AMS Assignment 3

Xiao Changrong

September 28, 2019

```
# Assignment 3
# Exercise 1
A = matrix(c(13, -4, 2, -4, 13, -2, 2, -2, 10), 3, 3)
C = cov2cor(A)
# a
print(C)
```

```
##            [,1]       [,2]       [,3]
## [1,]  1.0000000 -0.3076923  0.1754116
## [2,] -0.3076923  1.0000000 -0.1754116
## [3,]  0.1754116 -0.1754116  1.0000000
```

```
# b
print(eigen(C)$values)
```

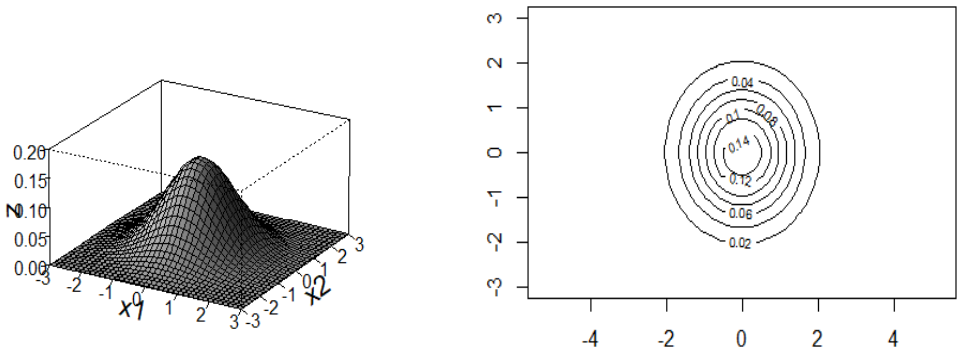
```
## [1] 1.4457487 0.8619436 0.6923077
```

```
print(eigen(C)$vectors)
```

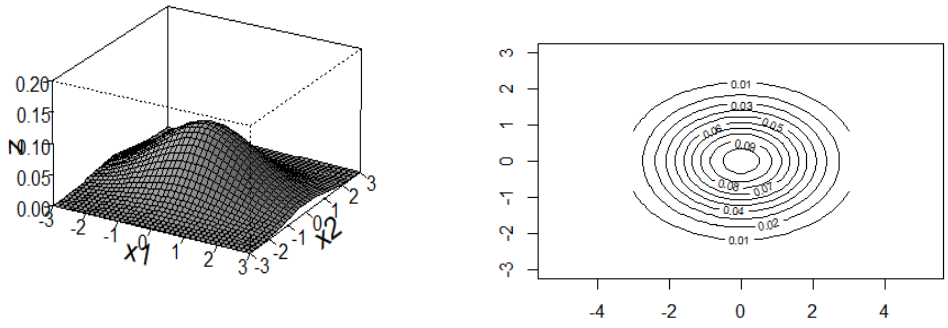
```
##            [,1]       [,2]          [,3]
## [1,]  0.6178686  0.3438582  7.071068e-01
## [2,] -0.6178686 -0.3438582  7.071068e-01
## [3,]  0.4862889 -0.8737981 -1.042297e-16
```

```
# Exercise 2
x1 = seq(-3, 3, length = 40)
x2 = x1
draw_distribution = function(mu1, mu2, sqr_sig11, sqr_sig22, rho12) {
    bivariate_normal = function(x1, x2) {
        1 / (2 * pi * sqr_sig11 * sqr_sig22 * sqrt(1 - rho12 ^ 2)) * ex
p(-1 / (2 * (1 - rho12 ^ 2)) * ((x1 - mu1) ^ 2 / (sqr_sig11 ^ 2) - 2 *
rho12 * (x1 - mu1) / sqr_sig11 * (x2 - mu2) / sqr_sig22 + (x2 - mu2) ^
2 / (sqr_sig22 ^ 2)))
    }
    z = outer(x1, x2, bivariate_normal)

    persp(x1, x2, z, main = "",
      cex.lab = 1.5, theta = 30, phi = 20, r = 50,
      d = 0.1, expand = 0.5, ltheta = 90, lphi = 180,
      shade = 0.75, ticktype = "detailed", nticks = 5, xlim = c(-3, 3),
      ylim = c(-3, 3), zlim = c(0, 0.2), asp = 1)
    contour(x1, x2, z, asp = 1)
}
```

