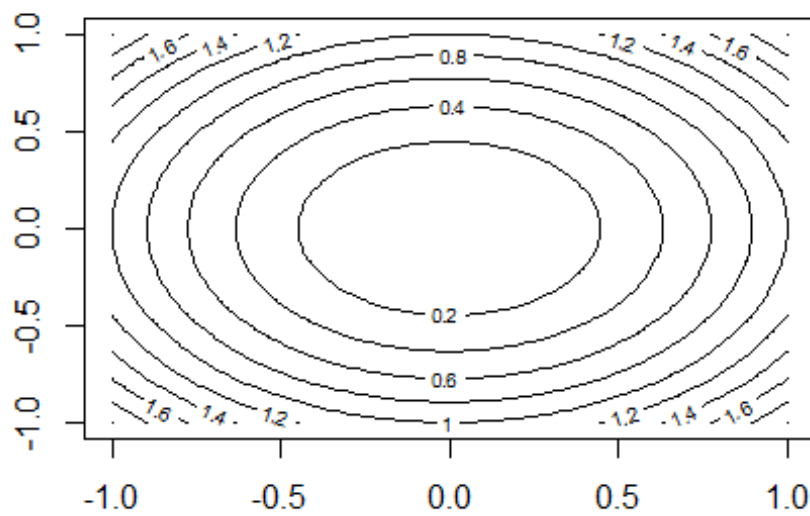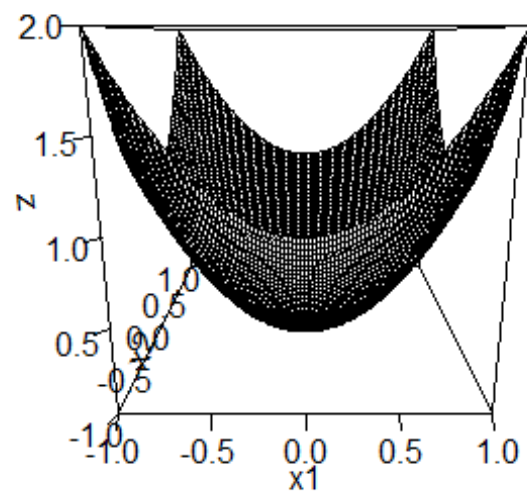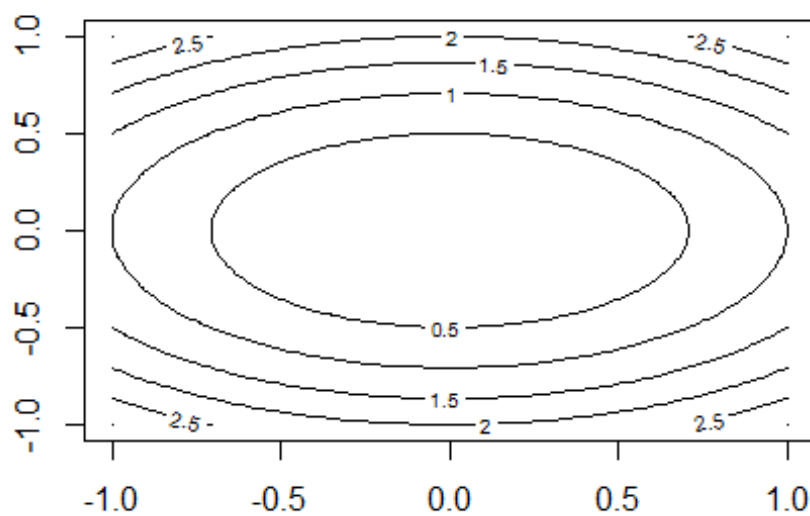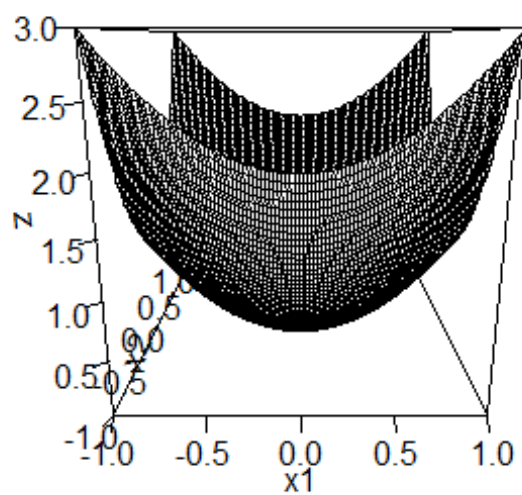# AMS Assignment 2

Xiao Changrong
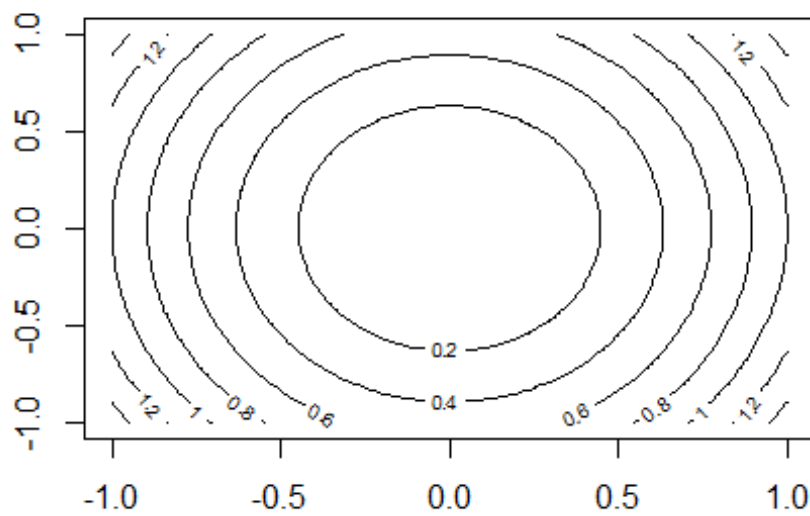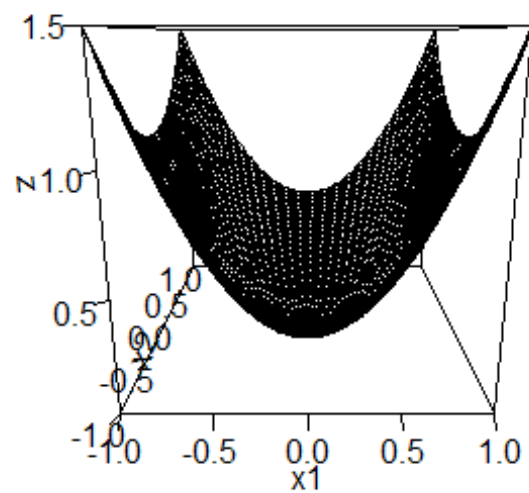
September 20, 2019

```r
# Assignment 2
# Exercise 3
draw_graphs = function(A) {
    f = function(x1, x2) {
        A[1, 1] * x1 ^ 2 + (A[2, 1] + A[1, 2]) * x1 * x2 + A[2, 2] * x2
 ^ 2
    }
    z = outer(x1, x2, f)
    persp(x1, x2, z, ticktype = "detailed")
