Data analysis of simulated soil erosion data

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```
rm(list = ls(all = T))
library(dplyr)
library(ggplot2)
library(patchwork)
## load model package
if(!require(saezero)) devtools::install_github("XiaodanLyu/saezero")
library(saezero)
## load pseudo data due to confidentiality
data(erosion)
glimpse(erosion)
```

```
## Observations: 646
## Variables: 10
## $ ctylab <chr> "Aurora", "Aurora", "Aurora", "Aurora", "Aurora", "Aurora", ...
## $ cty    <chr> "003", "003", "003", "003", "003", "003", "003", "003", "005...
## $ mukey  <dbl> 354836, 354837, 354841, 354844, 354847, 354851, 354866, 3548...
## $ crop   <chr> "others", "soybean", "others", "others", "others", "others",...
## $ logR   <dbl> 4.553877, 4.553877, 4.553877, 4.553877, 4.553877, 4.553877, ...
## $ logK   <dbl> -1.4271164, -1.4271164, -1.4271164, -0.9942523, -1.1394343, ...
## $ logS   <dbl> -0.3549698, -2.1465136, -1.0483327, -2.1465136, -1.0483327, ...
## $ crop2  <dbl> 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, ...
## $ crop3  <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, ...
## $ RUSLE2 <dbl> 0.091, 0.009, 0.007, 0.034, 0.087, 0.000, 0.006, 0.001, 0.00...
```

Figure 1

```
g1 <- erosion %>% ggplot(aes(x = RUSLE2)) +
  geom_histogram(aes(y = ..density..), binwidth = 0.01,
                 color = "black", fill = NA, size = rel(1)) +
  geom_rug(sides = "b") +
  xlab("RUSLE2") + theme_bw(base_size = 15)
erosion2 <- erosion %>% group_by(cty) %>%
  summarise(
    ybar = mean(log(RUSLE2)[RUSLE2>0]),
    pbar = mean(RUSLE2>0),
    count = n()) %>%
  filter(!is.na(ybar))
g2 <- erosion2 %>% ggplot() +
  geom_point(aes(x = ybar, y = pbar, size = count), color = "black") +
```

```
  labs(x = "Average of positive RUSLE2s in log scale",
       y = "Proportion of positive RUSLE2s",
       size = "Sample size", title = "Counties") +
  theme_bw(base_size = 15) + ylim(0, 1) +
  theme(plot.title = element_text(hjust = 0.5),
        plot.margin = margin(0, 0, 0, 1, "cm"))
g1 + g2 + plot_layout(widths = c(6, 4))
```

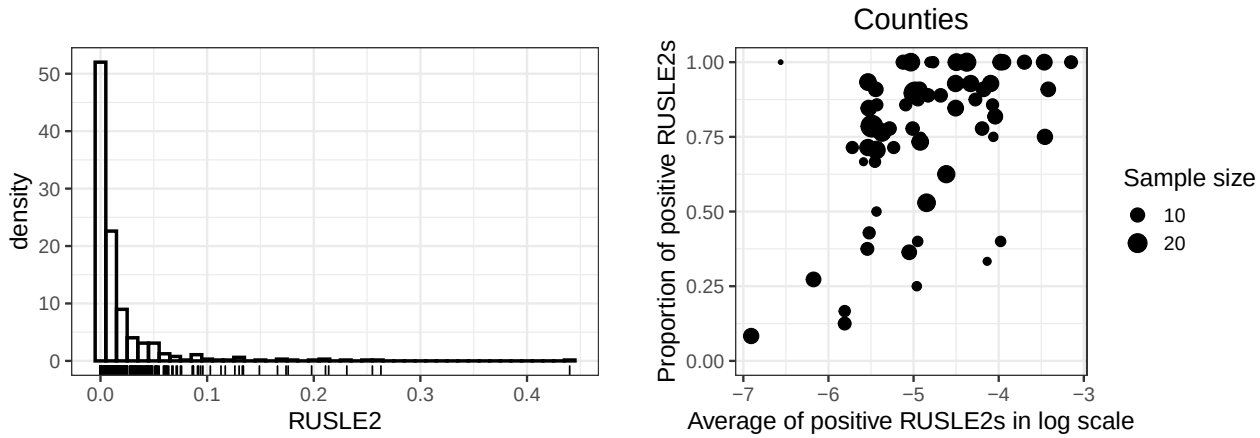


Table 4

```
## data pre-processing
erosion_2p <- as.2pdata(f_pos = RUSLE2~logR+logK+logS,
                        f_zero = ~logR+logS+crop2+crop3,
                        f_area = ~cty, data = erosion)
## Maximum Likelihood Estimates
fit <- mleLBH(erosion_2p)
```

```
## parametric bootstrap
## 2-3 hrs
system.time({
  set.seed(2020)
  est.store <- do.call(
    "rbind.data.frame",
    lapply(1:500, function(b){
      ys <- simLBH(fit, erosion, f_pos = ~logR+logK+logS,
                   f_zero = ~logR+logS+crop2+crop3, f_area = ~cty)
      erosion_boot <- erosion %>% mutate(RUSLE2 = ys)
      erosion_boot_2p <- as.2pdata(f_pos = RUSLE2~logR+logK+logS,
                                   f_zero = ~logR+logS+crop2+crop3,
                                   f_area = ~cty, data = erosion_boot)
      fit_boot <- mleLBH(erosion_boot_2p)
      c(unlist(fit_boot$fixed), fit_boot$refcor)
    }))
})
colnames(est.store) <- c(paste0("beta", 0:3), paste0("alpha", 0:4), "rho")
save(est.store, file = "data/theta_boot.RData")
```

