

SUPPLEMENTARY PROOF

1 PROOF OF THEOREM 1

Theorem 1. Under Assumptions 1, 2, and setting the desired parametric differences $\Delta_1^{[i,j]}$, $\Delta_2^{[i,j]}$ for $(i, j) \in \mathcal{E}$ to achieve the desired distributed maneuvering behavior (as illustrated in TABLE 1), the DGVF $\mathfrak{X}^{[i]}(\xi^{[i]})$ designed by (12) associated with the desired interception and target enclosing speeds (by setting $\lambda_{w_1} = (-1)^{n+2}\dot{w}_1^*$, $\lambda_{w_2} = (-1)^{n+1}\dot{w}_2^*$), solves Problem 1 globally.

Proof. We firstly give the definitions, i.e., $K^{[i]} := \text{diag}\{k_1^{[i]}, \dots, k_n^{[i]}\}$, $\mathbf{f}^{[i]'} := (f_1^{[i]'}, \dots, f_n^{[i]'})^\top$, $\mathbf{g}^{[i]'} := (g_1^{[i]'}, \dots, g_n^{[i]'})^\top$.

One has

$$\nabla \phi_j^{[i]\top} \text{vm} \chi^{[i]} = k_j^{[i]} \phi_j^{[i]} - f_j^{[i]'} \mathbf{f}^{[i]'\top} K^{[i]} \Phi^{[i]} - g_j^{[i]'} \mathbf{g}^{[i]'\top} K^{[i]} \Phi^{[i]}, \quad (1)$$

for $i \in \mathbb{Z}_1^N$. Since $j \in \mathbb{Z}_1^n$, we stack Eq. (1) as

$$\begin{bmatrix} \nabla \phi_1^{[i]\top} \text{vm} \chi^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{vm} \chi^{[i]} \end{bmatrix} = -K^{[i]} \Phi^{[i]} - \mathbf{f}^{[i]'} \mathbf{f}^{[i]'\top} K^{[i]} \Phi^{[i]} - \mathbf{g}^{[i]'} \mathbf{g}^{[i]'\top} K^{[i]} \Phi^{[i]} \quad (2)$$

Following the same way, we can obtain

$$\begin{bmatrix} \nabla \phi_1^{[i]\top} \text{cr} \chi_1^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{cr} \chi_1^{[i]} \end{bmatrix} = -c_1^{[i]}(\mathbf{w}) \begin{bmatrix} f_1^{[i]'} \\ \vdots \\ f_n^{[i]'} \end{bmatrix} = -c_1^{[i]}(\mathbf{w}) \mathbf{f}^{[i]'}(\mathbf{w}), \quad (3)$$

and

$$\begin{bmatrix} \nabla \phi_1^{[i]\top} \text{cr} \chi_2^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{cr} \chi_2^{[i]} \end{bmatrix} = -c_2^{[i]}(\mathbf{w}) \mathbf{g}^{[i]'}(\mathbf{w}). \quad (4)$$

With Eqs. (1),(2),(3), the time derivative of $\Phi^{[i]}$ is

$$\begin{aligned} \dot{\Phi}^{[i]} &= \begin{bmatrix} \nabla \phi_1^{[i]\top} \text{vm} \chi^{[i]} + k_{c1} \nabla \phi_1^{[i]\top} \text{cr} \chi_1^{[i]} + k_{c2} \nabla \phi_1^{[i]\top} \text{cr} \chi_2^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{vm} \chi^{[i]} + k_{c1} \nabla \phi_n^{[i]\top} \text{cr} \chi_1^{[i]} + k_{c2} \nabla \phi_n^{[i]\top} \text{cr} \chi_2^{[i]} \end{bmatrix} \\ &= -K^{[i]} \Phi^{[i]} - \mathbf{f}^{[i]'} \mathbf{f}^{[i]'\top} K^{[i]} \Phi^{[i]} - \mathbf{g}^{[i]'} \mathbf{g}^{[i]'\top} K^{[i]} \Phi^{[i]} - k_{c1} c_1^{[i]}(\mathbf{w}) \mathbf{f}^{[i]'}(\mathbf{w}) - k_{c2} c_2^{[i]}(\mathbf{w}) \mathbf{g}^{[i]'}(\mathbf{w}). \end{aligned} \quad (5)$$

