MC-ODG:Multi-category oversampling based on DBSCAN and Gaussian distribution*

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Abstract. Classification on imbalanced data is one of the most challenging classification tasks in machine learning. In the predicting task, the imbalanced distribution of data leads to the poor prediction ability for minority classes. According to the number of different classes, the classification problem can be divided into binary classification problems and multiclassification problems. Due to the complex relationship between different classes, the difficulity of multi-class problems on imbalance data is much greater. Many scholars have proposed disirable oversampling methods to solve the problem of classification on imbalance data, such as SMOTE and its derivatives. But these methods are easily affected by noise and overlapping data distributions, and cannot handle clusters form data. Therefore we propose new oversampling methods called ODG and MC-ODG, In these methods, DBSCAN and gaussian distribution were applied to generate new sample points. The experiment results show that our methods has high prediction ability compared to state-of-art methods.

Keywords: imbalance data · oversampling · DBSCAN · Gaussian distribution.

1 Introduction

Imbalanced data [6] refers to data sets that have an uneven amount of data between different classes. In the binary classification problem, we assume that the majority class are negative class and the minority class are positive class. The actual demand shows the importance and difficulty of learning on imbalanced data distributions. The imbalance occurs in many fields, such as medical diagnosis [30,36] and fault detection [41]. disease samples in medical diagnosis and fault samples in fault detection are minority samples. Although the sample size of minority class is small, we tend to be interested in minority samples because minority samples tend to contain more value. The disease samples need to be predicted more accurately than the normal samples, and the cost of misjudging fault samples is much higher than misjudging normal samples. The difficulty in solving imbalanced data sets lies in the fact that existing machine learning models tend to skew the prediction results towards majority class in the process of prediction [24], then the minority class cannot be classified correctly [3]. Therefore, More advanced algorithms need to be proposed to better prediction of minority samples.

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The methods dealing with imbalanced data sets can be roughly divided into two types: algorithm-level [29] and data-level [7]. Algorithm-based approaches include BalanceCascade [33], Adaboost [14], XGBoost [9], and more. But algorithm-based methods are limited to a single type of classifier [20] and cannot be extended to more general machine learning models. The data-level methods perform data preprocessing with the aim of reducing the imbalance ratio. Therefore the data can be processed by general machine learning models. Data-level methods are generally divided into undersampling [31] and oversampling [7]. undersampling decrease the number of majority samples,the price is losing part of the information, while oversampling enhances minority class by introducing new samples [18]. In our work, oversampling is used to preprocess the data.

Data-level methods address the imbalance by changing the data distributions and can be applied with general mechine learning methods. SMOTE is an improved method based on the random oversampling algorithm, which realizes oversampling by randomly generating new sample points on the line of minority sample points. Numerous modifications of SMOTE algorithm have been proposed in the literature, such as Borderline-SMOTE [15], ADASYN [16]. SMOTE and its derivatives have their weaknesses, Methods based on SMOTE are susceptible to noise points and are difficult to deal with overlapping. The generated minority points and majority points are covered, which affects the separating hyperplane of the classifier. In addition, $SMOTE$ method is also poor in the process of cluster data. SMOTE is unable to capture the cluster distributions if the monority sample points are distributed in clusters. Furthermore, the method of syntheticing new points by SMOTE is relatively single, so the generated data points are difficult to maintain the original data distributions [10]. After using $SMOTE$ to generate new data, the data distributions of minority sample points may change, which will affect the prediction progress.

SMOTE and its derivatives are less effective when dealing with multiple classification problems [5,20].The relationship between two classes is relatively simple, but in multi-classification problems, the relationship between classes becomes much more complex [4]. Especially when dealing with sample points near the multiple category boundaries, Create overlapping data distributions [17] is hard to avoid. The existing methods dealing with multi-classification include transforming the multi-classification problem into binary classification problems(OVO) [12], But processing two classes separately will ignore the rest classes' information. Multiple classification problems can also be divided into one-to-many problems(OVA) instead, Treat one class as a minority class and the rest as a majority class. However, this does not capture the relationships between majority classes effectively. New approach should to be adaptive to multi-classification problems.

If minority sample points are distributed in clusters, we could use cluster methods. Clustering algorithm [34,38] divides the same set of data into multiple sets, and the data in each set is similar. We can use the clustering algorithm to divide the data into multiple clusters and oversample data in each cluster. There are many kinds of clustering algorithms, among which the most famous one is K-MEANS [2] algorithm. However, K-MEANS algorithm is a distance-based algorithm. When there are irregular clusters, K-MEANS algorithm is difficult to handle, meanwhile the parameter K is difficult to set. Therefore, We use DBSCAN [25] algorithm based on density.

Our work is to propose a new oversampling method. The above mentioned methods is sensitive to noise, easy to produce overlapping and unable to deal effectively with cluster distribution data. To solve these problems, we proposed ODG and MC-ODG algorithms. The proposed algorithm can solve the above problems while the new sample points can retain the original data distributions. The ODG algorithm can be adaptive to determine how to generate new sample points and how many sample points are generated around what kinds of points. The new algorithms apply DBSCAN

clustering method and Gaussian distributions [35] to fit the data. The experiments resuls show that the proposed algorithms are superior to the existing oversampling methods on different metrics.

To summarize, our work makes the following contributions.

- ODG is an oversampling method that can effectively process clustered data.
- We explain the influence of noise data points on the oversampling process and ODG shows how to deal with noise data points.
- ODG preserves the original data distributions after oversampling.
- ODG is suitable for binary classification, and its extension MC-ODG is suitable for multi-classification problems.

The paper is organized as follows.The next section introduces the knowledge required for our ODG and MC-ODG algorithms, as well as the existing oversampling methods. Section 3 introduces our ODG and MC-ODG algorithms in details, and gives the specific flow of the algorithms and the corresponding complexity analysis. In section 4, we introduce a series of comparative experiments based on different models, and the analysis and results of the experiments are given. Section 5 gives the summary and our future works.

2 Related work

In this section, we propose the required knowledge and some related over-sampling methods dealing with imbalanced data.

2.1 Clustering method

Clustering algorithm is to find groups of similar data, and each group of similar data forms a cluster [34]. There may be multiple clusters in a single minority class data, we need to use clustering algorithm to identify multiple clusters and then process each of them. The most famous clustering methods include distance-based K-MEANS algorithm and density-based DBSCAN algorithm. K-MEANS algorithm is relatively easy, but the algorithm must determine the number of clusters K. Meanwhile, compared with density-based DBSCAN algorithm, K-MEANS cannot find clusters with various shapes. DBSCAN defines a cluster as the largest set of points connected by density and can find clusters of arbitrary shape. After the whole process, the algorithm classifies the sample points into core points, borderline points and noise points. Given the parameters eps and min_pts, we will get three kinds of points. Core points have no less than min_pts points in the eps neighborhood. Borderline points refer to that were partitioned into a cluster after DBSCAN, but not belong to core points. After the DBSCAN algorithm, The points were not divided into any cluster defined as noise points. The core points can best represent the distribution of the corresponding cluster.