```
load("data/theta_boot.RData")
## fixed effect coefficients and bootstrap standard errors
est <- c(beta = fit$fixed$p1[-1], alpha = fit$fixed$p0[-1])
se <- apply(t(est.store[,c(2:4, 6:9)]) - est, 1, sd)
coef <- sprintf("%.2f (%.2f)", est, se)
names(coef) <- names(est)
coef
```

```
##         beta1         beta2         beta3        alpha1        alpha2
## "1.83 (0.33)" "0.38 (0.15)" "0.52 (0.07)" "4.82 (0.65)" "0.17 (0.15)"
##        alpha3        alpha4
## "0.62 (0.31)" "1.42 (0.40)"
```

```
## variance components
c(sig2lu = fit$refvar1, sig2b = fit$refvar0,
  sig2le = fit$errorvar, rho = fit$refcor) %>%
  print(digit = 2)
```

```
## sig2lu  sig2b sig2le    rho
##   0.22   0.47   1.20   0.64
```

```
## 95% confidence interval of rho
round(quantile(est.store[,10], c(0.025, 0.975)), 2)
```

```
##  2.5% 97.5%
##  0.06  0.99
```

Shapiro-Wilk test of normality

```
## standardized marginal residuals
stderr.mar <- na.omit(fit$residuals$mar)/sqrt(fit$refvar1 + fit$errorvar)
## standardized conditional residuals
stderr.con <- na.omit(fit$residuals$con)/sqrt(fit$errorvar)
shapiro.test(stderr.mar)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  stderr.mar
## W = 0.99546, p-value = 0.1549
```

```
shapiro.test(stderr.con)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  stderr.con
## W = 0.99502, p-value = 0.1078
```

Figure 2

```
predictions <- ebLBH(Xaux, f_q = ~cnt, erosion_2p, fit, fullpop = TRUE)
predictions <- predictions %>% mutate(cv = sqrt(mse)/eb)
predictions %>% glimpse()
```

```
## Observations: 64
## Variables: 4
## $ area <fct> 003, 005, 007, 009, 011, 013, 015, 017, 021, 023, 025, 027, 02...
## $ eb   <dbl> 0.017522184, 0.021843915, 0.002520479, 0.020928505, 0.02743210...
## $ mse  <dbl> 3.508430e-05, 2.707652e-05, 2.700753e-06, 4.347515e-05, 5.3349...
## $ cv   <dbl> 0.3380401, 0.2382133, 0.6520176, 0.3150520, 0.2662602, 0.21297...
```

```
## parametric bootstrap estimates of M2
set.seed(2020)
b <- 0
eb_boot.store <- mmse_boot.store <- c()
## take several hours
system.time(repeat{
  b <- b + 1
  ys <- simLBH(fit, erosion, f_pos = ~logR+logK+logS,
               f_zero = ~logR+logS+crop2+crop3, f_area = ~cty)
  erosion_boot <- erosion %>% mutate(RUSLE2 = ys)
  erosion_boot_2p <- as.2pdata(f_pos = RUSLE2~logR+logK+logS,
                               f_zero = ~logR+logS+crop2+crop3,
                               f_area = ~cty, data = erosion_boot)
  fit_boot <- mleLBH(erosion_boot_2p)
  eb_boot <- ebLBH(Xaux, f_q = ~cnt, erosion_boot_2p, fit_boot, fullpop = TRUE)$eb
  mmse_boot <- ebLBH(Xaux, f_q = ~cnt, erosion_boot_2p, fit, fullpop = TRUE)$eb
  eb_boot.store <- rbind(eb_boot.store, eb_boot)
  mmse_boot.store <- rbind(mmse_boot.store, mmse_boot)
  if(b>=100) break
})
m2_boot <- colMeans((eb_boot.store - mmse_boot.store)^2)
save(eb_boot.store, mmse_boot.store, m2_boot, file = "data/eb_boot.RData")
```

```
load("data/eb_boot.RData")
## pooled standard error of direct estimator
direct <- erosion %>% group_by(cty) %>%
  summarise(nis = n(),
            ctymean = mean(RUSLE2),
            pool = sqrt(var(erosion$RUSLE2)/nis)) %>%
  mutate(group = cut(nis, c(0, 5, 10, 20, 30)))
se_all <- direct %>% mutate(
  ID = 1:n(),
  semiboot = sqrt(predictions$mse+m2_boot),
  onestep = sqrt(predictions$mse))
se_long <- se_all %>%
  tidyr::gather(method, SE, pool, semiboot, onestep) %>%
  mutate(method = forcats::fct_recode(
    method, "Direct" = "pool",
    "EB (Semi-Boot)" = "semiboot",
```

