Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
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An MCM Paper Made by Team 1234567

Summary

People around the world are complaining about the traffic jams every day. Some people may spend more than 3 hours on their way from office to home everyday on their cars, which makes them frustrated. And our problem is to investigate the methodsto optimize the design of toll plazas on highways, especially the area after merging,which is also a possible location where vehicles can be stuck in.

Before establishment of our models, we list some assumptions to make the real lifescenario easier to model.

And then we start to analyze the existing models, from which we conclude their strengths and weaknesses. By investigating their characteristics, we get the inspiration to form our two new models: Control Time Model (CTM) andWaiting Area Model(WAM). In these two new models, we introduced a way of conducting control of the departure time of the vehicles at the tollbooths. This kind of control is basically preventing the vehicles which are merging in the same lane from leaving the booths simultaneously, which may be a hidden danger for traffic accident or leads to a traffic jam. And we?ll continue to calculate the size and shape of the merging area according to our control methods and some assumptions. After that, we introduce a method for finding the optimal merging pattern based on both mathematical proof and computer simulations.

After that, we run some simulations to find out the throughput, risk and cost of different models, which are based on some statistical laws for real life situations. Then we compare these three models in all factors with the help of statistical hypothesis testing, and conclude that the CTM is the best in general. The next section is about some slight modification under different conditions such as including the self-driving car, and the different arrangements in terms of merging patterns when the proportion of different types of tollbooths vary.

For the following section, we test our model by investigating the sensitivity of construction cost and throughput (per hour) in terms of some variables included in our model, in order to justify the reliability of our model from different perspectives.

Finally, we end our report by the conclusion part followed by strengths and weaknesses analysis.

Contents

1 Introduction

1.1 Background

AIDS, also known as acquired immunodeficiency syndrome, is a diease whose virus attacks the human immune T cells, eventually destroying the human immune system, losing resistance to the disease and dying. AIDS is mainly transmitted through sexual transmission, mother-to-child transmission, and cross-use of drug syringes. Therefore, the transmission groups we studied include adults who have sex (including heterosexual and homosexual behaviors), pregnant women, and drug users. At present, there is almost no cure for AIDS, that is, there are no specific drugs and effective vaccines. Both the infection and the onset periods can be transmitted and transmitted. According to the stage chart of adults infected with AIDS given by WHO, the period from infection to death includes a three-month empty window period, an eight-year incubation period and a thirteen-month The onset period, and finally entered death. Patients generally die within a decade after contracting AIDS.

ARV therapy ARV therapy inhibits virus replication by interfering with or blocking key nodes of HIV-1 virus infection and replication. Antiretroviral therapy can prolong the survival of AIDS patients, but it has not yet cured the patients. In richer countries, the coverage of antiretroviral therapy is higher, while poorer, poorer countries cannot afford a large area of antiretroviral therapy, so coverage is lower. The effect of ARV therapy has higher requirements on whether to adhere to treatment. On the one hand, if continuous long-term treatment cannot be performed, the effect of ARV therapy will be greatly reduced; on the other hand, intermittent treatment is likely to produce resistance, which will make subsequent ARV treatment lose its effect. The cost of ARV therapy per person is about $1,100 per year, so for countries with a per capita GDP of less than $ 1,100, a significant number of AIDS patients cannot receive long-term effective ARV therapies, and the probability of developing resistance will be higher than in rich countries Patient. Once the patients who have received antiretroviral therapy have realized that they have AIDS, the probability of transmission of AIDS will decrease in the short term, such as reducing the number of sexual relations and reducing the number of cross-use syringes. Looking at it, because antiretroviral therapy prolongs the lifespan of AIDS patients, the total number of sexual relationships that occur during their lifetime will increase, and there is the possibility of increased transmission.

ARV resistance When antiretroviral therapy is not adhered to for a long period of time, patients may develop ARV resistance, and subsequent antiretroviral treatment will lose its effectiveness after resistance is developed. When a patient develops resistance, his / her follow-up treatment is terminated, and the treatment funds that should have been invested in him / her will be able to be used for newly occurred AIDS patients.

