

# SUPPLEMENTARY PROOF

## 1 PROOF OF THEOREM 1

**Theorem 1.** Under Assumptions 1, 2, and given constant desired parametric differences  $\Delta_1^{[i,j]}, \Delta_2^{[i,j]}$  for  $(i, j) \in \mathcal{E}$ , the DGVF  $\mathfrak{X}^{[i]}(t, \xi^{[i]})$  designed by (13) and choosing

$$\mathbf{v} = (-1)^{n+1}(0, \dots, 0, \dot{w}_2^*, -\dot{w}_1^*)$$

in (9), solve Problem 1 globally.

*Proof.* We firstly give the definitions, i.e.,  $K^{[i]} := \text{diag}\{k_1^{[i]}, \dots, k_n^{[i]}\}$ ,  $\mathbf{f}_\cdot^{[i]'} := (f_1^{[i]'}, \dots, f_n^{[i]'})^\top$ ,  $\mathbf{g}_\cdot^{[i]'} := (g_1^{[i]'}, \dots, g_n^{[i]'})^\top$ .

One has

$$\nabla \phi_j^{[i]\top} \text{vm} \chi^{[i]} = k_j^{[i]} \phi_j^{[i]} - f_j^{[i]'} \mathbf{f}_\cdot^{[i]'}{}^\top K^{[i]} \Phi^{[i]} - g_j^{[i]'} \mathbf{g}_\cdot^{[i]'}{}^\top K^{[i]} \Phi^{[i]}, \quad (1)$$

for  $i \in \mathbb{Z}_1^N$ . Since  $j \in \mathbb{Z}_1^n$ , we stack Eq. (1) as

$$\begin{bmatrix} \nabla \phi_1^{[i]\top} \text{vm} \chi^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{vm} \chi^{[i]} \end{bmatrix} = -K^{[i]} \Phi^{[i]} - \mathbf{f}_\cdot^{[i]'} \mathbf{f}_\cdot^{[i]'}{}^\top K^{[i]} \Phi^{[i]} - \mathbf{g}_\cdot^{[i]'} \mathbf{g}_\cdot^{[i]'}{}^\top K^{[i]} \Phi^{[i]} \quad (2)$$

Following the same way, we can obtain

$$\begin{bmatrix} \nabla \phi_1^{[i]\top} \text{cr} \chi_1^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{cr} \chi_1^{[i]} \end{bmatrix} = -c_1^{[i]}(\mathbf{w}) \begin{bmatrix} f_1^{[i]'} \\ \vdots \\ f_n^{[i]'} \end{bmatrix} = -c_1^{[i]}(\mathbf{w}) \mathbf{f}_\cdot^{[i]'}(\mathbf{w}), \quad (3)$$

and

$$\begin{bmatrix} \nabla \phi_1^{[i]\top} \text{cr} \chi_2^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{cr} \chi_2^{[i]} \end{bmatrix} = -c_2^{[i]}(\mathbf{w}) \mathbf{g}_\cdot^{[i]'}(\mathbf{w}). \quad (4)$$

With Eqs. (1),(2),(3), the time derivative of  $\Phi^{[i]}$  is

$$\begin{aligned} \dot{\Phi}^{[i]} &= \begin{bmatrix} \nabla \phi_1^{[i]\top} \text{vm} \chi^{[i]} + k_{c1} \nabla \phi_1^{[i]\top} \text{cr} \chi_1^{[i]} + k_{c2} \nabla \phi_1^{[i]\top} \text{cr} \chi_2^{[i]} \\ \vdots \\ \nabla \phi_n^{[i]\top} \text{vm} \chi^{[i]} + k_{c1} \nabla \phi_n^{[i]\top} \text{cr} \chi_1^{[i]} + k_{c2} \nabla \phi_n^{[i]\top} \text{cr} \chi_2^{[i]} \end{bmatrix} \\ &= -K^{[i]} \Phi^{[i]} - \mathbf{f}_\cdot^{[i]'} \mathbf{f}_\cdot^{[i]'}{}^\top K^{[i]} \Phi^{[i]} - \mathbf{g}_\cdot^{[i]'} \mathbf{g}_\cdot^{[i]'}{}^\top K^{[i]} \Phi^{[i]} - k_{c1} c_1^{[i]}(\mathbf{w}) \mathbf{f}_\cdot^{[i]'}(\mathbf{w}) - k_{c2} c_2^{[i]}(\mathbf{w}) \mathbf{g}_\cdot^{[i]'}(\mathbf{w}). \end{aligned} \quad (5)$$