```
    "EB (One-step)" = "onestep") %>%
      forcats::fct_relevel("Direct", "EB (One-step)"))
ggplot(se_long, aes(x = ID, y = SE)) +
  geom_line(aes(group = method, color = method, linetype = method)) +
  scale_linetype_manual(values = c(1,1,0)) +
  geom_point(aes(shape = method, color = method),
             size = rel(2), stroke = rel(1)) +
  facet_grid(.~group, scales = "free_x") +
  scale_color_manual(values = c("darkgrey", "black", "black")) +
  scale_shape_manual(values = c(8, 1, 4)) +
  labs(x = "County", y = "Standard Error",
       color = "", shape = "", linetype = "") +
  theme_bw(base_size = 18) +
  theme(legend.position = "bottom",
        axis.text.x = element_blank(),
        axis.ticks.x = element_blank(),
        legend.background = element_rect(colour = "black", size = 0.5))
```

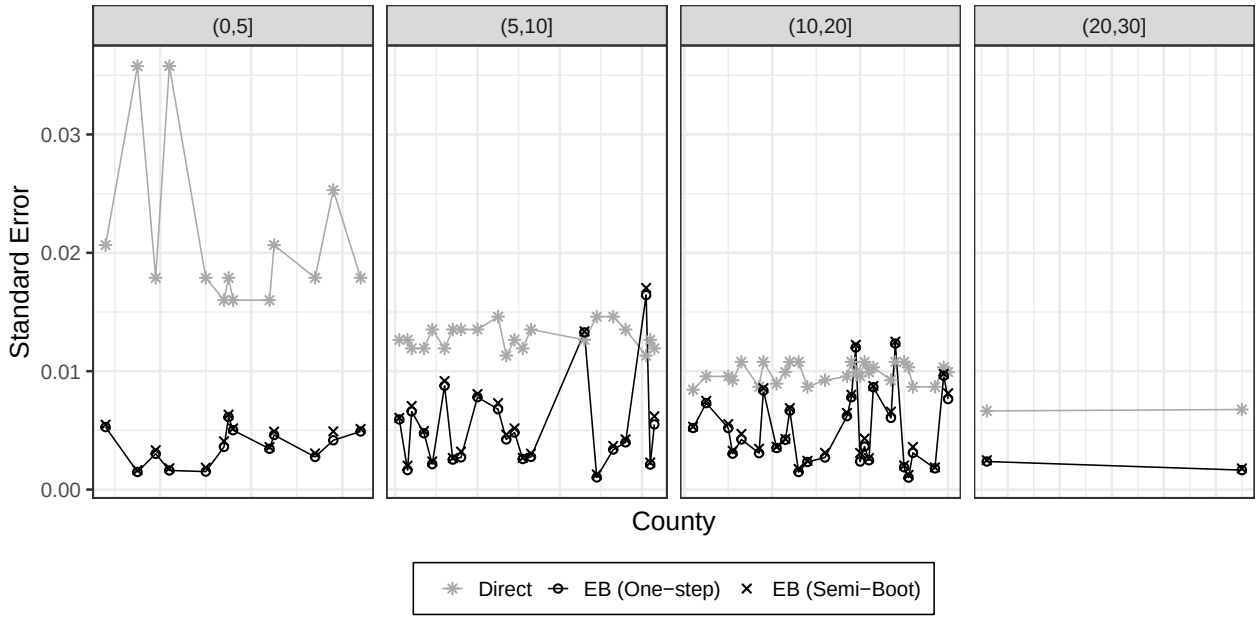
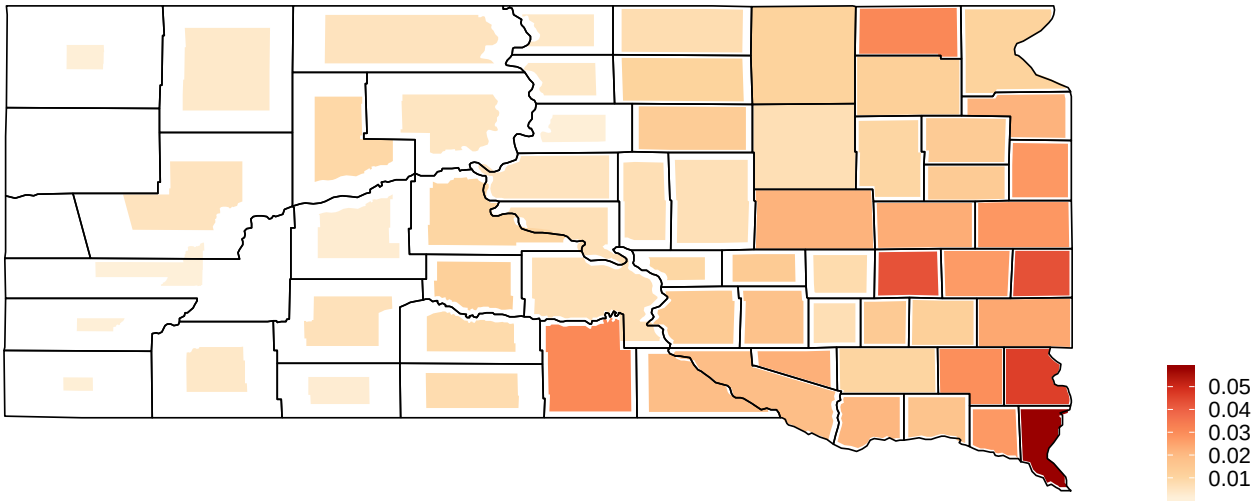


Figure 3

```
if(!require(ggmapr)) devtools::install_github("heike/ggmapr")
## EB predictions and one-step MSE estimates
map.eb <- map_sd %>% left_join(predictions, by = c("COUNTY" = "area"))
## scale by CV (coffecient of variance)
b <- (0.2-1)/diff(range(map.eb$cv, na.rm = T))
map.eb.df <- map.eb %>% mutate(s = b*(cv-min(cv, na.rm = T))+1)
scale_cty <- function(cty) {
  map.eb.df %>% filter(COUNTY == cty) %>% ggmapr::scale(scale = unique(.$s))
}
map.eb.scale <- do.call("rbind", lapply(unique(map.eb.df$COUNTY), scale_cty))
map.eb.scale %>%
```