1.2 Restatement of the problem

Our problem is divided into four tasks. For the first task, we need to model a country with the most severe epidemic among all continents, and give reasons for such selection. After the selection, we need to build a model to predict the trend of HIV infection rates in these countries from 2020 to 2060 without human intervention. Human intervention on the HIV infection rate includes two aspects, one is preventive intervention, including voluntary counseling and testing, promotion of condom use, school education promotion of AIDS, and prevention of mother-to-child transmission of HIV; on the other hand, Therapeutic interventions include treatment of other untreated sexually transmitted diseases, treatment of opportunistic infections, etc. In the next tasks we consider the intervention of antiretroviral therapy and the development of HIV vaccines. The second task is to determine the allocation of international aid funds between the two intervention strategies, and based on the model established by task1, the trend of HIV infection rates from 2020 to 2060 in three cases. The following questions are given to three cases: (1) only antiretroviral therapy; (2) only HIV vaccine research and development; (3) antiretroviral therapy and HIV vaccine research and development simultaneously. Among them, in the third case, we need to find the optimal allocation ratio of funds. In addition, not all countries are suitable for the discussion in these three cases. For example, some backward countries cannot afford the cost of ARV therapy, nor do they have sufficient funds and scientific research power for the development of HIV vaccines. Therapies have been widely used, and they don't need to invest more funds to promote them, they can participate in the research and development of vaccines. The third task is to take drug resistance into consideration. Drug resistance is an obstacle to the large-scale application of ARV therapy. Once resistance is developed, subsequent treatment will lose its effectiveness. Irregular medication or long-term adherence to medication may produce resistance. The requirement of the third task is to put resistance into the model and re-evaluate measures related to ARV therapy, namely (1) (3). In fact, for countries with insufficient funds, ARV therapy cannot cover all patients. Once a patient develops resistance, the cost of subsequent treatment can be transferred to newly emerged patients, so the total number of patients receiving ARV treatment is not Too much change; and for countries with sufficient funds, all patients can be treated with ARV, the more resistance develops, the more the overall treatment effect decreases. Finally, the fourth task is to write a letter to the United Nations, provide the results of our research, and give our suggestions based on our research results to some questions of great concern, such as resource allocation in the promotion of ARV therapy and the development of HIV vaccines, The weight of AIDS should be considered in policy and how to integrate AIDS donations.

1.3 .... Literature review ....

The optimal number of tollbooths needed to minimize the average waiting time is well-studied and simulated based on different real situations (Corwen et al, 2005). Though the definition of "optimal" varies, similar suggestions have been given. Tollbooths should be implemented conforming to encouraged behaviors, e.g. faster booths should be put on the left (Spann et al, 2005), which is incorporated in our model. Some literature suggested that tollbooths employ no barrier to ensure a relatively smooth flow (Kane, 2005), but we contend that the uplift of barrier takes negligible amount of time, the benefit of which cannot be compared with the chaos and potential risk if some vehicles go through toll plaza directly. Therefore, barriers are included in our model.

2 Assumptions

To make our problems easier to deal with, we make the following basic assumptions, each of which is properly justified.

- The number of people of a given age is evenly distributed. Since the age structure's data we got is segmented. For example, there are 14.18% of people are in age 0 14. So it's convenient for us to assume they are evenly distributed in every age.
- The study of sexual behavior divides the population into men, women and homosexuals. There is no mean to offense, in order to simplify our model. We assume that homosexuals

only make love with people who have the same gender with them.

- Lesbians are not considered in studies of the spread of HIV among homosexuals. According to *journal of youth and adolescence* on the website of springer. The risk of infecting AIDS is ranked as Female bisexuality $\gg$ gay $\gg$ heterosexual $\gg$ Lesbians, so in our model, Lesbians are not included..
- when think about population longevity, we use expectation of life as Age of death for people who did not get sick. And the longevity of infector is denied as the current age plus the incubation period plus the mean survival time after diagnosis, which differs when people receive treatment or not.

3 Notations

Notation	Definition	Unit
Q_T	Total number of infections	N
Q_0	Number of homosexuals	N
Q_e	Number of heterosexuals	N
Q_{ho}	The number of gay people infected	N
Q_{he}	The number of heterosexuals infected	N
Q_{nd}	Number of drug users in year n	N
Q_p	The number of pregnant women	N/A
N_y	Number of sexual encounters per year	N
P_{ucho}	Gay people who do not use condoms to get infected	N/A
P_i	The probability of getting infected without using a condom	N/A
P_{uche}	Heterosexual people do not use condoms to get infected	N/A
P_p	Proportion of pregnant women	N/A
P_a	The percentage of pregnant women infected	N/A
η	Percentage of drug users	N/A
ε	The rate of AIDS among drug users	N/A
δ	The percentage of babies infected	N/A

4 ... Models ...