Define

$$\begin{aligned} \mathfrak{F}_1 &:= \text{diag}\{\mathbf{f}_\cdot^{[1]'}, \dots, \mathbf{f}_\cdot^{[N]'}\} \in \mathbb{R}^{nN \times N} \\ \mathfrak{F}_2 &:= \text{diag}\{\mathbf{g}_\cdot^{[1]'}, \dots, \mathbf{g}_\cdot^{[N]'}\} \in \mathbb{R}^{nN \times N} \\ K &:= \text{diag}\{K^{[1]}, \dots, K^{[N]}\} \in \mathbb{R}^{nN \times nN} \\ \Phi &:= (\Phi^{[1]}, \dots, \Phi^{[N]})^\top \in \mathbb{R}^{nN}. \end{aligned} \quad (6)$$

Thereby, we have  $\dot{\Phi}$ . Now we define the composite error:

$$\dot{e} = \begin{bmatrix} \dot{\Phi} \\ D^\top \dot{\tilde{w}}_1^{[1]} \\ D^\top \dot{\tilde{w}}_2^{[1]} \end{bmatrix} = \begin{bmatrix} -K\Phi - \mathfrak{F}_1\mathfrak{F}_1^\top K\Phi - \mathfrak{F}_2\mathfrak{F}_2^\top K\Phi + k_{c1}\mathfrak{F}_1 L\tilde{w}_1^{[1]} + k_{c2}\mathfrak{F}_2 L\tilde{w}_2^{[1]} \\ D^\top \mathfrak{F}_1^\top K\Phi - k_{c1}D^\top L\tilde{w}_1^{[1]} \\ D^\top \mathfrak{F}_2^\top K\Phi - k_{c2}D^\top L\tilde{w}_2^{[1]} \end{bmatrix} \quad (7)$$

According to [1, Proof of Theorem 2], we conclude that  $\|\Phi^{[i]}(t)\| \rightarrow 0$  as  $t \rightarrow \infty$  for  $i \in \mathbb{Z}_1^N$ . This further implies that  $\|e\|$  converges to 0 globally under arbitrary initial error.

Following [1, Proof of Theorem 2] and with Assumptions 1 and 2, one can also obtain that the differences in virtual coordinates  $w_1^{[i]}$  and  $w_2^{[i]}$  of neighboring robots converge to the desired values, i.e.,  $\lim_{t \rightarrow \infty} w_1^{[i]}(t) - w_1^{[j]}(t) - \Delta_1^{[i,j]} \rightarrow 0$ ,  $\lim_{t \rightarrow \infty} w_2^{[i]}(t) - w_2^{[j]}(t) - \Delta_2^{[i,j]} \rightarrow 0$ . This implies that  $\chi_1^{[i]}(t, w_1^{[1]})$  and  $\chi_2^{[i]}(t, w_2^{[1]})$  converge to 0 as  $t \rightarrow \infty$ . Combining  $\lim_{t \rightarrow \infty} \|\Phi^{[i]}(t)\| \rightarrow 0$ , one has

$$\begin{aligned} & \lim_{t \rightarrow \infty} \dot{w}_1^{[i]}(t) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 1 \ 0] \left( {}^{\text{vm}}\chi^{[i]}(t, \xi^{[i]}) + k_{c1} {}^{\text{cr}}\chi_1^{[i]}(t, w_1^{[1]}) + k_{c2} {}^{\text{cr}}\chi_2^{[i]}(t, w_2^{[1]}) \right) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 1 \ 0] \cdot {}^{\text{vm}}\chi^{[i]}(t, \xi^{[i]}) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 1 \ 0] \cdot (-1)^n \cdot \begin{bmatrix} v_{n+2}f_1^{[i]'} - v_{n+1}g_1^{[i]'} \\ \vdots \\ v_{n+2}f_n^{[i]'} - v_{n+1}g_n^{[i]'} \\ v_{n+2} \\ -v_{n+1} \end{bmatrix} \\ &= (-1)^{n+1}(-1)^{n+1}\dot{w}_1^* = \dot{w}_1^*. \end{aligned}$$

Applying the same calculation to  $\lim_{t \rightarrow \infty} \dot{w}_2^{[i]}(t)$ , one has

$$\begin{aligned} & \lim_{t \rightarrow \infty} \dot{w}_2^{[i]}(t) \\ &= \lim_{t \rightarrow \infty} [0 \cdots 0 \ 1] \cdot {}^{\text{vm}}\chi^{[i]}(t, \xi^{[i]}) \\ &= (-1)^{n+1}v_{n+1} = (-1)^{n+1}(-1)^{n+1}\dot{w}_2^* = \dot{w}_2^*. \end{aligned}$$

In light of the above, Theorem 1 is proved to solve problem 1 globally.

## REFERENCES

- [1] W. Yao, H. G. de Marina, Z. Sun, and M. Cao, "Guiding vector fields for the distributed motion coordination of mobile robots," *IEEE Transactions on Robotics*, vol. 39, no. 2, pp. 1119–1135, 2023.