2.2 Algorithm-level methods

Algorithm-level methods are based on proposing new models to solve the problem of imbalanced data prediction [9, 40]. Algorithm-level methods can be broadly divided into cost-sensitive methods [27] and ensemble methods [8]. The cost-sensitive methods change the weights or loses to improve the performance of classifier, such as adaboost [14], which is a lifting algorithm, and the weights of the minority samples misclassfied will gradually increase. Ensemble classifier improves the learning

result by combining a group of basic classifiers together. The classification of ensemble methods are Boosting [13] and Bagging [23]. Boosting methods include SMOTEBoost [8], XGBoost [9], LogitBoost [40], and so on. Although some algorithm-level approaches have good classification ability, algorithm-level methods are limited to single-type classifiers [20]. This kind of solutions cannot be extended to more general machine learning models.

2.3 Data-level methods

unsersampling methods [6, 32] remove majority points so as to make the data balanced. Famous undersampling methods include random undersampling [41], *EasyEnsemble* [1] and $NearMiss$ [38]. *BalanceCascade* [33] mentioned earlier also adopts the idea of undersampling. But some information will be lost when majority points are discarded by undersampling. Oversampling, on the other hand, balances the distribution of data by generating new data points without losing information.

The fundamental idea of SMOTE is to generate new sample by interpolation between minority class samples [11]. Specifically, for a certain sample x_i, one of the K nearest neighbors is randomly selected as $x_i^{'}$. Use the equation1 to generate a new sample x_{new}. $\epsilon \in [0, 1]$ is a random number.

$$x_{new} = x_i + (x_i - x_i^{'}) \times \epsilon \tag{1}$$

We can easily find the weakness of SMOTE, Using the algorithm of SMOTE, Generating new sample points inside the minority sample points is easy, but new points are far from the classification surface, which has little influence on the classification prediction. SMOTE algorithm is easily affected by noise, the new synthetic samples may overlap with majority class samples. Borderline-SMOTE [15] is based on SMOTE, this algorithm classifies the minority points into three types, *noise*, *danger* and $safe$. All the K nearest neighbor samples of the *noise* class are majority points, more than half of the K nearest neighbor samples of the *danger* points are majority points, and all of K nearest neighbor samples of $safe$ points are minority points. Borderline-SMOTE uses *danger* points to generate new points. To a certain extent Borderline-SMOTE reduced the negative effect of noise and overlapping data distribution. ADASYN [16] is an adaptive composite sampling method, a mechanism is used to determine how many samples need to be synthesized for each minority sample point. In general, for a minority sample point, more majority sample points in its K nearest neighbors, the more minority class sample points will be generated near this point. There are also methods conbined oversampling and undersampling, such as SMOTE+ENN [36], This method first deleted majority points ,of which all of K nearest neighbors are minority points. Then use SMOTE to generate new samples.

The way of generating new data in SMOTE is too simple, Simply generating data on the line does not effectively preserve the original data distribution [20, 21]. In addition, for the problem of multiple classification, methods based on SMOTE could not make use of the relationship between multiple classes during the process of generation and expected effect cannot be achieved. CCR and MC-CCR algorithms are oversampling methods based on *energy*, In these methods, a spherical region is drawn at the center of each sample point of a minority class. The spherical region is constantly expanding, when meeting a majority point the energy is reduced. When the *energy* is exhausted, the region will no longer extend. New data points are randomly generated inside the spherical region. The number of generated data is inversely proportional to the radius of the spherical region. In addition, this method also proposed *cleaning* process, the majority points inside the spherical area will be shifted out to reduce overlapping. But the CCR approach also has its

drawbacks. In the process of generating data points, the CCR method generates data randomly in the region without considering the distribution of data points. In addition, the CCR method may synthetic too many sample points near the noise points. As the radius of the spherical region corresponding to a noise point is small, many sample points will be generated, which may affect the prediction results. However, the proposed algorithm can identify the noise of minority points and avoid generating data around the noise.

Algorithm 1 ODG

Input:
$M \leftarrow$ majority points
$m \leftarrow$ minority points
$N_{maj} \leftarrow$ number of sample points for majority class
$N_{min} \leftarrow$ number of sample points for minority class
$\alpha_{borderline} \leftarrow$ the ratio of samples generated by borderline samples
$noise\_ratio \leftarrow$ ratio threshold of noise sample points
$eps \leftarrow$ the parameters of DBSCAN
$min\_pts \leftarrow$ the parameters of DBSCAN

Output: Minority points and Majority points

m_{core},$m_{borderline}$,m_{noise}=DBSCAN(m,eps,min_pts)
$N_{oversampling}$=N_{maj}-N_{min}
if $len(m_{noise})/N_{min} < noise\_ratio$ **then**
    N_{noise}=0
else
    ratio=$len(m_{noise})/N_{min} < noise\_ratio$
    $\alpha_{noise} = \frac{0.9}{1-noise\_ratio^2} \times ratio^2 + 1 - \frac{0.9}{1-noise\_ratio^2}$
    $N_{noise} = \alpha_{noise} \times N_{oversampling}$
    $N_{noise} = min(multiple\_k \times K \times len(m_{noise}), N_{noise})$
$N_{oversampling}$=$N_{oversampling}$-N_{noise}
$N_{borderline}$=$N_{oversampling} \times \alpha_{borderline}$
N_{core}=$N_{oversampling}$-$N_{borderline}$
clusters $\leftarrow$ clustering of minority points
$cov\_mats \leftarrow$ Calculate the variance of the core points of each cluster,scale according to the number of core points
$new\_data \leftarrow$ an empty set.
$translations \leftarrow$ a zero matrix, the same shape as M.
$generate\_borderline(m_{boaderline}, N_{borderline}, translations, core\_points, new\_data, cov\_mats$,multiple_k)
$generate\_core(clusters, N_{core}, new\_data)$
$generate\_noise(N_{noise}, m_{noise}$,new_data)
$M \leftarrow M + translations$
$m \leftarrow m$ concatenate with new_data
return M,m