    contour(x1, x2, z)
}
x1 = seq(-1, 1, le = 100)
x2 = x1
# i
A1 = matrix(c(1, 0, 0, 1), 2, 2)
draw_graphs(A1)
```
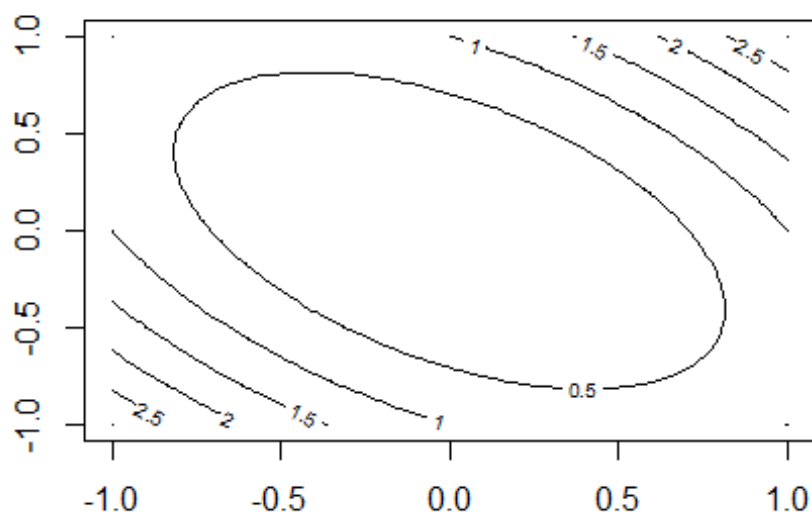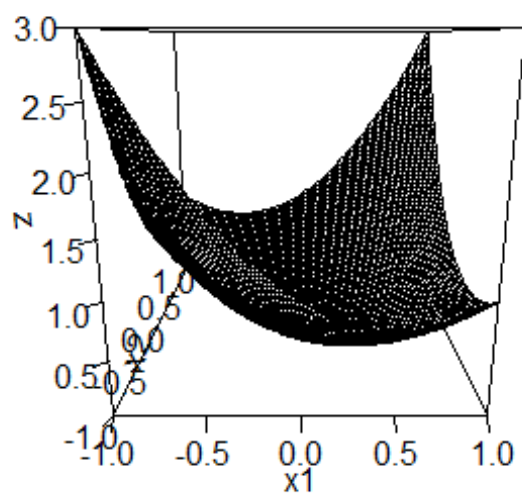
```
# ii
A2 = matrix(c(1, 0, 0, 2), 2, 2)
draw_graphs(A2)
```
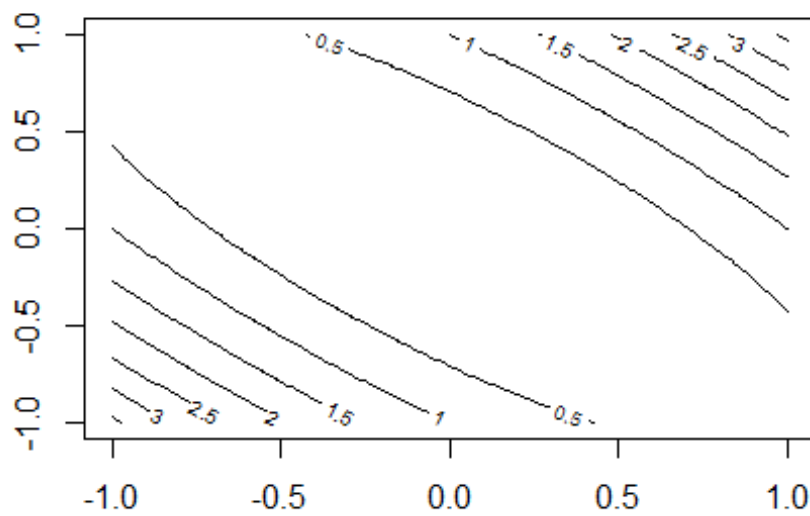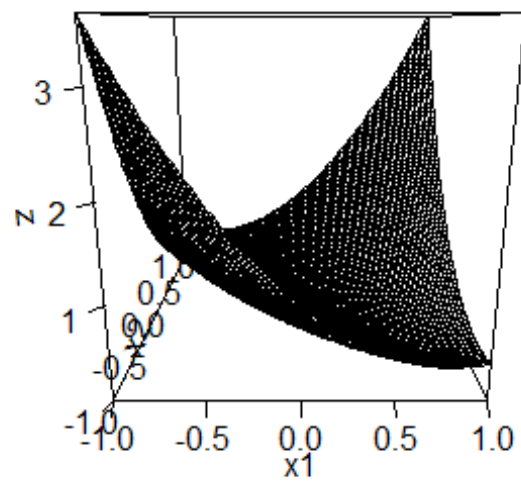
```
# iii
A3 = matrix(c(1, 0, 0, 0.5), 2, 2)
draw_graphs(A3)
```

```
# iv
A4 = matrix(c(1, 0.5, 0.5, 1), 2, 2)
draw_graphs(A4)
```
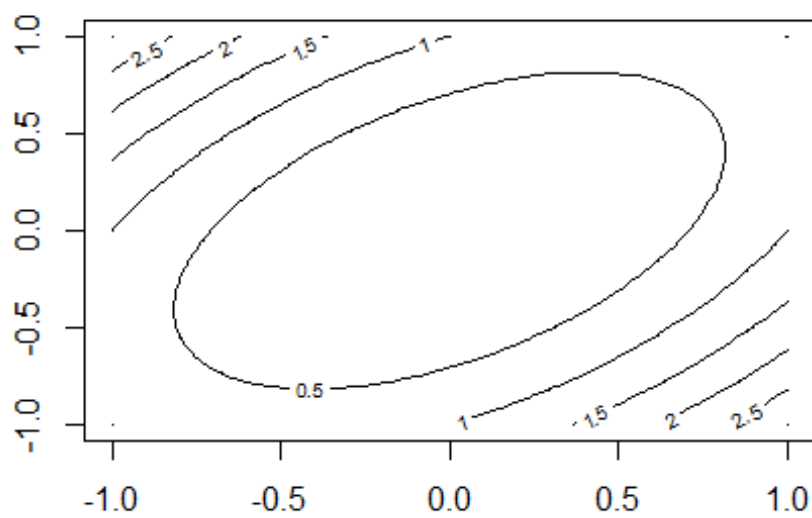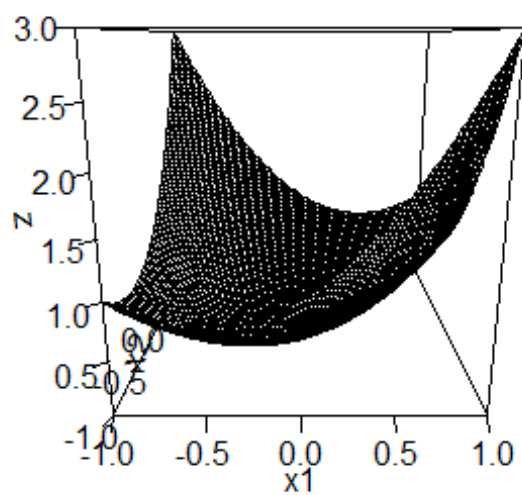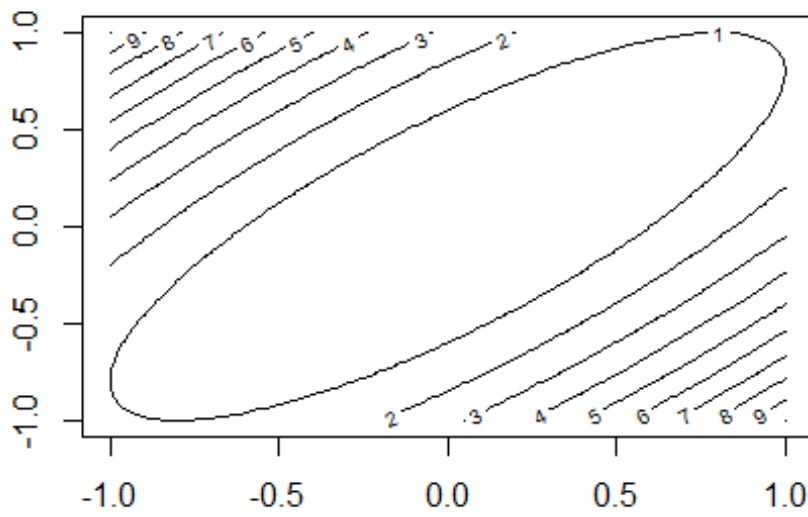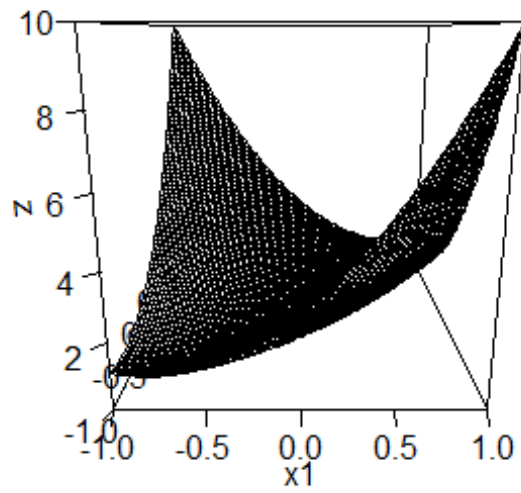