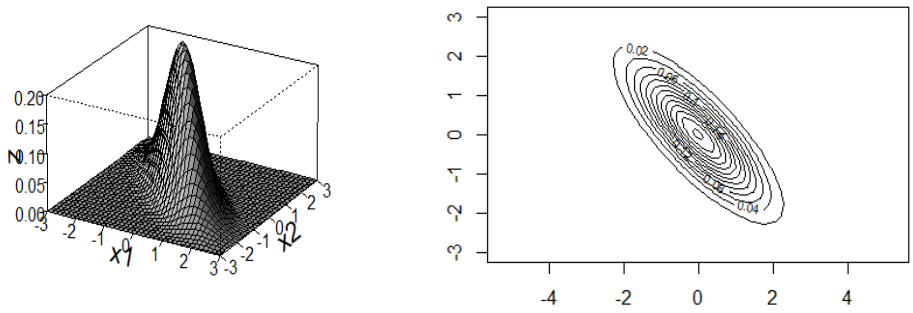
```r
# i
draw_distribution(0, 0, 1, 1, 0)
```



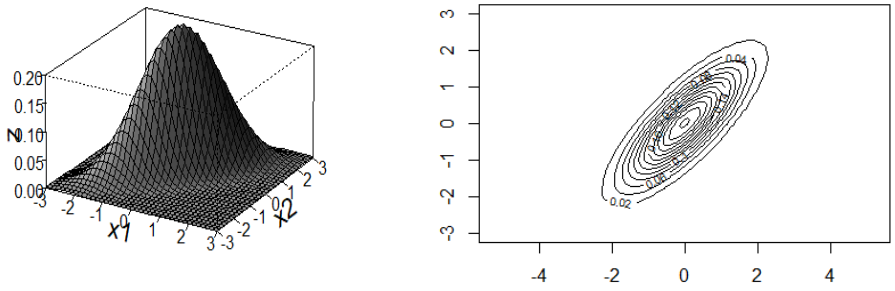

```r
# ii
draw_distribution(0, 0, 1.5, 1, 0)
```




```r
# iii
draw_distribution(0, 0, 1, 1, -0.8)
```




```
# iv
draw_distribution(0, 0, 1, 1, 0.8)
```




```
# Exercise 4
# a
mu = c(1, 0, -1, 2)
Sigma = matrix(c(1, 0.2, 0.4, -0.5, 0.2, 2, 0.8, 0, 0.4, 0.8, 2, 0, -0.
5, 0, 0, 1), 4, 4)
f = function(x) {
    1 / ((2 * pi) ^ 2 * (det(Sigma) ^ 0.5)) * exp(-0.5 * t(x - mu) %*%
solve(Sigma) %*% (x - mu))
}
# i
x1 = c(0, 0, 0, 0)
print(f(x1))

##              [,1]
## [1,] 4.705344e-05

# ii
x2 = c(1, 1, 1, 1)
print(f(x2))
```

```
##             [,1]
## [1,] 0.00202044
```

```r
# iii
x3 = c(1, 0, 1, 0)
print(f(x3))
```

```
##              [,1]
## [1,] 0.0001671811
```

```r
# b
f_chi = function(chi) {
    1 / ((2 * pi) ^ 2 * (det(Sigma) ^ 0.5)) * exp(-0.5 * chi)
}
# i
print(f_chi(qchisq(0.95, 4)))
```

```
## [1] 0.0001470645
```