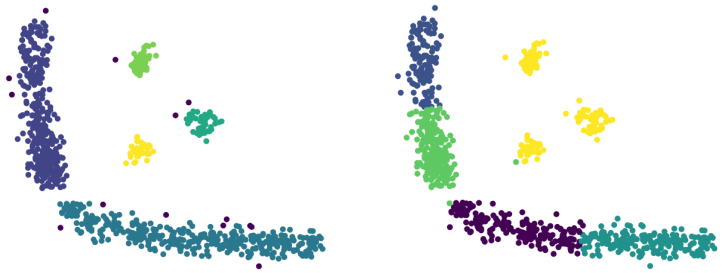
3 ODG Algorithm and MC-ODG Algorithm

In this section we will introduce ODG algorithm and MC-ODG algorithm. The methods based on SMOTE are not able to effectively deal with noise points and cluster distribution data. The noise in the data will lead to overlapping data distributions after oversampling. Also, the data point generated by SMOTE cannot preserve the original distributions. MC-CCR randomly generates points in a certain range without retaining the original distributions of a minority points. Furthermore, this method may generate too many samples of minority points near the noise, which will adversely affect the prediction results. Aiming at the shortcomings of the above methods, we propose ODG and MC-ODG to avoid the problems mentioned above.

3.1 Binary inbalanced classification

We first give the ODG algorithm, which is used to solve the binary classification problems. The whole ODG algorithm process can be roughly divided into clustering minority sample points and generating new samples near borderline points, core points and noise points respectively. The flow of ODG is shown in the algorithm 1.

Clustering We cluster the minority sample points firstly. The results of K-MEANS and DBSCAN



(a) $K-means$ (b) $DBSCAN$

Fig. 1. The comparision of $K-means$ and $DBSCAN$

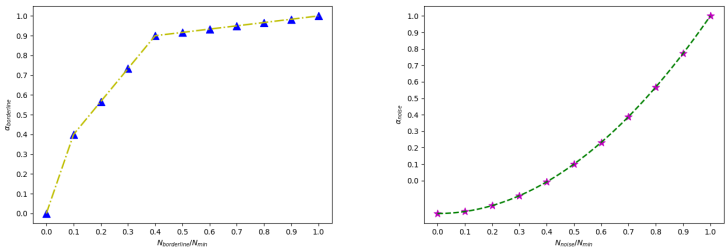
are shown in figure1. The same color represents the same cluster. Obviously, the result of DBSCAN is more consistent with the data distribution. Therefore we use DBSCAN as the clustering method. After DBSCAN, the minority sample points are divided into core points, borderline points and noise points. Next, we need to generate new points around different kinds of points respectively. We assume that the number of majority sample points is N_{maj}, and the number of minority sample points is N_{min}, then the number of oversample points $N_{oversampling}$ is shown as equation 2.

$$N_{oversampling} = N_{maj} - N_{min} \tag{2}$$

After obtaining the total number of oversampled samples, we need to determine the proportion of generated sample points near the core points, borderline points and noise points. Sample points

should not be generated near the noise unless it is impossible to form effective clusters on minority points. The failure to form clusters means that most of the sample points are marked as noise points after the DBSCAN algorithm , which we will deal with specially. In general, N_{noise} is zero. we note the ratio of generated new sample points at borderline points is $\alpha_{borderline}$, Then the number of generated samples $N_{borderline}$ and N_{core} are shown in equation 3.

$$\begin{aligned} N_{borderline} &= N_{oversampling} \times \alpha_{borderline} \\ N_{core} &= N_{oversampling} - N_{borderline} \end{aligned} \tag{3}$$



(a) $\alpha_{borderline}$ (b) α_{noise}

Fig. 2.

The parameter $\alpha_{borderline}$ can be specified as a hyperparameter and can be adaptive based on the ratio of borderline points. In general, $\alpha_{borderline}$ is greater than the ratio of the boundary points because we prefer to generate data near borderline points, then the classification surface is biased to minority points. Figure 2(a) shows the relation with $\alpha_{borderline}$ and $\frac{N_{borderline}}{N_{min}}$

Algorithm 2 *generate_borderline*

Input: $m_{borderline}$,$N_{borderline}$,$translations$,$core\_points$,new_data cov_mats,$multiple_k$

for point in $m_{borderline}$ **do**

Calculate K nearest neighbors of point.

$cov\_mat \leftarrow$ Find the covariance matrix corresponding to the point from cov_mats

$d\_max \leftarrow$ maximum distance of knearest neighbors.

$n_{borderline} = min(\frac{n\_knearest\_maj}{\sum_{i \in minority} n\_knearest\_maj} \times N_{boardline}, multiple\_k \times K)$

$tem\_data = multivariate\_normal(mean(core\_points), cov\_mat, n_{borderline})$

add tem_data to new_data.

for for m_j in $k\_nearest\_maj$ **do**

$translations_j$=$translations_j$+$\frac{d_{max}-d_j}{d_j} \times (m_j - point)$

Generate data near borderline points In order to make the distribution of the generated new sample points consistent with the core points corresponding to the borderline point, gaussian

distribution is adopted. We first need to determine $n_{boardline}$ for a particular borderline point. We calculate the K nearest neighbors of each borderline point. If the K nearest neighbor of the minority point have more majority sample points, means the point is closer to the decision boundary, and more sample points should be generated near this point. We set the number of majority sample points is $n\_knearest\_maj$, then $n_{boardline}$ can be obtained in equation 4.

$$n = \frac{n\_knearest\_maj}{\sum_{i \in minority} n\_knearest\_maj} \times N_{boardline} \tag{4}$$

We need to limit the maximum value of $n_{boardline}$ to avoid generating too many minority samples around one single point because extreme distribution may exist in some of datasets. If too many points are generated near one sample point. the original data distribution is changed and the training of the model is adversely affected. Therefore we limit the value of $n_{boardline}$. We define the parameter $multiple\_k$, Limit the maximum of $n_{borderline}$ to $multiple\_k \times K$, as shown in equation5.

$$n_{boardline} = min(multiple\_k \times k, n_{boaderline}) \tag{5}$$

(a) (b) (c)

Fig. 3. (a)is initial dataset (b) and (c) are oversampling results, (c) has limited the number of generation.

In Figure3,(a) is initial dataset (b) is the data distribution obtained without limiting the number of individual sample points generated (c) limited the number of generation. We can find that too many sample points are generated near one sample point in the lower right of Figure3(b) , thus affecting the overall distribution of minority points. Obviously Figure3(c) is more consistent with the distribution of the original data.