```
# v
A5 = matrix(c(1, 0.8, 0.8, 1), 2, 2)
draw_graphs(A5)
```
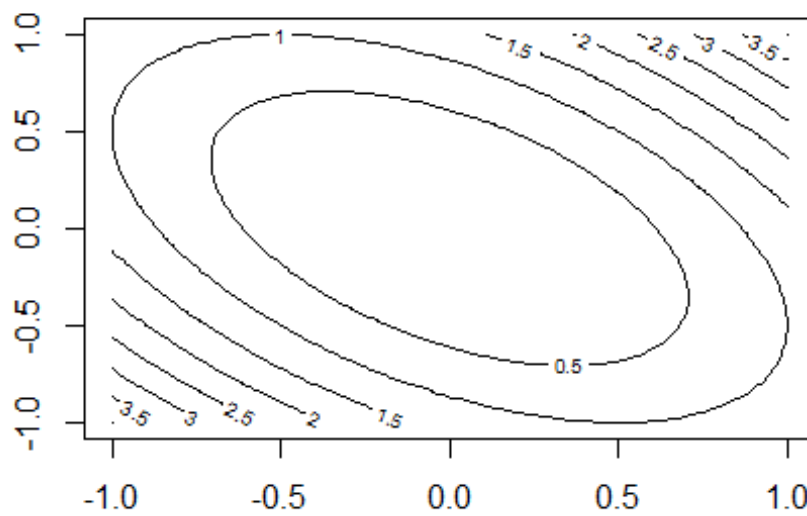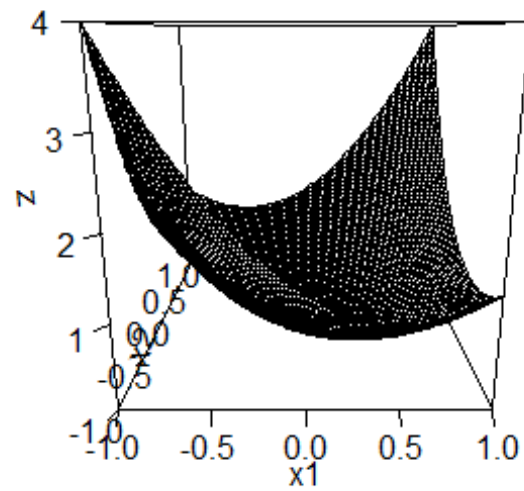
```
# vi
A6 = matrix(c(1, -0.5, -0.5, 1), 2, 2)
draw_graphs(A6)
```

```
# vii
A7 = solve(A5)
draw_graphs(A7)
```

```
# viii
A8 = solve(A6)
draw_graphs(A8)
```

```
# Exercise 5
draw_contour_picture = function(A, c2) {
    f = function(x1, x2) {
        A[1, 1] * x1 ^ 2 + (A[2, 1] + A[1, 2]) * x1 * x2 + A[2, 2] * x2
```

```