```
  ggplot(aes(x = long, y = lat, group = factor(group))) +
  geom_polygon(aes(fill = eb), colour = NA) +
  geom_path(data = map_sd) +
  scale_fill_distiller("", palette = "OrRd", direction = 1) +
  scale_alpha(trans = "reciprocal", range = c(0.3, 1)) +
  ggthemes::theme_map(base_size = 18) +
  theme(legend.position = "right") +
  coord_equal()
```

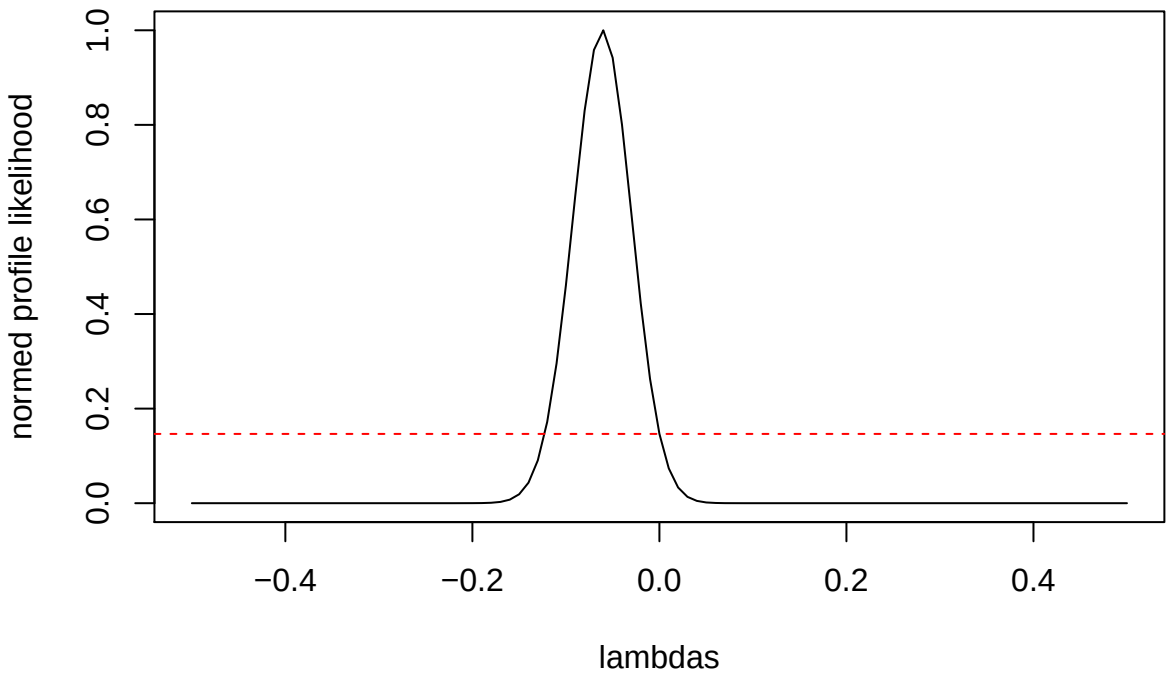


Appendix B. Link Function Analysis

```
## positive: Box-Cox
lambdas <- seq(-0.5, 0.5, by = 0.01)
profile_lam <- function(lambda){
  mleLBH(as.2pdata(f_pos = RUSLE2~logR+logK+logS,
                   f_zero = ~logR+logS+crop2+crop3,
                   f_area = ~cty, data = erosion,
                   transform = "BoxCox", lambda = lambda))$loglik
}
## 30-40 minutes
system.time(loglike_lambda <- sapply(lambdas, profile_lam))
(lambdahat <- lambdas[which.max(loglike_lambda)])
## 2-3 minutes each
rend <- uniroot(function(lambda)
  exp(profile_lam(lambda)-max(loglike_lambda))-exp(-0.5*qchisq(0.95, 1)), c(lambdahat, 0.2))
lend <- uniroot(function(lambda)
  exp(profile_lam(lambda)-max(loglike_lambda))-exp(-0.5*qchisq(0.95, 1)), c(-0.2, lambdahat))
## binary: Aranda-Ordaz
phis <- 0:50/12.5
profile_phi <- function(phi){
  mleLBH(as.2pdata(f_pos = RUSLE2~logR+logK+logS,
                   f_zero = ~logR+logS+crop2+crop3,
                   f_area = ~cty, data = erosion),
         link = glmx::ao2(phi))$loglik
}
```

```
system.time(loglike_phis <- sapply(phis, function(phi) try(profile_phi(phi))))
(phihat <- phis[which.max(loglike_phis)])
rend_phi <- uniroot(function(phi)
  exp(profile_phi(phi)-max(loglike_phis))-exp(-0.5*qchisq(0.95, 1)), c(1, 2))
## save results
save(lambdas, loglike_lambda, lend, rend, phis, loglike_phis, rend_phi,
     file = "data/link_analysis.RData")
```