After clustering, each borderline point corresponds to a certain cluster, and the core points of this cluster can best represent the distribution of this cluster. We calculate the covariance matrix corresponding to the core points, then use gaussian distribution to generate sample points at the corresponding borderline points after scaling it. We set the particular borderline point is *point*, *cov* is the function to calculate the covariance matrix, and the number of core points is num_core_points, The function to generate a new sample point using gaussian distribution is $multivariate\_normal$, New sample points new_data can be obtained by equation6.

$$\begin{aligned} cov\_mat &= cov(core\_points)/num\_core\_points \\ new\_data &= multivariate\_normal(point, cov\_mat, n_{boardline}) \end{aligned} \tag{6}$$

The algorithm for generating new sample points at borderline points is algorithm2.

Algorithm 3 *generate_core*

Input: $clusters$,N_{core},new_data
for cluster in clusters **do**
  $n_{core} = \frac{num\_data\_cluster}{\sum_{i \in cluster} num\_data\_cluster} \times N_{core}$
  $core\_points \leftarrow$ core of cluster
  $cov\_mat = cov(core\_points)$
  $tem\_data = multivariate\_normal(mean(core\_points), cov\_mat, n_{core})$
  add tem_data to new_data

Generate data near core points We generate new data from the mean and variance of the core points of each cluster using gaussian distribution. After DBSCAN, the number of generation near the core points of each cluster depends on the number of sample points in the cluster. The number of sample points in the cluster is $num\_data\_cluster$, the number of generalization is n_{core}, as shown in 7.

$$n_{core} = \frac{num\_data\_cluster}{\sum_{i \in cluster} num\_data\_cluster} \tag{7}$$

The generation method is similar to that near the boundary points. We assume the distribution of sample points in a single cluster obeys gaussian distribution. The equation8 is to generate new sample points near core points.

$$\begin{aligned} cov\_mat &= cov(core\_points) \\ new\_data &= multivariate\_normal(mean(core\_points), cov\_mat, n_{core}) \end{aligned} \tag{8}$$

The algorithm for generating new sample points at core points is algorithm3.

Algorithm 4 *generate_noise*

Input: N_{noise},m_{noise},new_data
for i from 1 to N_{noise} **do**
  point $\leftarrow$ random choice from m_{noise}
  $new\_point \leftarrow$ using SMOTE to generate a new sample on point
  add new_point to new_data

Generate data near noise points we use the most basic SMOTE for generation near the noise points. In general, no new sample points will be synthesized near noise points. Only when minority points cannot form effective clusters, that is, most of the sample points are marked as noise points, such as when the ratio of noise points $ratio$ exceeds a certain super-parameter $noise\_ratio$(generally set as 0.5), part of new samples will be synthesized near noise points. As the amount of noise sample

point data increases, the ratio of generated data near noise points also increases. The proportion of samples generated near noise point is α_{noise} is shown in equation 9.

$$\alpha_{noise} = \frac{0.9}{1 - noise\_ratio^2} \times radio^2 + 1 - \frac{0.9}{1 - noise\_ratio^2} \tag{9}$$

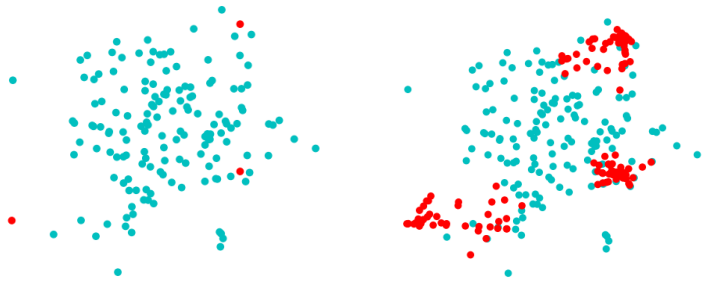
The reason for using equation9 is that when $ratio$ is exactly $noise\_ratio$, the generated ratio is 0.1. When the $noise\_ratio$ is 0.5, α_{noise} is shown in2(b) We also need to limit the number of generation near a single noise point. The number of generation N_{noise} is equation10.

$$\begin{aligned} N_{noise} &= \alpha_{noise} \times N_{oversampling} \\ N_{noise} &= min(multiple\_k \times k \times N^{'}_{noise}, N_{noise}) \end{aligned} \tag{10}$$

We determine N_{noise} first, and then determine the value of $N_{borderline}$ and N_{core}.

Gaussian distribution cannot be used for generation because effective clusters cannot be generated. Therefore SMOTE is used. A single sample is randomly selected from the K nearest neighbors of the minority point, and sample points are randomly generated on the line between the two points. We set $\gamma \in [0, 1]$ is a random value, the two points are $point1$ and $point2$, then the generation equation is 11.

$$new\_point = point1 + \gamma \times (point2 - point1) \tag{11}$$



(a) (b)

Fig. 4. (a) and (b) are distribution of original data and distribution of oversampled data. Minority samples in the dataset are too rare and far apart to form effective cluster. We use SMOTE to generate new points.

As shown in Figure4,4(a) and 4(b) are distribution of original data and distribution of oversampled data. Minority samples in the dataset are too rare and far apart to form effective cluster, we cannot use gaussian distribution to generate new points, we use SMOTE instead. Algorithm $generate\_noise$ 4 is used to generate new sample points near the noise point.

Translation of majority points new samples are generated according to the distribution characteristics of minority points. At the same time we also clean the majority points near minority points to alleviate the overlapping data distributions. We use a similar approach to the model CCR [21]. When dealing with borderline points, the K nearest neighbors of each minority sample is calculated.

With this sample point as the center and the maximum distance in the K neighbors as the radius, a spherical region is drawn. We can move the majority points in these K neighbors out of this region.

(a) Original (b) SMOTE (c) CCR (d) ODG

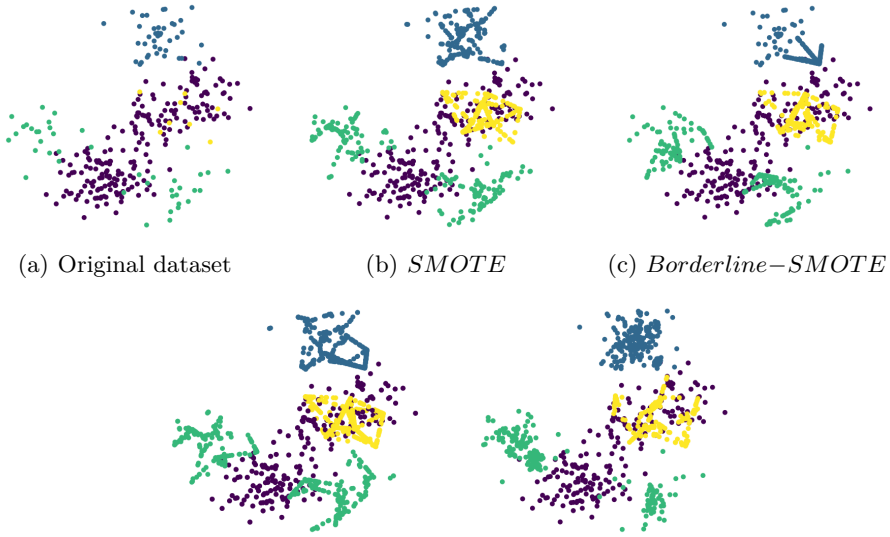
Fig. 5. (a) is Original dataset (b),(c),(d) are the oversampling results of SMOTE, CCR and ODG.

