$$P(+m) = P(+m|+g) \times P(+g) + P(+m|-g) \times P(-g)$$
$$= 0.8 \times 0.25 + 0.1 \times 0.75$$
$$= 0.2 + 0.075$$
$$= 0.275$$

$$P(+g|+m) = \frac{P(+g, +m)}{P(+m)} = \frac{P(+m|+g) \times P(+g)}{P(+m)}$$

$$= \frac{0.8 \times 0.25}{\frac{7}{24}} = \frac{0.2}{\frac{7}{24}} = 0.2 \times \frac{24}{7} = \frac{4.8}{7} = \frac{24}{35}$$

G	M	B	C	P(G, M, B, C)	0.25 0.8 0.1 0.2
+	+	+	+	0.012	P(G)P(M|G) P(B|M) P(C|M) : 0.012
+	+	+	−	0.048	0.25 0.8 0.3 0.8
+	+	−	+	0.028	
+	+	−	−	0.112	
+	−	+	+	0.005	0.25 0.7 0.2 0.5
+	−	+	−	0.005	
+	−	−	+	0.02	0.25 0.2 0.8 0.5
+	−	−	−	0.02	
−	+	+	+	0.0045	0.75 0.1 0.3 0.2
−	+	+	−	0.018	
−	+	−	+	0.0105	
−	+	−	−	0.042	0.75 0.1 0.7 0.8
−	−	+	+	0.0675	
−	−	+	−	0.0675	
−	−	−	+	0.27	0.75 0.9 0.8 0.5
−	−	−	−	0.27	

$P(+b \mid +m) = 0.3$

$P(+b \mid +m, \pm g) = 0.3$ since $B \perp\!\!\!\perp G \mid M$

$P(+b) = \sum P(G, M, +b, C)$

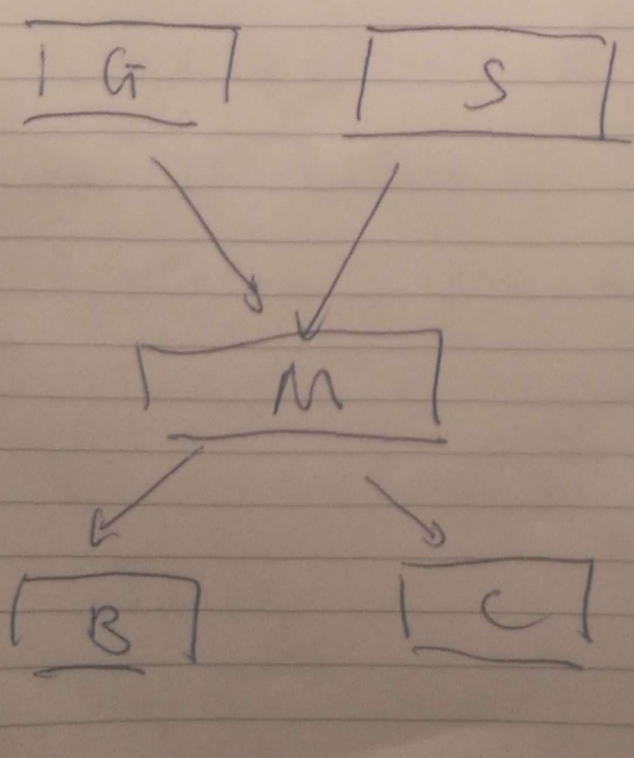
$= 0.012 + 0.048 + 0.005 + 0.005 + 0.0045 + 0.018 + 0.0675 + 0.0675$