In this section, we first introduce and analyze the existing model, and then two new models invented by us. And the focus for the new models will only be the establishment of the first new model (control time model) and second new model (waiting area model) only deviates from the first one slightly.

4.1 Free propagation model(FPM)

We consider the influence of several factors and establish a Free propagation model to reveal the development principle of the AIDS under on intervention.

4.1.1 Factors

Death

According to our hypothesis, AIDS patients cannot be cured, and the number of patients and potential patients can only be reduced through death. In both populations, the cause of death among AIDS patients was AIDS or AIDS-related diseases, and deaths in susceptible populations were determined based on life expectancy.

Mother-to-child transmission

Mother-to-child transmission depends on the proportion of pregnant women. Healthy pregnant women give birth to healthy babies. Patients who give birth to pregnant women may give birth to patients or healthy babies.

Sexual transmission

Sexual transmission includes same-sex transmission and heterosexual transmission. The number of new patients who catch AIDS through heterosexual transmission each year depends on the number of heterosexual acts per person per year, the proportion of sexual acts without condoms, and the probability of sexual transmission without condoms. The number of new patients who catch AIDS through homosexual transmission each year depends on the number of homosexual acts per person per year, the proportion of sexual acts without condoms, and the probability of sexual transmission without condoms.

Cross-use of syringes by drug users

The number of new patients transmitted by drug users through the use of syringes depends on the total number of people in the population, the proportion (constant) of drug users in the population, and the proportion of people infected with HIV among drug users.

4.1.2 Free propagation model construction

In task 1, there is no consideration about the intervention, so when we construct model about the expected rate of change in the number of AIDS, we only take the factors above. And the longevity of the man who is diagnosed with AIDS is relatively short. This issue will be discussed in the following. By exploring the influence of those factors mentioned above, we can predict the future rate of change in the number of AIDS in South Africa.

$$\frac{dQ_i}{dt} = \sum_{n}^{n+1} Q_o + \sum_{n}^{n+1} Q_e + \sum_{n}^{n+1} Q_{nd} - Q_{(n-1)d} + \sum_{n}^{n+1} Q_p\delta - \sum_{n}^{n+1} P_{ua}$$

In this model :

- Q_o indicate the number of gay people infected in this year
- Q_e indicate the number of heterosexuals infected in this year
- Q_n Q_{n-1} indicate the number of people in year n andyear n-1, we will construct model about the Natural population growth in the following part.
- Q_p indicate the number of baby born in this year
- δ is the percentage of the babies who are born with HIV

Form the expression, we can find that

$\sum_n^{n+1} Q_o + \sum_n^{n+1} Q_e$ indicate the infected member increased by sexual contact

$\sum_n^{n+1} Q_{nd} - Q_{(n-1)d}$ indicate the infected member increased by injecting drug

$\sum_n^{n+1} Q_p \delta$ indicate the infected member increased by mother-to-fetus transmission

$\sum_n^{n+1} P_{ua}$ indicate the infected member reduced because death, since there is no way to cure AIDS now, so dying is the only way to decrease the infected member.

Number of sexually transmitted infections

The nunmber of sexually transmitted infections an be expressed as

$$Q = Q_o + Qe = \sum_n^{n+1} Q_{ho} * (1 - (1 - P_{ucho} P_i)^{N_y}) + \sum_n^{n+1} Q_{he} * (1 - (1 - P_{uche} P_i)^{N_y})$$

In this exprssion:

- Q_{ho} indicate the number of homosexuals
- Q_{he} indicate the number of homosexuals.
- N_y is the number of sexual encounters per year, which is assumed to be same between homosexuals and homosexuals. We will change it in the sensitivity analysis part.
- P_{ucho} P_{uche}indicates the probabilities gay people and heterosexual people who do not use condoms to get infected. According the research, P_{ucho}, the percentage is so high that it means people use condoms only aiming for preventing them from having babies. And we can assume the percentage can be relatively higher in the gay people. We will also test this in the sensitivity analysis part
- P_i is the the probability of getting infected without using a condom.