  ^ 2
    }
    z = outer(x1, x2, f)
    contour(x1, x2, z, levels = c2, asp = 1)
    len1 = sqrt(c2 / eigen(A)$values[1])
    len2 = sqrt(c2 / eigen(A)$values[2])
    arrows(0, 0, len1 * eigen(A)$vectors[1, 1], len1 * eigen(A)$vectors
[2, 1], length = 0.1, asp = 1)
    arrows(0, 0, len2 * eigen(A)$vectors[1, 2], len2 * eigen(A)$vectors
[2, 2], length = 0.1, asp = 1)
}
x1 = seq(-1.5, 1.5, le = 100)
x2 = x1
A = matrix(c(5, 4, 4, 5), 2, 2)
draw_contour_picture(A, 2)
```
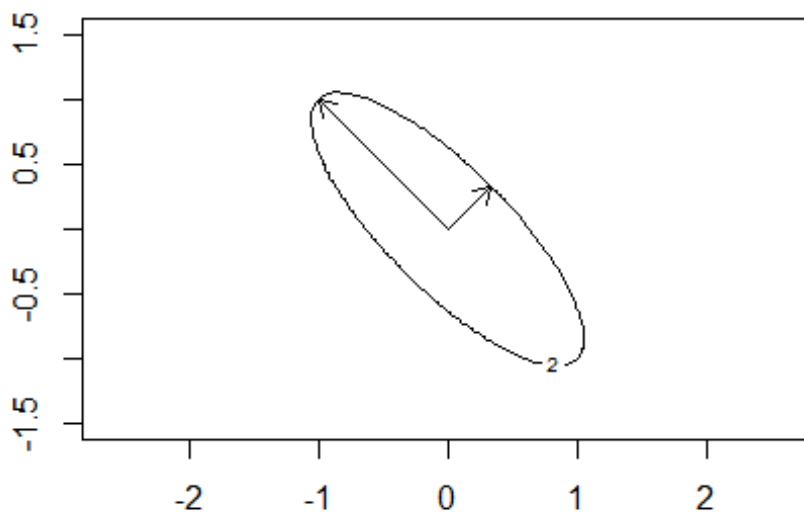


```