$= 0.2275$

$$P(+c \mid +b) = \frac{P(+c, +b)}{P(+b)} = \frac{\sum P(G, M, +b, +c)}{P(+b)}$$

$$= \frac{0.012 + 0.005 + 0.0045 + 0.0675}{0.2275}$$

≈ 0.391209



$P(M \mid G)$
should be modified
to $P(M \mid G, S)$

$B \perp\!\!\!\perp G$ False

$B \not\perp\!\!\!\perp C$ False

$C \perp\!\!\!\perp G \mid M$ True

$B \perp\!\!\!\perp C \mid G$ False

$G \perp\!\!\!\perp S$ True

$G \perp\!\!\!\perp S \mid B$ False

1. $f_2(X, T, W) = \sum_{v} P(X \mid v)\, P(T \mid v)\, f_1(v, W)$
2. $P(+y \mid W, x),\ P(Z \mid T).\ f_2(X, T, W)$
3. $f_3(+y, X, T) = \sum_{w} P(+y \mid w, X)\, f_2(X, T, w)$
4. $P(Z \mid T)\ f_3(+y, X, T)$
5. $f_4(+y, X, Z) = \sum_{t} P(Z \mid t)\, f_3(+y, X, t)$
6. $f_4(+y, X, Z)$
7. $f_5(+y, Z) = \sum_{x} f_4(+y, x, Z)$
8. $f_5(+y, Z)$
9. $P(+y, Z) = f_5(+y, Z)$ $\qquad P(Z \mid +y) = \dfrac{f(+y, Z)}{\sum_z f_5(+y, z)}$

10. f_2 is the largest factor which contains 3 binary domains $2^3 = 8$

11.

Value Elimination	Factor Generated
U	$f_1(W, V)$
T	$f_2(V, Z)$
X	$f_3(+y, W, V)$
V	$f_4(+y, W, Z)$
W	$f_5(+y, Z)$

$P(S_3 = ct \mid +GC) = 0.4889 \times 0.2$

3. HMM

	cg	at
T	0.2	0.2
F	0.8	0.8

1. Initial distribution $P(cg)=0.5$ $P(at)=0.5$

Transition $P(cg_{t+1}|cg)=0.8$ $P(cg_{t+1}|at_t)=0.2$ $P(at_{t+1}|cg_t)=0.2$

$P(at_{t+1}|at_t)=0.8$

Observation probabilities

$P(C|cg)=0.4$ $P(G|cg)=0.4$ $P(A|cg)=0.1$ $P(T|cg)=0.1$

$P(A|at)=0.3$ $P(T|at)=0.3$ $P(C|at)=0.2$ $P(G|at)=0.2$

2. $P(S_0,S_1,S_2)=P(S_0)P(S_1|S_0)P(S_2|S_1)$

Suppose $S_0=at$ then $P(S_0,S_1,S_2)=P(at)P(cg|at)P(at|cg)$

$S_1=cg$

$S_2=at$ $\quad P(S_0,S_2)=\sum_{S_1}P(S_0,S_1,S_2)$

$\Rightarrow P(S_0)=0.5$

$P(S_0,S_1)=P(S_0)P(S_1|S_0)$

$=P(at)P(cg|at)=0.5\times0.2=0.1$

$P(S_0,S_1,S_2)=P(S_1)P(S_2|S_1)$

$=0.1\times P(at|cg)$

$=0.1\times0.2$

$=0.02$

$P(S_0,S_2)=\sum_{S_1}P(S_0,S_1,S_2)$

$=P(S_0,S_1=at,S_2)+P(S_0,S_1=cg,S_2)$

$=0.5\times0.8\times0.8+0.02$

$=0.34$

$P(S_2)=$

$=\sum_{S_0}P(S_0,S_2)$

$=P(S_0=at,S_2)$

$+P(S_0=cg,S_2)$

$=P(at,at,at)$

$+P(at,cg,at)$

$+P(cg,at,at)$

$+P(cg,cg,at)$

$=0.5\times0.8\times0.8+0.5\times0.2\times0.2+0.5\times0.2\times0.8+0.5\times0.8\times0.2$

$=0.5$

$P(S_0)\times P(S_2)=0.02\times0.5\neq0.34$

$\therefore$ $S_0 \not\perp S_2$

$P(s_0 = at)$ $\quad P(s_0 = cg)$

3. $e = TGCACA$ $\quad P(s_1 = at) = P(s_0) \times P(at \mid at) + P(at \mid cg) = 0.5$ $\quad P(s_1 = cg) = 0.5$

$$P(s_1 = at \mid T) = \frac{P(s_1 = at, T)}{P(at)P(T \mid at) + P(cg)P(T \mid cg)} = \frac{0.5 \times 0.3}{0.5 \times 0.3 + 0.5 \times 0.1} = 0.75$$

$$P(s_1 = cg \mid T) = \frac{P(s_1 = at, T)}{\sim} = \frac{0.5 \times 0.1}{\sim} = 0.25$$