Every sex act between susceptible people can cause the spread of AIDS. And the probability of infection is equal to $P_{ucho} * P_i$, After having sex a hundred times a year, we can conclude that the probability of not contracting AIDS is equal to $(1 - P_{ucho} * P_i)^{N_y}$, This is a binomial distribution experiment, And the probability of getting AIDS is$1 - (1 - P_{ucho} P_i)^{N_y}$

N_y is a constant. Considering the social pressure and different social status, these two group can act diffrently theoretically speaking. and in the futher work, we will introduce social factors to estimate people's eager for sexual life. The higher the pressure is, the more likely they want turn to their sexual partner to have relax. And although homosexuality is recognized in South Africa,Gay people still don't get a fair deal in many social situations. We can see later, weather the change of N_y is vital to our model.

the infected member increased by injecting drug

the infected member increased by injecting drug can be expressed as

$$\sum_n^{n+1} (Q_n - Q_{n-1}) \eta \varepsilon$$

In this exprssion:

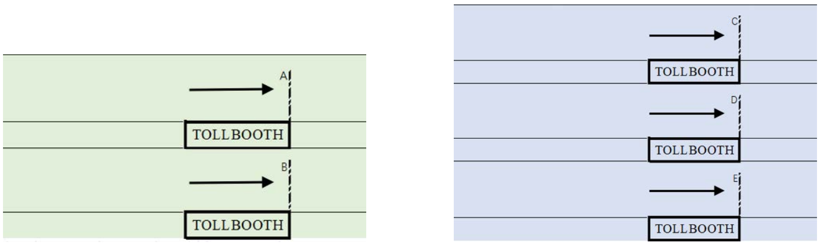
- η is the percentage of drug users,
- ε is the rate of AIDS among drug users

Drugs spread AIDS mainly by sharing syringes. However, studies on individuals or small groups exchanging syringes are too complicated and lack relevant data.

4.2 Control Time Model (CTM)

Given aforementioned deficiencies of existing merging pattern, we propose a new model, partially based on the current one. Instead of having all the vehicles moving and merging at their own discretion, control time model will control the departure time of vehicles to ensure a smooth and safe emerging process.

Specifically, for situations where two booths merge into one lane, the second vehicle will only be allowed to proceed t0 seconds after the first vehicle moves forward. The time t0 is defined as the control time. Similarly, for situations where three booths merge into one lane, the third vehicle will be allowed to proceed t0 seconds after the second vehicle moves forward, whilst the second vehicle t0 seconds after the first vehicle, as shown in Figure 1

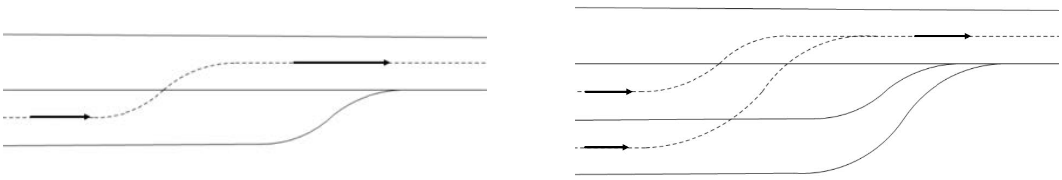


(a) Either A or B is open. (b) Either C, D or E is open.

Figure 1

In this way, the regulated merging of the vehicles into another lane would be more efficient than the situation where vehicles are proceeding without regulation, for drivers should take time to make decision when multiple booths merge into one lane simultaneously, let alone the risk for doing so.

We model that vehicles start with constant acceleration as until reach the maximum speed vm in the straight path. They then immediately starts merging into their prescribed road in two consecutive tangent circle arcs.



(a) (b)

Figure 2: Two or three tollbooth egress lanes merge into one lane

Now we evaluate the "appropriate" control time. When emergency happens, one vehicle take severe brake action with acceleration ab, after response time tres, the posterior vehicle take severe brake action with the same acceleration. Consider the distance of the two vehicles (t = 0 is the time when emergencies happen):

$$d = v_m t_{rea} + \int_0^t v_{posterior}(t)dt - \int_0^t v_{previous}(t)dt$$

where

$$v_{previous} = \begin{cases} v_m, & t \leq t_{rea} \\ v_m - a_b(t - t_{rea}), & t_{rea} < t \leq t_{rea} + \frac{v_m}{a_b} \\ 0, & t_{rea} + \frac{v_m}{a_b} \leq t \end{cases} \tag{1}$$

Solve the equation $d \geq 0$, we get $t_0 \geq tres$. By saying "appropriate" control time, we take t_0= $1s \geq t_{res} = 0.8s$ (Lee et al, 2002).

After describing the control time model, we first calculate the throughput of the toll plaza.

Let i_k be the number of booths corresponding with the k^{th} lane. For example, if the first three booths merge into the first lane, we say $i_1 = 3$.