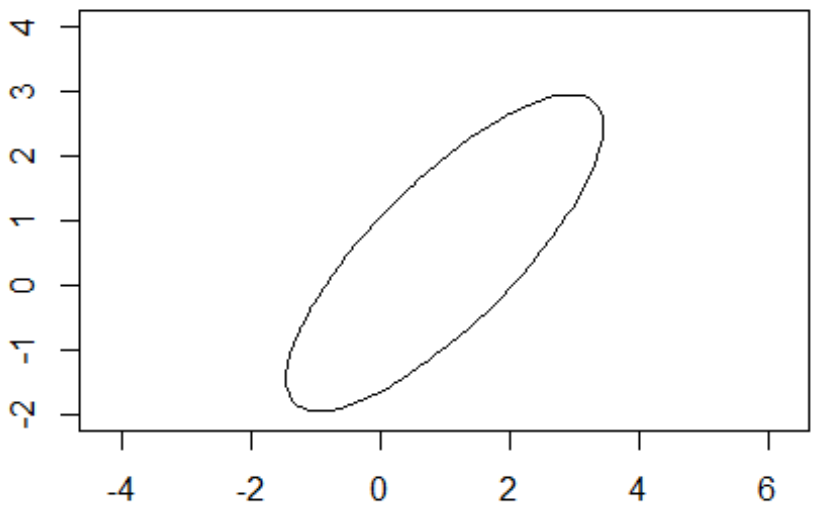
```r
# ii
print(f_chi(qchisq(0.9, 4)))
```

```
## [1] 0.000345508
```

```r
# iii
print(f_chi(qchisq(0.8, 4)))
```

```
## [1] 0.0008459224
```

```r
# Exercise 5
mu = c(1, 0.5)
Sigma = matrix(c(1, 0.8, 0.8, 1), 2, 2)

x1 = seq(-2, 4, length = 40)
x2 = x1
# a
f = function(x1, x2) {
    1 / (2 * pi * sqrt(1 - 0.8 ^ 2)) * exp(-1 / (2 * (1 - 0.8 ^ 2)) *
((x1 - 1) ^ 2 - 2 * 0.8 * (x1 - 1) * (x2 - 0.5) + (x2 - 0.5) ^ 2))
}
z = outer(x1, x2, f)
level = 1 / (2 * pi * sqrt(1 - 0.8 ^ 2)) * exp(-0.5 * qchisq(0.95, 2))
contour(x1, x2, z, levels = level, asp = 1, drawlabels = FALSE)
```




```r
# b
c2 = qchisq(0.95, 2)
print(c2)
```

```
## [1] 5.991465
```

```r
# c
len1 = sqrt(c2 / eigen(solve(Sigma))$values[1])
len2 = sqrt(c2 / eigen(solve(Sigma))$values[2])
print(len1)
```

```
## [1] 1.094666
```

```r
print(len2)
```

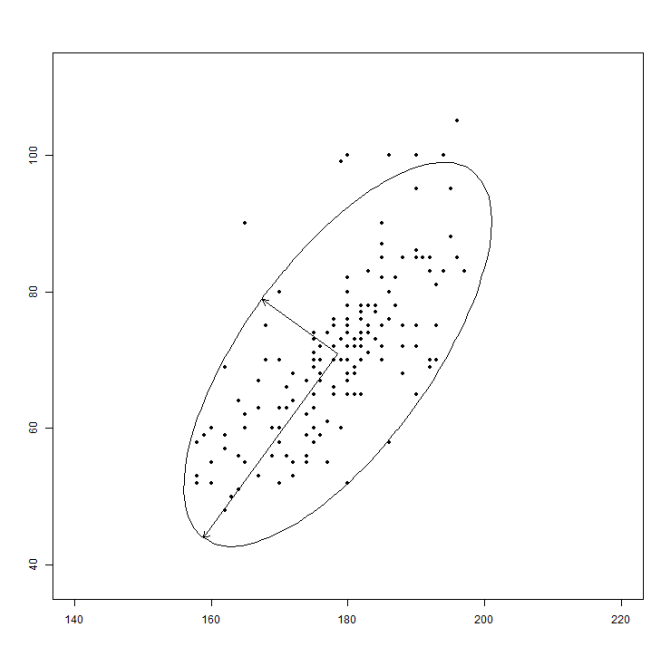
```
## [1] 3.283997
```

Exercise 7

```r
students2008 = read.table(file = "students2008.txt", header = T, dec = ",")

attach(students2008)

heightweight = data.frame(height, weight)

heightweight = na.omit(heightweight)
```