$$P(s_2 = at \mid T) = P(s_1 = at \mid T)P(s_2 = at \mid s_1 = at) + P(s_1 = cg \mid T)P(s_2 = at \mid s_1 = cg)$$
$$= 0.75 \times 0.8 + 0.25 \times 0.2 = 0.65$$

$$P(s_2 = cg \mid T) = P(s_1 = cg \mid T)\,P(s_2 = cg \mid s_1 = cg) + P(s_1 = cg \mid T)P(s_2 = cg \mid s_1 = at)$$
$$= \cancel{0.5 \times 0.} \; 0.25 \times 0.8 + 0.75 \times 0.2 = 0.35$$

$$P(s_2 = at \mid TG) = \cancel{P(s_2 = at \mid TG) = P(s_2 = at \mid T)}$$

$$= \frac{P(s_2 = at \mid T)P(G \mid at)}{P(s_2 = at \mid T)P(G \mid at) + P(s_2 = cg \mid T)\,P(G \mid cg)}$$

$$= \frac{0.65 \times 0.2}{0.65 \times 0.2 + 0.35 \times 0.4} \quad \cancel{\frac{4}{9} \approx 0.4815} \quad \approx 0.4815$$

$$P(s_2 = cg \mid TG) = \frac{P(s_2 = cg \mid T)P(G \mid cg)}{0.65 \times 0.2 + 0.35 \times 0.4} \approx 0.5185$$

$$P(s_3 = at \mid TG) = P(s_2 = at \mid TG)\,\cancel{P(s_3 \mid s_2)}\,P(s_3 = at \mid s_2 = at)$$
$$+ P(s_2 = cg \mid TG)\cancel{P(s_3 \mid s_2)}\,P(s_3 = at \mid s_2 = cg)$$
$$= 0.4815 \times 0.8 + 0.5185 \times 0.2$$
$$= 0.4889$$

$$P(s_3 = cg \mid TG) = 0.4815 \times 0.2 + 0.5185 \times 0.8 = 0.5111$$

$$P(S_3=at \mid TGC) = \frac{0.4889\times 0.2}{0.4889\times 0.2+0.5111\times 0.4} = \cancel{0.324401}\ 0.323539$$

$$P(S_3=cg \mid TGC) = 0.676461$$

$$P(S_4=at \mid TGC) = 0.323539\times 0.8+0.676461\times 0.2 = 0.394123$$

$$P(S_4=cg \mid TGC) = 0.323539\times 0.2+0.676461\times 0.8 = 0.605877$$

$$P(S_4=at \mid TGCA) = \cancel{0.245426}\ \frac{0.394123\times 0.3}{0.394123\times 0.3+0.605877\times 0.1} = 0.642218$$

$$P(S_4=cg \mid TGCA) = \cancel{0.754574}\ \frac{0.605872\times 0.1}{0.394123\times 0.3+0.605872\times 0.1} = \cancel{0.642218}\ 0.35779$$

$$P(S_5=at \mid TGCA) = \cancel{0.347256}\ 0.642218\times 0.8+0.35779\times 0.2 = 0.585326$$

$$P(S_5=cg \mid TGCA) = \cancel{0.652744}\ 0.642215\times 0.2+0.35779\times 0.8 = 0.414674$$

$$P(S_5=at \mid TGCAC) = \cancel{0.210108}\ \frac{0.585326\times 0.2}{0.585326\times 0.2+0.414674\times 0.4} = \cancel{0.413715}\ 0.413753$$

$$P(S_5=cg \mid TGCAC) = \cancel{0.789892}\ \frac{0.414674\times 0.4}{0.585326\times 0.2+0.414674\times 0.4} = 0.586247$$

$$P(S_6=at \mid TGCAC) = \cancel{0.321010}\ \cancel{0.352065}\ 0.413753\times 0.8+0.58624\times 0.2 = 0.448252$$

$$P(S_6=cg \mid TGCAC) = \cancel{0.647935}\ 0.551748$$

$$P(S_6=at \mid TGCACA) = \cancel{0.197329}\ \frac{0.448252\times 0.3}{0.448252\times 0.3+0.551748\times 0.1} = 0.709071$$

$$P(S_6=cg \mid TGCACA) = \cancel{0.302671}\ 0.290929$$