We compare the result of SMOTE, CCR and ODG. We assume that some noise points exist in the original data set, as well as clustered minority points. The result shows in Figure5. We can find that after SMOTE, there are overlapping regions, many generated minority points overlap with majority points. Although CCR algorithm avoided overlapping, but some minority points were synthesized near the noise, which may produce some interference to predicting results.

Computational complexity analysis We present the time complexity of the oversampling method ODG. We define the number of majority points is N_{maj}, the number of minority points is N_{min}, the total number of samples is N, and the number of attributes on the dataset is m. In the DBSCAN algorithm, the time complexity can be controlled at $O(mN_{min}\log(N_{min})$. which can be simplied to $O(mNlog(N))$. When calculating the K nearest neighbors of minority points, The distance between sample points needs to be calculated and sorted, and the complexity of this step is $O(mN_{min}N + N_{min}Nlog(N))$, it can be simplied as $O(mN^2)$. Therefore the time complexity of ODG is $O(mN(N + log(N)))$.

3.2 Multiple classification problem

Multiclassification on imbalance dataset is more difficult to solve than that of binary problem. In the binary classification, it is easy to distinguish the majority class and the minority class. However, in the multi-classification problem, the relationship between classes is more complex and difficult to deal with. It is possible to have multiple majority classes or minority classes. A single class can act as a majority towards some classes, a minority towards others. In addition, overlapping and noise problems will be more serious. The traditional OVA and OVO methods have their own disadvantages, [20, 22] presents a method to deal with multiple classification problems. We iterate over multiple classes. When we oversample each class, A subset is selected as a majority class from the already processed classes, and the class is treated as a minority class by oversampling methods. We first sort the classes in reverse order according to the number of samples, and for each minority class, from the ones that have been processed, select a subset of the majority classes, and mark the minority class as the majority class after processing the ODG algorithm. The overall flow of the MC-ODG algorithm is shown in the algorithm5.



(a) Original dataset (b) $SMOTE$ (c) $Borderline-SMOTE$

(d) $SMOTE + ENN$ (e) $MC - ODG$

Fig. 6. (a) is Original dataset (b)(c)(d)(e) are oversampling result of SMOTE, Borderline-SMOTE, SMOTE+ENN and MC-ODG.

Algorithm 5 MC-ODG

Input: $X \leftarrow$ a data matrix,which has multiple different labels.
$C \leftarrow$ different classes **Output:** $X \leftarrow$ A new data matrix which has been translated and oversamped.

for $i \leftarrow 1$ to $|C|$ **do**
  $n_{classes} \leftarrow$ number of classes with higher number of points than C_i
  if $n_{classes}>0$ **then**
    $X_{min} \leftarrow X^{C_i}$
    $X_{maj} \leftarrow \emptyset$
    for $j \leftarrow 1$ to $n_{classes}$ **do**
      add $\lfloor \frac{|X^{C_1}|}{n_{classes}} \rfloor$ randomly chosen points from $X^{(j)}$ to X_{maj}
    $X^{'}_{maj}, S = ODG(X_{maj}, X_{min})$
    $X^{C_i} \leftarrow X^{C_i} \cup S$
    substitute points used to construct X_{maj} with $X^{'}_{maj}$.
return X

Figure6 shows the original dataset and the oversampling results of SMOTE, Borderline-SMOTE, SMOTE+ENN, MC-ODG. We can find that the original data distributions can be retained after oversampling using the $MC-ODG$ algorithm.

Computational complexity analysis We calculate the time complexity if MC-ODG. We also define the total number of samples is N, and the number of attributes on the dataset is m. The number of classes is c. The entire MC-ODG process is equivalent to c-1 times ODG. Therefore the time complexity of MC-ODG is $O(cmN(N+log(N)))$.

4 Experiments

In this section, we will detail a series of experiments on ODG and MC-ODG. We compared the ODG, MC-ODG algorithms with the existing oversampling methods, We found that our algorithm ODG and MC-ODG can achieve better results on different metrics. We also compared the influence of changing the ratio generated near borderline points on predicting results. We published the code of our model on github[4].

4.1 Evaluation matrics

We evaluated different oversampling methods using multiple metrics. Prediction accuracy is not a good metric on imbalanced problems. Due to the imbalance of data, the accuracy will naturally incline to the majority class. The predictive ability of the model for minority classes cannot be reflected by accuracy.

Binary classification We labeled the majority class as 0(Negative) and the minority class as 1(Positive). Precision, Recall, F-value and Auc are more reasonable metrics. Table1 gives the confusion matrix for the binary classification problems. Using the confusion matrix, we can calculate

Table 1. Confusion matrix for the two-class problem

Actual label	Predicted positive	Predicted negative
Positive	TP	FP
Negative	FP	FN

the Precision, Recall and F-value. The calculation formulas are as follows:

$$Precision = \frac{TP}{TP+FP} \tag{12}$$

$$Recall = \frac{TP}{TP+FN} \tag{13}$$

[4] https://github.com/Xiaoctw/MC-ODG

$$F - value = \frac{(1+\beta^2) \times Precision \times Recall}{\beta^2 \times Precision + Recall} \tag{14}$$

The value of Auc is defined as the area bounded by the curve of *roc* and the coordinate axis. In general, the value of Auc is greater than 0.5. The closer the value is to 1, the better the prediction result is.

Multiple classification The evaluation of multi-classification is more complex, and there is not a unified evaluation standard and it is still an open problem. In addition to the above mentioned Precision, Recall and F-value, mGM15 and CBA16 are also introduced in [4].

$$CBA = \frac{\sum_{i=1}^{M} \frac{mat_{i,j}}{max(\sum_{j=1}^{M} mat_{i,j}, \sum_{j=1}^{M} mat_{j,i})}}{M} \tag{15}$$

$$mGM = \sqrt[M]{\Pi_{i=1}^{M} Recall_i} \tag{16}$$

4.2 Dataset

The dataset used in the experiment is the classification datasets on UCI[5], datasets' details are in tables 2 and 3.