```
load("data/link_analysis.RData")
## positive part transformation family
plot(x = lambdas, y = exp(loglike_lambda-max(loglike_lambda)),
     type = "l", ylab = "normed profile likelihood")
abline(h = exp(-0.5*qchisq(0.95, 1)), col = "red", lty = "dashed")
```

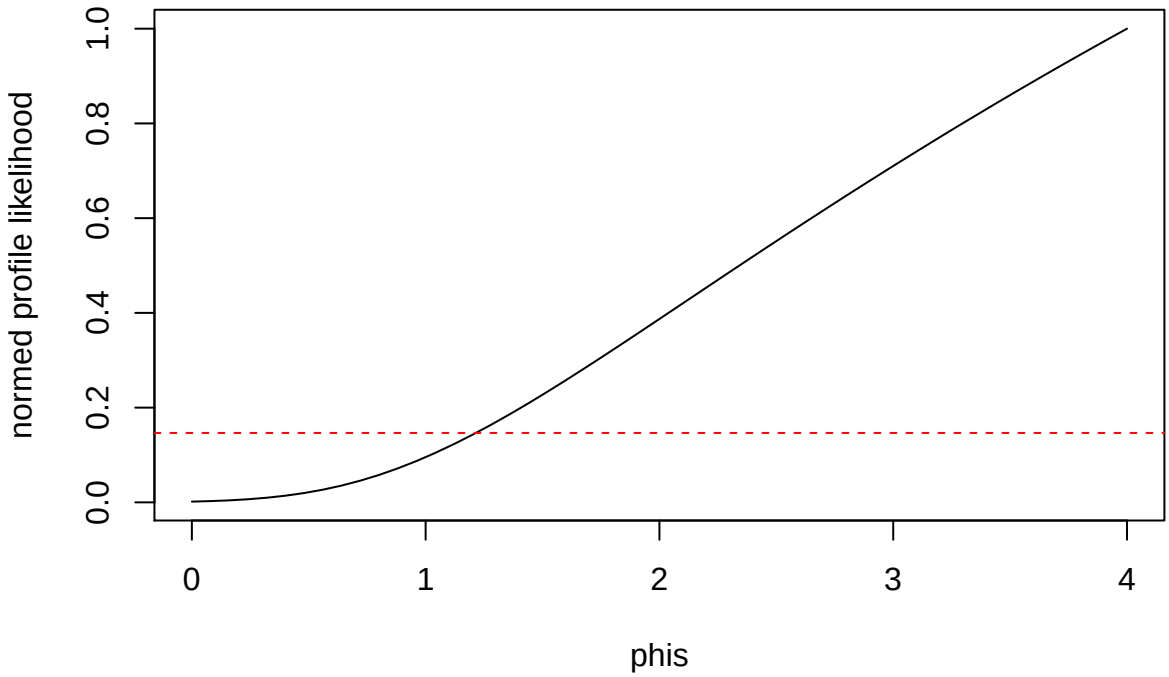



```
sprintf("(%.3f, %.3f)", lend$root, rend$root)
```

```
## [1] "(-0.123, 0.000)"
```

Remark: The following profile liklihood function of ϕ in the Aranda-Ordaz transformation family in the binary part is obtained using the simulated erosion data as contained in the R package `saezero`. (We admit the profile likelihood in the following figure may not be normed by the globally maximized likelihood.) The same R code gives a MLE of $\hat{\phi} = 0$ and a 95% confidence interval containing $\phi = 1$ (the logit link) using the real erosion data. We suspect this discrepancy is due to sampling randomness in the process of simulating erosion data. The profile likehood function based on another simulated erosion data (not shown here) is close to what we obtained using the real erosion data.

```
## binary part transformation family
plot(x = phis, y = exp(loglike_phis-max(loglike_phis)),
     type = "l", ylab = "normed profile likelihood")
abline(h = exp(-0.5*qchisq(0.95, 1)), col = "red", lty = "dashed")
```




```
sprintf("%.3f", rend_phi$root)
```

```
## [1] "1.215"
```

Figure S2

```
cty <- erosion %>% group_by(ctylab = tolower(ctylab)) %>%
  summarise(nis = n(), qi = mean(RUSLE2==0)) %>%
  mutate(sizegroup = cut(nis, c(0, 5, 10, 20, 30)),
         qigroup = cut(qi, seq(0, 1, 0.25)))
levels(cty$qigroup) <- c(levels(cty$qigroup), "0")
levels(cty$sizegroup) <- c(levels(cty$sizegroup), "0")
cty <- cty %>% mutate(qigroup = replace(qigroup, is.na(qigroup), "0"),
                      qigroup = relevel(qigroup, "0"))
cty2 <- left_join(map_sd, cty, by = c("subregion" = "ctylab"))
cty2 <- cty2 %>% mutate(sizegroup = replace(sizegroup, is.na(sizegroup), "0"),
                        sizegroup = relevel(sizegroup, "0"))
p1 <- cty2 %>% ggplot() +
  geom_polygon(aes(x = long, y = lat, group = factor(group), fill = qigroup), alpha = 0.6) +
  scale_fill_brewer(name = "", palette = "Greens", direction = -1) +
  geom_path(aes(x = long, y = lat, group = factor(group)))
p2 <- cty2 %>% ggplot() +
  geom_polygon(aes(x = long, y = lat, group = factor(group),
                   fill = sizegroup), alpha = 0.6) +
  scale_fill_brewer(name = "", palette = "Blues") +
  geom_path(aes(x = long, y = lat, group = factor(group)))
(p1 / p2) & coord_equal() &
  ggthemes::theme_map(base_size = 18) &
  theme(legend.position = "right")
```

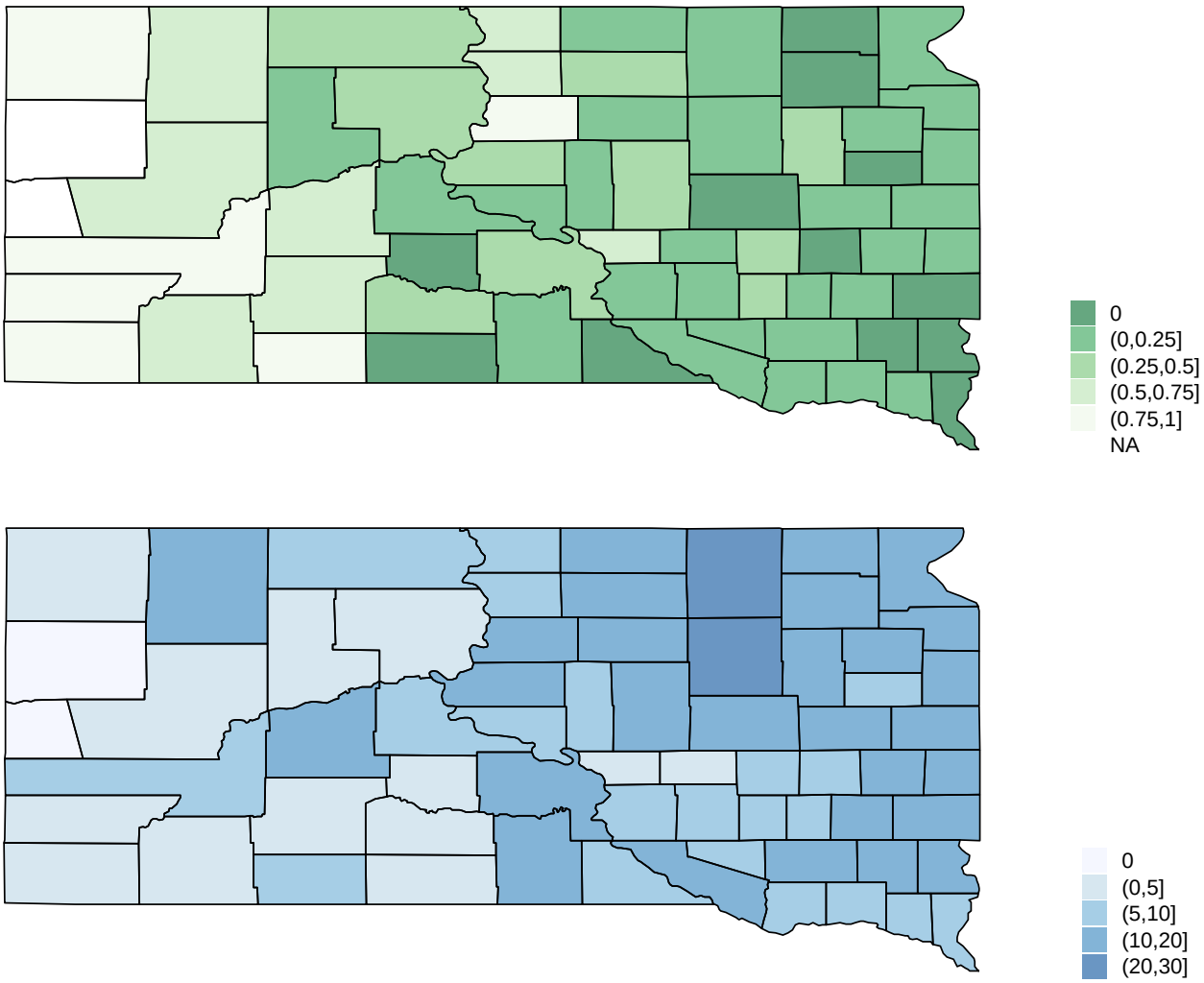


Figure S3