Define

$$\begin{aligned} \mathfrak{F}_1 &:= \text{diag}\{\mathbf{f}^{[1]'}, \dots, \mathbf{f}^{[N]'}\} \in \mathbb{R}^{nN \times nN} \\ \mathfrak{F}_2 &:= \text{diag}\{\mathbf{g}^{[1]'}, \dots, \mathbf{g}^{[N]'}\} \in \mathbb{R}^{nN \times nN} \\ K &:= \text{diag}\{K^{[1]}, \dots, K^{[N]}\} \in \mathbb{R}^{nN \times nN} \\ \Phi &:= (\Phi^{[1]}, \dots, \Phi^{[N]})^\top \in \mathbb{R}^{nN}. \end{aligned} \quad (6)$$

Thereby, we have $\dot{\Phi}$. Now we define the composite error:

$$\dot{e} = \begin{bmatrix} \dot{\Phi} \\ D^\top \dot{\tilde{w}}_1^{[1]} \\ D^\top \dot{\tilde{w}}_2^{[1]} \end{bmatrix} = \begin{bmatrix} -K\Phi - \mathfrak{F}_1\mathfrak{F}_1^\top K\Phi - \mathfrak{F}_2\mathfrak{F}_2^\top K\Phi + k_{c1}\mathfrak{F}_1 L\tilde{w}_1^{[1]} + k_{c2}\mathfrak{F}_2 L\tilde{w}_2^{[1]} \\ D^\top \mathfrak{F}_1^\top K\Phi - k_{c1}D^\top L\tilde{w}_1^{[1]} \\ D^\top \mathfrak{F}_2^\top K\Phi - k_{c2}D^\top L\tilde{w}_2^{[1]} \end{bmatrix} \quad (7)$$

According to [1, Proof of Theorem 2], we conclude that $\|\Phi^{[i]}(t)\| \rightarrow 0$ as $t \rightarrow \infty$ for $i \in \mathbb{Z}_1^N$. This further implies that $\|e(t)\|$ converges to 0 globally under arbitrary initial error.

Following [1, Proof of Theorem 2] and with Assumptions 1 and 2, one can also obtain that the differences in virtual coordinates $w_1^{[i]}$ and $w_2^{[i]}$ of neighboring robots converge to the desired values, i.e., $\lim_{t \rightarrow \infty} w_1^{[i]}(t) - w_1^{[j]}(t) - \Delta_1^{[i,j]} \rightarrow 0$, $\lim_{t \rightarrow \infty} w_2^{[i]}(t) - w_2^{[j]}(t) - \Delta_2^{[i,j]} \rightarrow 0$. This implies that $\chi_1^{[i]}(t, w_1^{[1]})$ and $\chi_2^{[i]}(t, w_2^{[1]})$ converge to 0 as $t \rightarrow \infty$. Combining $\lim_{t \rightarrow \infty} \|\Phi^{[i]}(t)\| \rightarrow 0$, one has

$$\begin{aligned} & \lim_{t \rightarrow \infty} \dot{w}_1^{[i]}(t) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 1 \ 0] \left({}^{\text{vm}}\chi^{[i]}(t, \xi^{[i]}) + k_{c1} {}^{\text{cr}}\chi_1^{[i]}(t, w_1^{[1]}) + k_{c2} {}^{\text{cr}}\chi_2^{[i]}(t, w_2^{[1]}) \right) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 1 \ 0] \cdot {}^{\text{vm}}\chi^{[i]}(t, \xi^{[i]}) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 1 \ 0] \cdot (-1)^n \cdot \begin{bmatrix} \lambda_{w_1} f_1^{[i]'} - \lambda_{w_2} g_1^{[i]'} \\ \vdots \\ \lambda_{w_1} f_n^{[i]'} - \lambda_{w_2} g_n^{[i]'} \\ \lambda_{w_1} \\ -\lambda_{w_2} \end{bmatrix} \\ &= (-1)^{n+1} (-1)^{n+1} \dot{w}_1^* = \dot{w}_1^*. \end{aligned}$$

Applying the same calculation to $\lim_{t \rightarrow \infty} \dot{w}_2^{[i]}(t)$, one has

$$\begin{aligned} & \lim_{t \rightarrow \infty} \dot{w}_2^{[i]}(t) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 0 \ 1] \cdot {}^{\text{vm}}\chi^{[i]}(t, \xi^{[i]}) \\ &= (-1)^{n+1} \lambda_{w_2} = (-1)^{n+1} (-1)^{n+1} \dot{w}_2^* = \dot{w}_2^*. \end{aligned}$$

In light of the above, Theorem 1 is proved to solve problem 1 globally.

REFERENCES

- [1] W. Yao, H. G. de Marina, Z. Sun, and M. Cao, "Guiding vector fields for the distributed motion coordination of mobile robots," *IEEE Transactions on Robotics*, vol. 39, no. 2, pp. 1119–1135, 2023.