By computer simulation (details are included in Appendix), we have t_{ik} close to $\frac{T}{i_k}$ when $i_k \ll \frac{T}{t_0}$ (especially when the variance of service time is small), and t_{ik} increases as i_k increases, and the increase speed is quite large when $i_k \geq \frac{T}{t_0}$. where tik is the averaged timefor a car to come into k_{th} lane.

Therefore, to maximize the efficiency, we'll keep i_k as small as possible, whereas the throughput comes suboptimal if $i_k \geq \frac{T}{t_0}$ for some k. The optimal throughput is closed to $\frac{B}{T}$ car(s) per second.

By substituting B by 8, the expectation of human service time T by 15. According to our optimization method above, when we keep the i_k as small as possible, which is in other words, let them distributed evenly(e.g. 2,3,3). We will get the optimal throughput closed to $\frac{B}{T} \approx 0.533$ car per second.

Then we calculate the size of the plaza, by doing so we discuss the value of D.

Define f : $\{1, 2, \dots ,\mathrm{B}\} \rightarrow \{1, 2, \dots ,\mathrm{L}\}$ which is non-decreasing, and $f(i) = j$ if and only if vehicles from i_{th} booth merge into j_{th} lane. It obviously follows that $i_k = \| f^{-1}(\{\mathrm{k}\})\|$. We call the mapping f a "merging pattern".

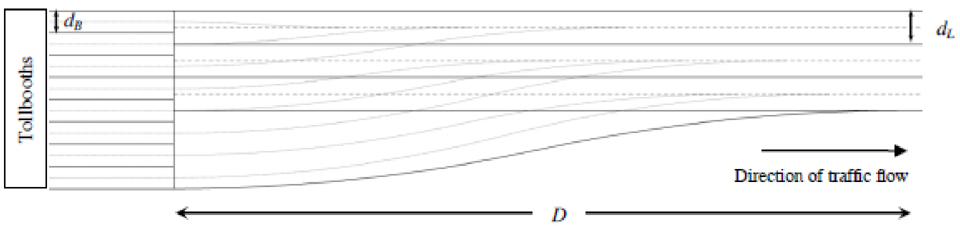


Figure 3: B = 8 tollbooth egress lanes merge into L = 3 lanes

By geometric relations, we have

$$(\frac{D - \frac{v_m^2}{2a}}{2})^2 + (r - \frac{(i - \frac{1}{2})d_B - (j - \frac{1}{2})d_L}{2})^2 = r^2$$

Observed from above relation, a certain (i, j) prescribes a lower bound of D, and for a certain merging pattern, the distance between the tollbooth and the end of the plaza that we eventually employ is the greatest lower bound of D. The less the greatest lower bound is, the smaller the size will be, and so will the cost of plaza. By property of quadratic function, for a certain j, the greater i is, the greater D is. We have

$$\sum_{k=1}^{j} i_k d_B - j d_L \geq (\sum_{k=1}^{j} i_k + 1) d_B - (k+1) d_L$$

since $d_L \geq d_B$. That implies if j is changed into j + 1 whilst i is changed into i + 1, D will get smaller. Therefore, the global maximum must be obtained at one of the local maximum points, and then we let x be

$$x = \max_{j\in\{1,2,...,L\}} \left\{ \frac{(\sum_{k=1}^{j} i_k - \frac{1}{2})d_B - (j-\frac{1}{2})d_L}{2} \right\}$$

And for given B and L, the optimal choice of merging pattern f would be the case if we can minimize x to be

$$x_{min} = \min_{\{i_k\}\in\{\|f^{-1}(\{k\})\|:\forall f\}} x$$

After trying out some pairs of small (B,L), we induce that for any given small number B, L (which can be put in practical use) satisfying $B \geq L$, there exists a merging pattern f_m such that

$$x = \frac{(B-\frac{1}{2})d_B - (L-\frac{1}{2})d_l}{2} := x_0$$

Since for any ik we have the relation $f(B) = L$, which implies above expression will appear in every calculation of x for all i_k, hence $x_{min} \leq x_0$, hence the existence of x_0 guarantees "=", providing us with the optimal merging pattern to minimize D.

By substituting $B = 8$, $L = 3$, $d_B = 2.5$m, $d_L = 3.75$m, $r = 115m$, $v_m = 15m/s$, $a = 2.78m/s^2$, the minimum of D is $105.5m$, when $(i1, i2, i3) \in \{(3, 3, 2), (3, 2, 3), (2, 3, 3)\}$.