```
fitted.mar <- erosion_2p$lys - na.omit(fit$residuals$mar)
fitted.con <- erosion_2p$lys - na.omit(fit$residuals$con)
ggplot(data.frame(r = stderr.mar)) + stat_qq(aes(sample = r)) +
  geom_abline(slope = 1, intercept = 0) +
  labs(x = "theoretical quantile", y = "sample quantile") -> g1
ggplot(data.frame(x = fitted.mar, y = stderr.mar)) +
  geom_point(aes(x, y)) +
  labs(x = "Fitted Value", y = "Marginal Residual") +
  geom_hline(yintercept = 0) -> g2
ggplot(data.frame(r = stderr.con)) + stat_qq(aes(sample = r)) +
  geom_abline(slope = 1, intercept = 0) +
  labs(x = "theoretical quantile", y = "sample quantile") -> g3
ggplot(data.frame(x = fitted.con, y = stderr.con)) +
  geom_point(aes(x, y)) +
  labs(x = "Fitted Value", y = "Conditional Residual") +
  geom_hline(yintercept = 0) -> g4
(g1 | g2) / (g3 | g4) & theme_bw(base_size = 18)
```

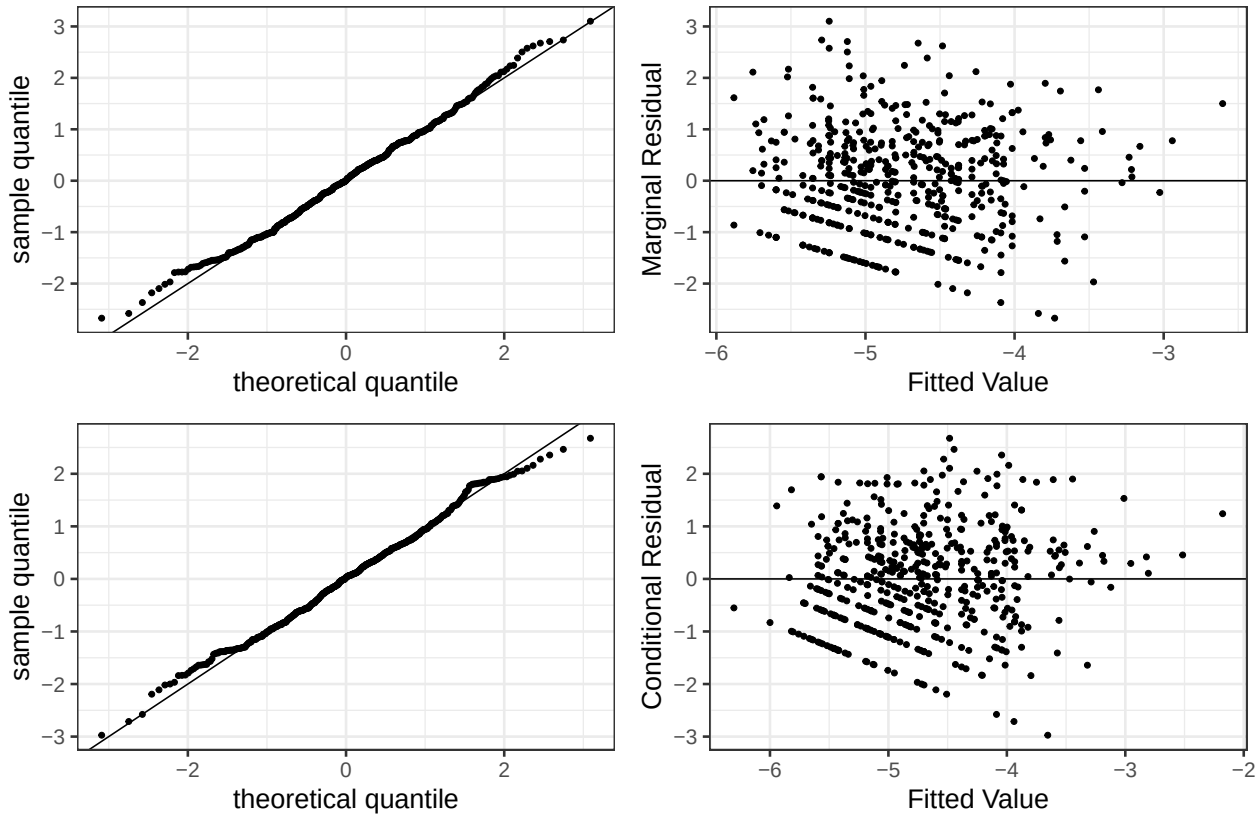


Figure S4

