligned} \tag{16}
$$

References

[1] G. Duplančić and B. Melić. “J. High Energ. Phys. 2015, 138”. In: *Journal of High Energy Physics* 2015 (2015), p. 138. DOI: `https://doi.org/10.1007/JHEP11(2015)138`.

[2] Gunnar Källén. “On the Definition of the Renormalization Constants in Quantum Electrodynamics”. In: *Helvetica Physica Acta* 25 (1952), p. 417. DOI: `https://doi.org/10.5169/seals-112316`.

[3] Harry Lehmann. “Über Eigenschaften von Ausbreitungsfunktionen und Renormierungskonstanten quantisierter Felder”. In: *Nuovo Cimento* 11.4 (1954), pp. 342–357. ISSN: 0029-6341. DOI: `https://doi.org/10.1007/bf02783624`.

[4] I. I. Balitsky and V. M. Braun. “Nucl. Phys. B 311, 541”. In: *Nuclear Physics B* 311 (1989), p. 541.

[5] V. M. Braun and I. E. Filyanov. “Z. Phys. C 48, 239”. In: *Zeitschrift für Physik C* 48 (1990), p. 239.

[6] Patricia Ball, Vladimir M. Braun, and Alexander Lenz. “Higher-twist distribution amplitudes of the K meson in QCD”. In: *Journal of High Energy Physics* 2006 (2006), JHEP05(2006)004. DOI: `10.1088/1126-6708/2006/05/004`.