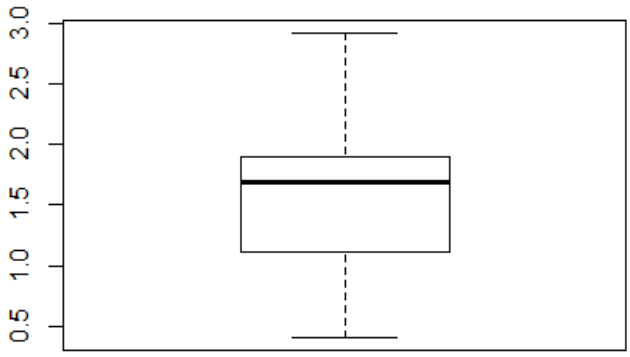
```
detach(students2008)

attach(heightweight)

# a

X = cbind(height, weight)

Sigma = cov(X)

mu = apply(X, 2, mean)


x1 = seq(140, 220, length = 40)

x2 = seq(40, 110, length = 40)


f = function(x1, x2) {

   rho12 = (Sigma[1, 2] / sqrt(Sigma[1, 1] * Sigma[2, 2]))

   1 / (2 * pi * sqrt(det(Sigma))) * exp(-1 / (2 * (1 - rho12 ^ 2)) * ((x1 - mu[1]) ^ 2 / (Sigma[1, 1]) - 2 * rho12 * (x1 - mu[1]) / sqrt(Sigma[1, 1]) * (x2 - mu[2]) / sqrt(Sigma[2, 2]) + (x2 - mu[2]) ^ 2 / (Sigma[2, 2])))

}

z = outer(x1, x2, f)

level = 1 / (2 * pi * sqrt(det(Sigma))) * exp(-0.5 * qchisq(0.95, 2))

contour(x1, x2, z, levels = level, asp = 1, drawlabels = FALSE)

# b

points(height, weight, pch = 20, cex = 0.5, asp = 1)


c2 = qchisq(0.95, 2)

A = solve(Sigma)


len1 = sqrt(c2 / eigen(A)$values[1])

len2 = sqrt(c2 / eigen(A)$values[2])
```

```
arrows(mu[1], mu[2],
      mu[1] + len1 * eigen(A)$vectors[1, 1], mu[2] + len1 * eigen(A)$vectors[2, 1],
      length = 0.1, asp = 1)
arrows(mu[1], mu[2],
      mu[1] + len2 * eigen(A)$vectors[1, 2], mu[2] + len2 * eigen(A)$vectors[2, 2],
      length = 0.1, asp = 1)
```

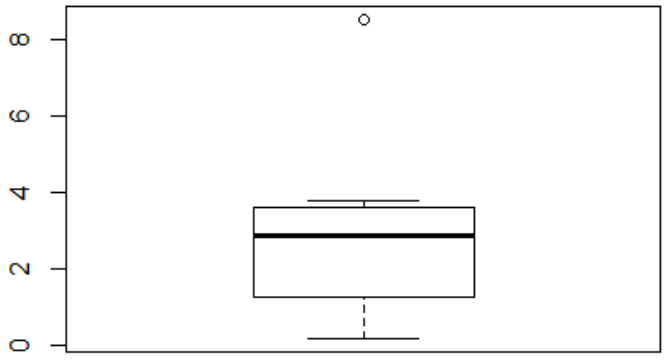



```
# c
count = 0
for (i in mahalanobis(X, mu, Sigma)) {
   if (i < c2)
      count = count + 1
}
ratio = count / length(height)
print(ratio)
##  [1] 0.9433962
```

```r
# Exercise 8
X = read.table(file = "outlier3dim.txt", header = T, dec = ",")
mu = apply(X, 2, mean)
Sigma = cov(X)
# a
d2 = mahalanobis(X, mu, Sigma)
d1 = sqrt(d2)
# i
boxplot(d1)
```




```r
# ii
boxplot(d2)
```



```r
# b
print(max(d1))

## [1] 2.917057

max_index = which(max(d1) == d1)
print(max_index)

## [1] 10

# c
y = rep(1, 16)
y[max_index] = 2
Y = cbind(X, y)

pairs(Y[1:3], pch = 20, asp = 1, col = c("black", "red")[unclass(Y$y)])
```