```
## Hosmer-Lemeshow Goodness of Fit Test
library(ResourceSelection)
etahat <- erosion_2p$Xs0 %*% fit$fixed$p0 +
  fit$random$p0[as.numeric(as.factor(erosion_2p$area))]
phat <- exp(etahat)/(1+exp(etahat))
pvalues <- sapply(5:15, function(i) hoslem.test(erosion_2p$deltas, phat, g = i)$p.value)
ggplot(data.frame(x = 5:15, y = pvalues), aes(x, y)) +
  geom_point(shape = 19, size = rel(2)) +
  geom_hline(yintercept = 0.1, linetype = "dashed")+
  ylim(0, 1) +
  labs(x = "number of groups", y = "p-value") +
  theme_bw(base_size = 18)
```

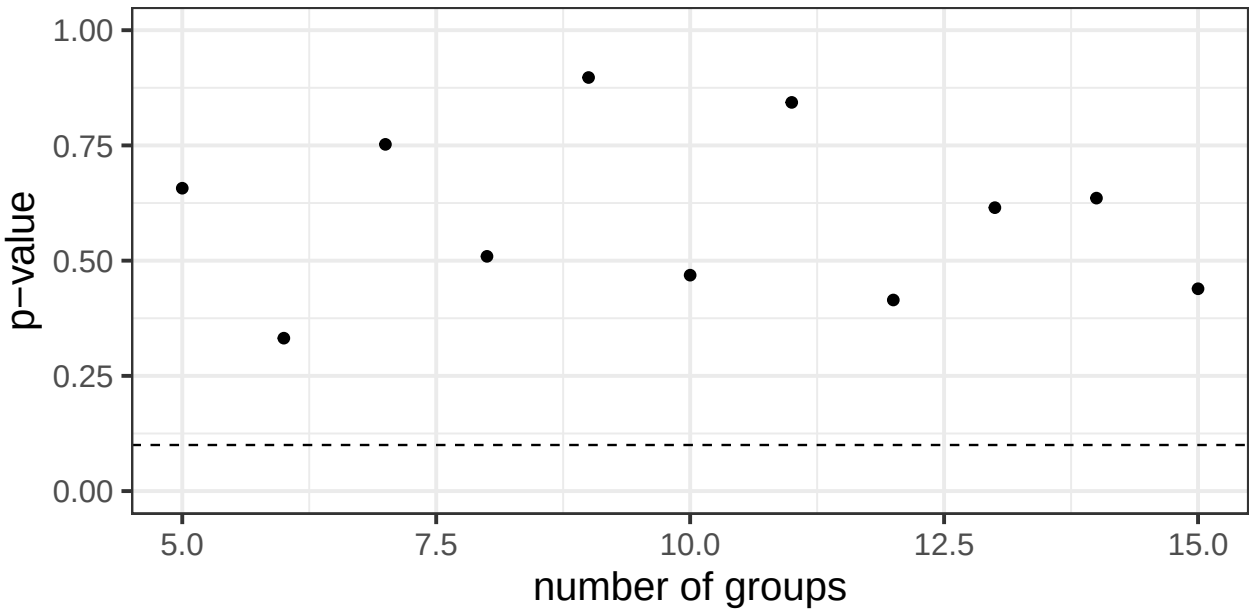
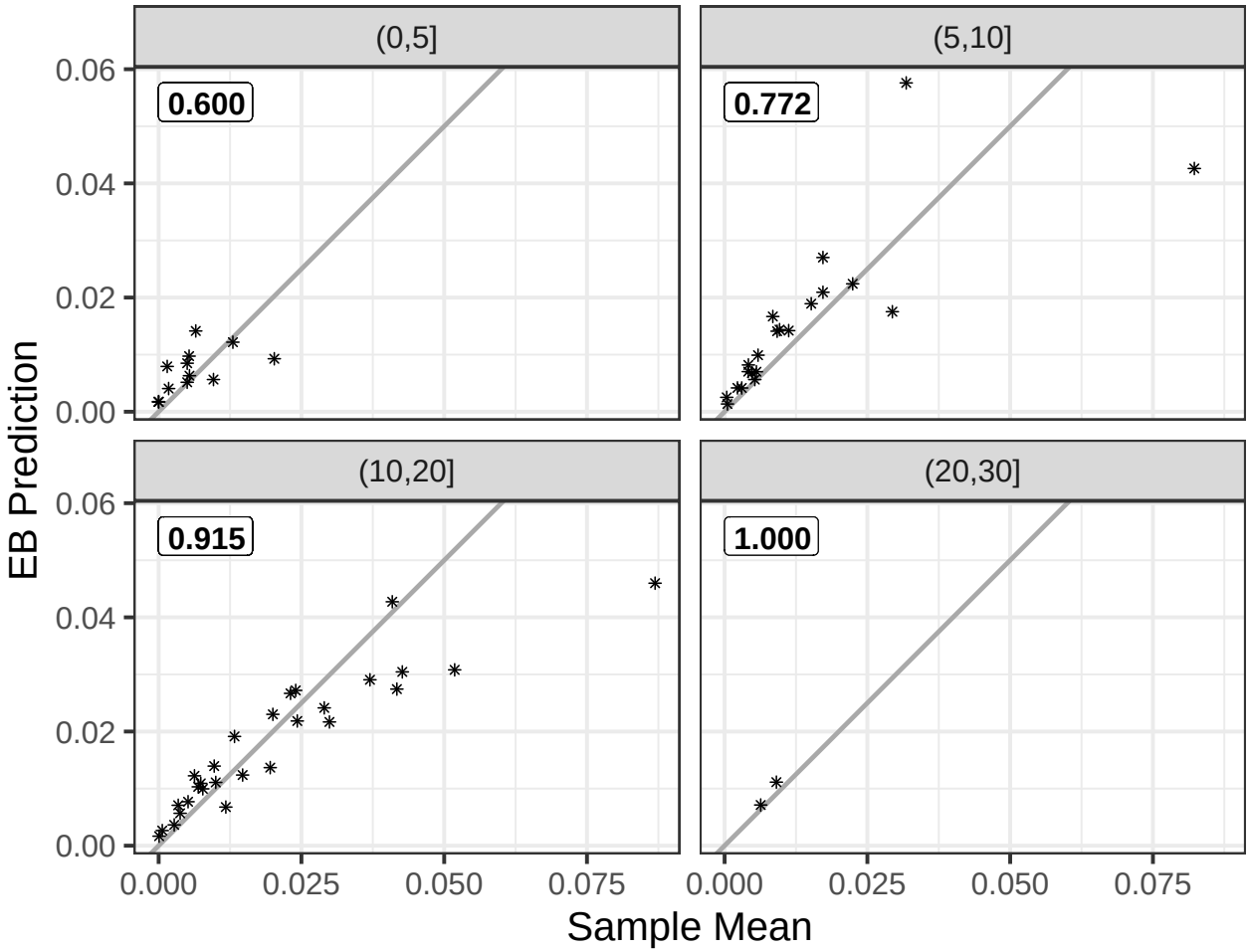


Figure S5