4.3 Waiting Area Model(WAM)

After completing the above model, something seems to be deficient also appears in our model. In the control time model, vehicles may need to wait at the booths even after the service finished, while there are a lot of vehicles queuing after it. So we wonder that whether it is better to build a waiting area at a plaza (as shown in Figure 5), so that the vehicles whose service has finished can go to the waiting area, then the consecutive vehicle can be serviced. So in this model, the staff spending more time serving instead of doing nothing while the vehicles are just waiting at the booths

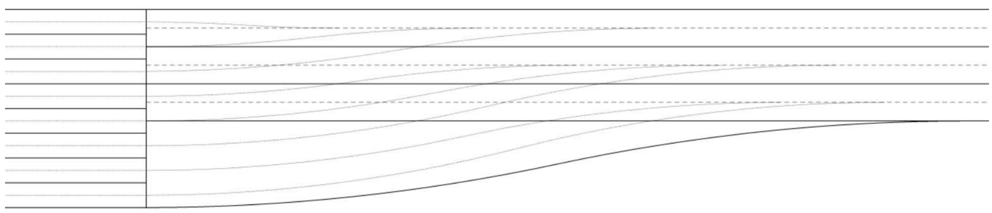


Figure 4: B = 8 tollbooth egress lanes merge into L = 3 lanes

We will test and compare this model with control time model in the following sections,to see whether it is efficient or not.

5 Analysis and Results

5.1 Throughput

Throughput (θ) here is defined to be the number of vehicles per hour passing the point where the end of the plaza joins one of the the L outgoing traffic lanes. Here we discuss only one outgoing lane merged from two lanes in human-staffed tollbooth area in heavy traffic where vehicles come continuously during heavy traffic so that the cashier work continually, as shown in Figure 6.

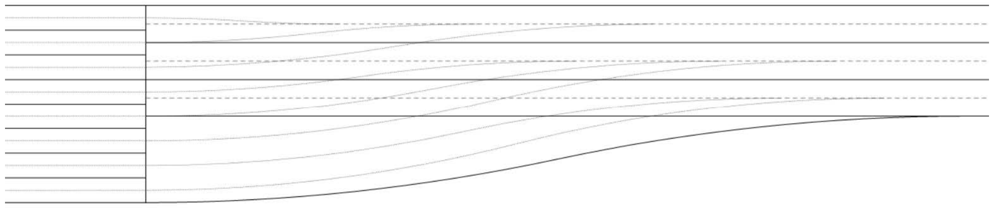


Figure 5: 8 tollbooth egress lanes merge intoL = 3 lanes in heavy traffic

5.1.1 Throughput of existing model

The toll collection service time is a random variable which can be seen as a sum of random variables including time needed by driver to search money or credit vehicled, time needed by cashier to take related record, etc. According to the central limit theorem, we can assume that the service time of ith vehicle ($t_{h,i}$) of the each lane follow normal distribution with $\mu = 15$ and $\sigma^2 = 4$ independently and identically.

6 Strengths and Weaknesses

6.1 Strengths

- First one...
- Second one ...

6.2 Weaknesses

- Only one ...

Memorandum

To: Heishan Yan
From: Team XXXXXXX
Date: October 1st, 2019
Subject: A better choice than MS Word: LaTeX

In the memo, we want to introduce you an alternate typesetting program to the prevailing MS Word: **LaTeX**. In fact, the history of LaTeX is even longer than that of MS Word. In 1970s, the famous computer scientist Donald Knuth first came out with a typesetting program, which named TeX ...

Firstly, ...

Secondly, ...

Lastly, ...

According to all those mentioned above, it is really worth to have a try on LaTeX!

References

[1] Abdel-Aty, M. A., & Radwan, A. E. (2000). *Modeling traffi c accident occurrence andinvolvement*. Accident Analysis & Prevention, 32(5), 633-642.

[2] Corwin, I., Ganatra, S., & Rozenblyum, N. (2005). *A Single-vehicle Interaction Model of Traffic for a Highway Toll Plaza*. The UMAP Journal, 26(223), 299-315.

[3] Friedman, D. A., & Waldfogel, J. (1995). *The administrative and compliance cost of manual highway toll collection: evidence from Massachusetts and New Jersey*. National Tax Journal, 217-228.

Appendix A: Further on LaTeX

To clarify the importance of using LaTeX in MCM or ICM, several points need to be covered, which are . . .

To be more specific, . . .

All in all, . . .

Anyway, nobody **really** needs such appendix . . .