Manual calculation of the intrinsic matrix of a projector

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1 Introduction

This document will describe the process of calculating the intrinsic matrix of a projector by taking manual measurements of its projected image. This process assumes that the projector can be modelled by a pinhole camera model. Since most consumer projectors are typically not subject to significant spherical or tangential distortion in the projected image, distortion coefficients are not calculated.

In order to calculate the intrinsic matrix of a projector, you will need to construct a jig to mount the projector at a fixed location, mark the corners of the projected image, take several measurements, calculate the focal length and principal point, and place these in the matrix.

2 Measurement jig

Figure 1: Our measurement jig was constructed from extruded aluminum sections and mounted the projector 1 m from a piece of posterboard.

The measurements needed should be taken with the projector mounted a fixed distance from a flat surface, such that the optical axis of the projector is perpendicular to the flat surface. It is important that this jig is rigid and dimensionally accurate to mitigate measurement error. Our jig was constructed from extruded aluminum sections and is shown in figure 1.

3 Measurement and calculation

First, mark the corners of the image as well as the point at which the optical axis intersects the image plane. Our jig was constructed such that the projector will always be placed along the same optical axis, and thus this intersection happens at a fixed point which can be measured from some reference geometry.

To calculate the focal length f and principle point (c_x, c_y), you will need to measure:

- W and H, the width and height of the projected image
- Z, the distance along the optical axis from the focal point to the intersection of the optical axis and the image plane (the distance from the projector to the flat surface)
- w and h, the width and height of the image in pixels
- (C_x, C_y), the position on the image plane of the intersection of the optical axis and the image plane w.r.t. the origin

From this, you can calculate the focal length f in both the x and y directions. For the lenses found in most consumer projectors, these should work out to be the same.

$$f_x = w\frac{Z}{W} \quad (1) \qquad f_y = h\frac{Z}{H} \quad (2)$$

The principal point is calculated by

$$c_x = w\frac{C_x}{W} \quad (3) \qquad c_y = h\frac{C_y}{H} \quad (4)$$

Finally, these values are placed into an intrinsic camera matrix K.

$$K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

The projection matrix P can then be constructed from the values of f_x, f_y, w, and h.

$$P = \begin{bmatrix} f'_x & 0 & c'_x & 0 \\ 0 & f'_y & c'_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (6)$$

Where $f'_x = f_x$, $f'_y = f_y$, $c_x = \frac{x}{2}$, and $c_y = \frac{y}{2}$.

4 Limitations

Measurement error in this process is unavoidable, and great care must be taken to mitigate this by making the measurement jig very rigid and dimensionally accurate. Although this method has proven sufficient for our projection mapping applications, automatic projector-camera calibration tools offer far greater accuracy.