Table 2. binary classification dataset

Datasets	Instances	Features	Class distribution	IR
Transfusion(TR)	748	5	570/178	3.2
Wisconsin(WIS)	699	25	458/241	1.9
Adult(AD)	32561	108	24720/7841	3.2
Haberman(HA)	336	3	225/81	2.8

Table 3. multiple classification dataset

Datasets	Instances	Features	Classes	Class distribution	IR
Glass(GL)	213	8	6	76/69/29/17/13/9	8.4
Automobile(AU)	159	25	6	48/46/29/13/3	16
Wine(WI)	157	12	3	71/58/48	1.5
Yeast(YE)	1484	7	10	463/429/244/163/51/44/35/30/20/4	115.6
Ecoli(EC)	336	6	8	143/77/52/35/20/5/2/2	71

[5] http://archive.ics.uci.edu/ml/datasets.php

Table 4. The parameters of ODG and MC-ODG

Parameters	default value
k	5
eps	0.08
min_pts	4
fit_borderline_ratio	true
borderline_ratio	0.6
noise_ratio	0.1
multiple_k	4

4.3 Experiment contents

Compare the prediction results of different oversampling models In this experiment, three widely used machine learning models, C4.5 and K nearest neighbors, and logistic regression were used. We compared different oversampling methods, SMOTE, Borderline-SMOTE, ADASYN, SMOTE+ENN, MC-CCR and ODG or MC-ODG. we adopted the cross validation method of K folding, set K=5, repeated the experiment for 10 times, and took the mean value as the final result. Parameters of ODG and MC-ODG are shown in table 4. *fit_borderline_ratio* means being adaptive to determine the *borderline_ratio* or not. Parameter settings of the other models are shown in table 5

Table 5. Parameters

Algorithm	Parameters
KNN	k∈[3,5,7]
C4.5	max_depth=5 min_samples_split=3
MC-CCR	$energy \in [0.25, 0.5, 1]$ cleaning strategy:translation selection strategy: proportional multi-class decomposition method:sampling
SMOTE	k_neighbors ∈ [3,5,7]
Borderline-SMOTE	kind=”borderline-1” k_neighbors∈[3,5,7]
ADASYN	n_neighbors∈[3,5,7]
SMOTE+ENN	k_neighbors ∈ [3,5,7]

In ODG and MC-ODG algorithms, DBSCAN algorithm is used for clustering. The parameters DBSCAN needs to determine are EPS and min_pts. *borderline_ratio* of some datasets is artificially specified. Parameter settings on different datasets are shown in table 6.

The effect of *borderline_ratio* on the experimental results In order to improve the prediction ability of our model, more minority points need to be generated near the borderline points.

Table 6. Parameters of ODG, MC-ODG

Datasets	Parameters
TR	eps=0.15 min_pts=3 borderline_ratio=0.5
WIS	eps=0.5 min_pts=3 borderline_ratio=0.7
AD	eps=1.6 min_pts=5 borderline_ratio=0.7
HA	eps=0.14 min_pts=3
GL	eps=0.15 min_pts=3
AU	eps=1.8 min_pts=3 borderline_ratio=0.4 noise_ratio=0.2
WI	eps=0.36 min_pts=2
EC	eps=0.14 min_pts=3 borderline_ratio=0.7
YE	eps=0.13 min_pts=3 borderline_ratio=0.2 noise_ratio=0.9

The parameter *borderline_ratio* plays a crucial role. We choose dataset TR, WIS, AU and EC, and used matrics F1-score, Auc and mGM to observe the effect of *borderline_ratio* on experiment results. We did the experiment five times and took the average.

4.4 Experiment results

Compare the prediction results of different oversampling models We give a comparison of different oversampling methods on different datasets based on different machine learning models. For binary datasets, KNN, C4.5 and LR results on multiple metrics are shown in table7, table 8, table 9 For multiclassification datasets, KNN, C4.5 and LR results in table 10, table 11 and table 12.

Table 7. KNN model deals with binary classification datasets

Datasets	Precision				Recall				F1-score				Auc			
	TR	WIS	AD	HA	TR	WIS	AD	HA	TR	WIS	AD	HA	TR	WIS	AD	HA
None	0.823	0.934	0.823	0.426	0.646	0.921	0.839	0.282	0.717	0.926	0.83	0.332	0.837	0.955	0.913	0.604
SMOTE	0.843	0.926	0.819	0.465	0.604	0.935	0.837	0.337	0.7	0.93	0.827	0.381	0.824	0.954	0.912	0.617
Borderline-SMOTE	0.853	0.93	0.824	0.421	0.606	0.91	0.839	0.269	0.706	0.919	0.83	0.319	0.827	0.949	0.912	0.621
ADASYN	**0.873**	**0.937**	0.819	**0.452**	0.618	0.912	0.843	0.36	**0.721**	0.924	0.829	0.383	0.834	0.95	0.913	0.619
SMOTE+ENN	0.71	0.921	0.797	0.412	0.677	0.953	0.876	0.537	0.69	0.936	0.831	0.46	0.806	0.958	0.912	0.64
CCR	0.738	0.925	0.794	0.368	**0.808**	0.94	0.862	**0.626**	0.709	0.931	0.825	0.457	0.834	0.958	0.909	0.622
ODG	0.8	0.921	**0.831**	**0.452**	0.75	**0.956**	**0.879**	0.517	0.711	**0.938**	**0.84**	**0.476**	**0.846**	**0.965**	**0.916**	**0.667**

Table 8. C4.5 model deals with binary classification datasets

Datasets	Precision				Recall				F1-score				Auc			
	TR	WIS	AD	HA	TR	WIS	AD	HA	TR	WIS	AD	HA	TR	WIS	AD	HA
None	0.841	0.936	0.819	0.419	0.6	0.918	0.848	0.305	0.697	0.926	0.832	0.34	0.823	0.959	0.926	0.623
SMOTE	**0.844**	0.93	0.823	0.448	0.625	0.923	0.845	0.316	**0.714**	0.935	0.832	0.359	0.835	0.957	0.923	0.605
Borderline-SMOTE	0.839	0.931	0.812	**0.482**	0.587	0.922	0.852	0.315	0.684	0.925	0.83	0.373	0.816	0.958	0.924	0.594
ADASYN	0.843	0.928	0.815	0.475	0.596	0.924	0.848	0.338	0.694	0.924	0.83	0.388	0.822	0.957	0.924	0.627
SMOTE+ENN	0.697	0.919	0.789	0.41	0.673	0.954	**0.882**	0.552	0.678	0.936	0.834	0.449	0.799	0.959	0.927	0.628
CCR	0.751	0.918	0.79	0.367	**0.809**	**0.954**	0.88	**0.628**	0.688	0.935	0.832	**0.465**	0.836	0.965	0.924	0.635
ODG	0.755	**0.938**	**0.824**	0.429	0.681	0.951	0.859	0.524	0.672	**0.943**	**0.84**	0.455	**0.843**	**0.968**	**0.928**	**0.656**