```
est_all <- direct %>% mutate(eb = predictions$eb) %>%
  group_by(group) %>% mutate(cor = cor(ctymean, eb))
ggplot(est_all, aes(x = ctymean, y = eb)) +
  geom_abline(slope = 1, size = rel(1), color = "darkgrey") +
  geom_point(shape = 8, size = rel(1.5)) +
  geom_label(aes(x = min(ctymean), y = max(eb),
                 label = sprintf("%.3f", cor)),
             hjust = 0, vjust = 1,
             size = 5, fontface = "bold") +
  theme_bw(base_size = 18) +
  labs(x = "Sample Mean", y = "EB Prediction") +
  coord_equal() + facet_wrap(~group, nrow = 2)
```



System Configurations

```
sessionInfo()
```

```
## R version 3.6.1 (2019-07-05)
## Platform: x86_64-apple-darwin15.6.0 (64-bit)
## Running under: macOS Catalina 10.15.4
##
## Matrix products: default
## BLAS:   /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/3.6/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] ResourceSelection_0.3-5 ggmapr_0.1.0            saezero_0.1.0
## [4] patchwork_1.0.0.9000    ggplot2_3.2.1           dplyr_0.8.3
```

```
##
## loaded via a namespace (and not attached):
##  [1] statmod_1.4.34     tidyselect_1.0.0   xfun_0.11          reshape2_1.4.3
##  [5] purrr_0.3.3        ggthemes_4.2.0     splines_3.6.1      lattice_0.20-38
##  [9] colorspace_1.4-1   vctrs_0.2.2        htmltools_0.4.0    yaml_2.2.0
## [13] utf8_1.1.4         rlang_0.4.4        pillar_1.4.3       nloptr_1.2.1
## [17] foreign_0.8-71     glue_1.3.1         withr_2.1.2        RColorBrewer_1.1-2
## [21] sp_1.3-2           plyr_1.8.4         lifecycle_0.1.0    stringr_1.4.0
## [25] munsell_0.5.0      gtable_0.3.0       evaluate_0.14      labeling_0.3
## [29] knitr_1.28         forcats_0.4.0      maptools_0.9-9     sae_1.3
## [33] fansi_0.4.1        Rcpp_1.0.3         scales_1.1.0       farver_2.0.1
## [37] lme4_1.1-21        digest_0.6.23      stringi_1.4.5      grid_3.6.1
## [41] cli_2.0.1          tools_3.6.1        magrittr_1.5       lazyeval_0.2.2
## [45] tibble_2.1.3       crayon_1.3.4       tidyr_1.0.0        pkgconfig_2.0.3
## [49] MASS_7.3-51.4      ellipsis_0.3.0     Matrix_1.2-17      assertthat_0.2.1
## [53] minqa_1.2.4        rmarkdown_2.1      R6_2.4.1           boot_1.3-22
## [57] nlme_3.1-140       compiler_3.6.1
```