# Exercise 6
A = matrix(c(13, -4, 2, -4, 13, -2, 2, -2, 10), 3, 3)
# a
E = eigen(A)
e = E$vectors
lambda = E$values
a = diag(lambda)
print(e)

##              [,1]        [,2]       [,3]
## [1,]   0.6666667 -0.7453560 0.0000000
```

```
## [2,] -0.6666667 -0.5962848 0.4472136
## [3,]  0.3333333  0.2981424 0.8944272
```

```r
print(a)
```

```
##      [,1] [,2] [,3]
## [1,]   18    0    0
## [2,]    0    9    0
## [3,]    0    0    9
```

```r
# b
f = function(i) {
    lambda[i] * e[, i] %o% e[, i]
}
# i
print(f(1))
```

```
##      [,1] [,2] [,3]
## [1,]    8   -8    4
## [2,]   -8    8   -4
## [3,]    4   -4    2
```

```r
# ii
print(f(1) + f(2))
```

```
##      [,1] [,2] [,3]
## [1,]   13 -4.0  2.0
## [2,]   -4 11.2 -5.6
## [3,]    2 -5.6  2.8
```

```r
# iii
print(f(1) + f(2) + f(3))
```

```
##      [,1] [,2] [,3]
## [1,]   13   -4    2
## [2,]   -4   13   -2
## [3,]    2   -2   10
```

```r
# c
print(e %*% sqrt(a) %*% t(e))
```

```
##             [,1]       [,2]       [,3]
## [1,]  3.5522847 -0.5522847  0.2761424
## [2,] -0.5522847  3.5522847 -0.2761424
## [3,]  0.2761424 -0.2761424  3.1380712
```