Table 9. LR model deals with binary classification datasets

Datasets	Precision				Recall				F1-score				Auc			
	TR	WIS	AD	HA	TR	WIS	AD	HA	TR	WIS	AD	HA	TR	WIS	AD	HA
None	0.872	**0.962**	0.848	**0.691**	0.571	0.946	0.84	0.284	0.568	0.953	0.832	0.261	0.847	**0.996**	0.952	0.663
SMOTE	0.892	**0.962**	0.843	0.673	0.549	0.941	0.82	0.322	0.536	0.951	0.829	0.309	0.848	0.995	0.96	0.642
Borderline-SMOTE	0.871	0.961	**0.857**	0.653	0.553	0.946	0.824	0.286	0.549	0.953	0.837	0.313	0.845	0.992	0.961	0.663
ADASYN	**0.921**	0.958	0.852	0.658	0.553	0.95	0.828	0.321	0.646	0.953	**0.839**	0.309	0.847	0.995	0.96	0.644
SMOTE+ENN	0.779	0.951	0.775	0.536	0.701	0.962	0.895	**0.569**	0.663	0.956	**0.839**	0.443	0.847	**0.996**	0.957	0.658
CCR	0.69	0.951	0.779	0.525	0.763	0.967	0.884	0.445	**0.665**	0.958	0.832	0.396	0.848	0.965	0.957	0.635
ODG	0.655	0.956	0.775	0.543	**0.781**	**0.968**	**0.906**	0.457	0.657	**0.959**	0.834	**0.457**	**0.853**	**0.996**	**0.959**	**0.668**

Table 10. KNN model deals with multi-classification datasets

Datasets	Precision					Recall					F1-score					mGM					CBA				
	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE
None	**0.765**	0.84	0.57	0.962	0.5	0.718	0.78	0.575	0.96	0.492	0.716	0.767	0.555	0.954	0.465	0.713	0.761	0.651	0.958	0.467	0.643	0.717	0.497	0.92	0.419
SMOTE	0.742	0.839	0.572	0.962	0.506	0.695	0.793	0.554	0.969	0.496	0.692	0.802	0.541	**0.963**	**0.492**	0.643	0.762	0.593	**0.968**	0.484	0.618	0.748	0.484	0.931	0.45
Borderline-SMOTE	0.716	**0.843**	0.578	0.957	0.498	0.677	0.789	0.562	0.963	0.491	0.667	0.802	0.547	0.957	0.486	0.599	0.736	0.633	0.961	0.483	0.591	0.741	0.486	0.924	0.444
ADASYN	0.716	**0.843**	0.576	0.955	0.496	0.668	0.793	0.562	0.961	0.494	0.661	0.807	0.547	0.955	0.489	0.621	0.768	0.632	0.959	0.488	0.583	0.756	0.488	0.922	0.449
SMOTE+ENN	0.588	0.823	0.391	0.947	**0.51**	0.563	0.815	0.466	0.956	0.485	0.538	0.81	0.401	0.947	0.466	0.481	0.8	0.562	0.953	0.465	0.473	**0.757**	0.349	0.903	0.41
MC-CCR	0.719	0.774	0.603	0.955	0.455	0.737	0.803	0.642	0.967	0.5	0.706	**0.811**	0.597	**0.963**	0.482	0.713	0.775	0.652	0.958	**0.496**	0.626	0.717	0.525	0.934	0.445
MC-ODG	0.738	0.805	**0.637**	**0.963**	0.481	**0.75**	**0.815**	**0.664**	**0.97**	**0.511**	**0.721**	0.805	**0.622**	**0.963**	0.489	**0.725**	**0.803**	**0.658**	0.967	0.484	**0.641**	0.745	**0.546**	**0.935**	**0.453**

Table 11. C4.5 model deals with multi-classification datasets

Datasets	Precision					Recall					F1-score					mGM					CBA				
	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE
None	0.738	0.762	0.658	0.911	**0.541**	0.7	0.736	0.63	0.906	0.478	0.695	0.731	0.625	0.904	0.479	0.7	0.723	0.59	0.901	0.574	0.626	0.68	0.567	0.859	0.434
SMOTE	0.702	0.8	0.662	0.934	**0.541**	0.685	0.761	0.617	0.918	0.499	0.675	0.768	0.617	0.916	0.5	0.686	0.731	0.639	0.915	0.583	0.614	0.715	0.558	0.873	0.387
Borderline-SMOTE	0.741	**0.802**	0.67	0.915	0.532	0.718	0.75	0.631	0.908	0.497	0.709	0.759	0.622	0.908	0.498	0.679	0.728	0.594	0.905	0.57	0.642	0.701	0.552	0.869	0.439
ADASYN	0.749	0.789	0.646	0.915	0.539	0.716	0.748	0.619	0.915	0.501	0.708	0.755	0.611	0.912	0.504	0.695	0.739	0.619	0.912	0.576	0.637	0.695	0.543	0.881	0.449
SMOTE+ENN	0.577	0.761	0.45	0.906	0.491	0.564	0.739	0.528	0.907	0.442	0.524	0.726	0.464	0.901	0.443	0.542	0.71	**0.677**	0.902	0.523	0.495	0.658	0.411	0.853	0.387
MC-CCR	**0.773**	0.788	0.658	0.923	0.435	**0.748**	**0.788**	0.675	0.931	0.492	0.731	0.777	0.638	0.929	0.491	0.73	0.771	0.64	0.928	**0.605**	0.666	0.717	0.567	0.891	0.443
MC-ODG	0.762	0.797	**0.684**	**0.945**	0.501	0.738	0.786	**0.694**	**0.944**	**0.536**	**0.742**	**0.781**	**0.663**	**0.943**	**0.504**	**0.749**	**0.773**	0.634	**0.943**	0.584	**0.676**	**0.726**	**0.592**	**0.913**	**0.45**

Table 12. LR model deals with multi-classification datasets

Datasets	Precision					Recall					F1-score					mGM					CBA				
	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE	AU	EC	GL	WI	YE
None	0.668	0.657	**0.642**	0.984	**0.523**	0.62	0.607	0.575	0.98	0.416	0.617	0.679	0.615	0.973	0.423	0.709	0.702	0.583	0.984	0.521	0.594	0.538	0.519	0.963	0.471
SMOTE	**0.741**	0.674	0.622	0.984	0.548	0.676	0.601	0.594	0.985	0.425	0.681	0.675	0.608	0.984	0.436	0.678	0.716	0.584	0.985	0.533	0.618	0.544	0.525	0.969	0.48
Borderline-SMOTE	0.733	0.663	0.63	0.985	0.502	0.665	0.605	0.592	0.984	0.411	0.669	0.66	0.618	0.984	0.437	0.629	0.719	0.603	0.984	0.591	0.603	0.548	0.509	0.967	0.472
ADASYN	0.707	0.687	0.626	0.984	0.502	0.636	0.602	0.610	0.984	0.415	0.648	0.656	0.603	0.985	0.44	0.626	0.706	0.610	0.983	0.591	0.592	0.558	0.531	0.971	0.474
SMOTE+ENN	0.57	0.551	0.61	0.965	0.434	0.558	0.543	0.622	0.975	0.41	0.528	0.62	0.605	0.967	0.407	0.537	0.768	**0.679**	0.97	**0.67**	0.478	0.502	0.506	0.946	0.39
MC-CCR	0.718	0.679	0.624	0.983	0.493	0.711	0.693	**0.673**	**0.99**	0.515	0.701	0.672	0.63	0.983	0.537	0.691	0.866	0.652	0.985	0.581	0.644	0.628	0.566	0.869	0.517
MC-ODG	0.7	0.723	0.633	**0.988**	0.473	**0.74**	0.758	0.659	0.986	**0.52**	**0.705**	**0.723**	**0.657**	**0.989**	**0.558**	**0.763**	**0.882**	0.671	**0.99**	0.586	**0.646**	**0.663**	**0.577**	**0.979**	**0.525**

In the oversampling of binary-classification, we found that the ODG algorithm can achieve the best results on multiple metrics of most datasets. Best results can be obtained because we consider that the data may be distributed in clusters, and we adopt different oversampling methods for data points in different positions. Minority sample data are generated retaining the distribution characteristics of the original data, enhancing the ability of machine learning. It is obvious that the MC-ODG and MC-CCR are superior to other oversampling methods in multi-classification tasks. The reason is that both approaches take into account the relationships between different classes and can avoid overlapping data distribution. Meanwhile, the MC-ODG algorithm is superior to MC-CCR, because MC-ODG algorithm can retain the original data distribution when generating data.

We also found the following additional conclusions:

- Good results can be achieved on the Precision metric without any oversampling progress. the best results can be obtained on the data set WIS without oversampling. When we generate large numbers of minority sample points, the model will be inclined to predict the results as minority samples. In this way, the value of TP+FP will become larger, while the value of TP is limited, then the Precision will decrease.
- Different datasets have different distribution characteristics and are suited to different machine learning methods. For example, the LR model on WIS can get good prediction results, while KNN is poor. And on the HA dataset The prediction result of KNN algorithm is much better than that of LR algorithm.
- Different oversampling methods are suitable for different datasets. Such as CCR method is very suitable for TR, SMOTE+ENN is suitable for AD.
- CCR method can obtain good results, the reason is that the CCR method can effectively avoid overlapping and handle noise data.

The effect of *borderline_ratio* on the experimental results Figure7 shows the influence of the change with *borderline_ratio* on the model prediction results. 7(a),7(b) represents the change trend of Auc and F1-score on the binary data set. 7(c),7(d) represents the trend of F1-score and mGM on multiclassification datasets. It was observed that the variation trend of the predicted results remained the same on different metrics. The prediction results are optimal near a certain ratio. Except for the AU data set, *boaderline_ratio* is greater than 0.5 when the best result are obtained. It shows that in order to make the machine learning model more inclined to the learning of minority points, we need to generate more sample points near the boundary points. We find that after reaching the peak, as the *borderline_ratio* increases, The rate of decline is slower than the rate of rise before reaching the peak. This also indicates that the generation of minority sample points near the boundary is conducive to enhancing the prediction results.

5 Conclusions and future work

This paper presents a new oversampling method. The ODG algorithm is used to solve the binary classification problem while the MC-ODG algorithm is used to solve the multi-classification problem. In the ODG algorithm, we first use the DBSCAN algorithm to cluster minority points and get the core points, borderline points and noise points. Different oversampling methods are used to deal with different points. The key is generating new samples near the borderline points. When new samples are generated, the distribution of the generated data depends on the variance of the core

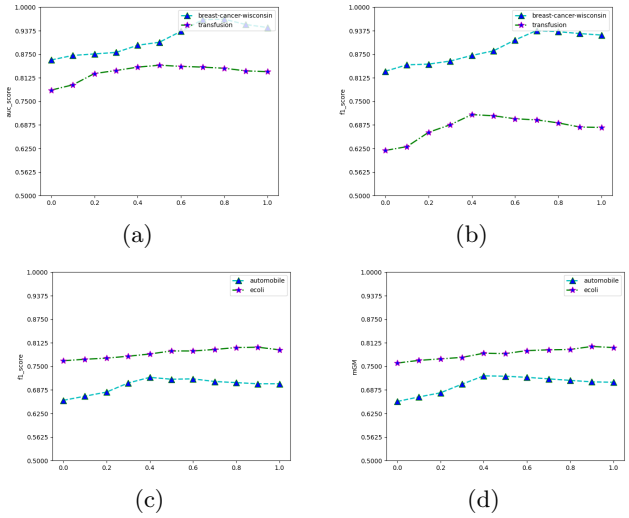


Fig. 7. Effect of *borderline_ratio* on experiment results

points. The number generated is positively correlated with the number of majority points near the boundary point. Meanwhile majority samples in the K nearest neighbors are moved out of the maximum distance in the K neighbors to avoid overlapping. MC-ODG takes an iterative approach for each class. In each oversampling process, the distribution information of other majority classes is used. Compared with previous proposed oversampling methods, the method to generate data points retains the original data distribution, can effectively avoid overlapping and handle noise data points. Experimental results show that it is an effective method.

In our future work, we will consider how to implement oversampling in a simpler way. We find that lots of parameters to be processed in ODG and MC-ODG. we will consider how to reduce the parameters, which can also achieve relatively good results.

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