I. View generation

The proposed method starts by generating the finest-scale view, where each sample is treated as a granular ball. From there, it iteratively produces coarser-scale views by merging granular balls from the previous finer-scale view, continuing this process until all samples are combined into a single granular ball, forming the coarsest-scale view.

II. Relationship between the number of views and samples

The number of views varies depending on the dataset and is inherently influenced by the number of samples. While the exact number of views differs across datasets, it generally increases with the number of samples. To explore this relationship, both theoretical analyses and experiments are conducted.

Let's assume the relationship between the number of granular balls in the k-th view and the $(k+1)$-th view follows that

$$\lim_{|U|\to\infty} \mathrm{E}[\frac{|GBS_{k+1}|}{|GBS_k|}] = q,$$

where $|GBS_k|$ is the number of granular balls in the k-th view, q $(0 < q < 1)$ is a constant determined by the given data, and $|U|$ is the number of all samples.

In simpler terms, when dealing with an infinitely large dataset, the ratio of granular balls between the $(k+1)$-th view and the k-th view eventually settles into a constant value, q. However, this value can vary depending on the dataset and is unique to each one. This means that, as view generated, the number of granular balls decreases at a steady rate. Since the initial number of granular balls, $|GBS_1|$, matches the total number of samples, $|U|$, we can introduce some constraints based on relationship as follows:

$$\begin{aligned} &\lim_{|U|\to\infty} \mathrm{E}[\frac{|GBS_{k+1}|}{|GBS_k|}] = q, (0 < q < 1) \\ &\text{s.t.} |GBS_k| \in N_+, 1 \le k \le K, \end{aligned}$$

where $K = |GBSV|$ is the number of generated views.

Further, the relationship between the number of views $K = |GBSV|$ and the number of sample $|U|$ can be deduced as

$$\begin{aligned} &|GBS_{k+1}| = q|GBS_k|, (0 < q < 1), \\ &\text{s.t.} |GBS_k| \in N_+, 1 \le k \le K, \\ \Longleftrightarrow\ &|GBS_k| = q^{k-1}|GBS_1|, (0 < q < 1), \\ &\text{s.t.} |GBS_k| \in N_+, 1 \le k \le K, \\ \Longleftrightarrow\ &|GBS_k| = q^{k-1}|U|, (0 < q < 1), \\ &\text{s.t.} |GBS_k| \in N_+, 1 \le k \le K. \end{aligned}$$

Since there is only one granule ball in the last view, i.e., $|GBS_K| = 1$. Then, we have $1 \le k \le -\log_q |U| + 1$ and $K \sim -\log_q |U| + 1 (0 < q < 1)$.

To explore how the number of views relates to the sample size, we created datasets with sample sizes ranging from 50 to 20,000, increasing by 50 samples at each step. We then recorded both the number of granular balls and the number of views for each dataset.

$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K
50	3	2550	7	5050	8	7550	8	10050	8	12550	8	15050	9	17550	8
100	5	2600	7	5100	8	7600	8	10100	9	12600	8	15100	8	17600	8
150	4	2650	7	5150	7	7650	8	10150	8	12650	8	15150	8	17650	9
200	5	2700	7	5200	8	7700	7	10200	8	12700	8	15200	8	17700	9
250	6	2750	7	5250	7	7750	8	10250	8	12750	8	15250	8	17750	9
300	5	2800	7	5300	8	7800	8	10300	8	12800	8	15300	8	17800	8
350	5	2850	7	5350	8	7850	8	10350	8	12850	7	15350	7	17850	8
400	6	2900	7	5400	8	7900	8	10400	8	12900	8	15400	8	17900	9
450	5	2950	7	5450	8	7950	8	10450	8	12950	8	15450	8	17950	8
500	6	3000	7	5500	7	8000	8	10500	8	13000	8	15500	8	18000	8
550	5	3050	7	5550	8	8050	8	10550	8	13050	8	15550	9	18050	9
600	6	3100	7	5600	8	8100	8	10600	8	13100	8	15600	9	18100	8
650	6	3150	7	5650	7	8150	8	10650	8	13150	8	15650	8	18150	8
700	6	3200	8	5700	8	8200	8	10700	8	13200	8	15700	8	18200	8
750	6	3250	7	5750	7	8250	8	10750	8	13250	8	15750	8	18250	9
800	6	3300	7	5800	8	8300	8	10800	8	13300	8	15800	8	18300	9
850	7	3350	7	5850	8	8350	8	10850	8	13350	8	15850	8	18350	9
900	6	3400	7	5900	8	8400	8	10900	8	13400	8	15900	8	18400	8
950	6	3450	7	5950	7	8450	8	10950	8	13450	8	15950	8	18450	8
1000	7	3500	7	6000	8	8500	8	11000	8	13500	8	16000	8	18500	9

$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K	$\lvert U\rvert$	K
1050	6	3550	8	6050	8	8550	8	11050	9	13550	8	16050	9	18550	8
1100	7	3600	7	6100	8	8600	8	11100	8	13600	9	16100	8	18600	8
1150	7	3650	7	6150	8	8650	8	11150	8	13650	8	16150	8	18650	9
1200	7	3700	8	6200	8	8700	8	11200	8	13700	8	16200	8	18700	9
1250	6	3750	7	6250	7	8750	8	11250	8	13750	8	16250	8	18750	9
1300	7	3800	7	6300	8	8800	8	11300	8	13800	8	16300	9	18800	9
1350	6	3850	7	6350	8	8850	8	11350	8	13850	8	16350	8	18850	9
1400	7	3900	8	6400	8	8900	8	11400	8	13900	8	16400	8	18900	9
1450	7	3950	7	6450	8	8950	8	11450	8	13950	8	16450	9	18950	9
1500	6	4000	7	6500	8	9000	8	11500	9	14000	9	16500	8	19000	8
1550	7	4050	7	6550	8	9050	8	11550	8	14050	8	16550	8	19050	8
1600	6	4100	7	6600	8	9100	8	11600	8	14100	8	16600	9	19100	9
1650	6	4150	7	6650	7	9150	7	11650	8	14150	8	16650	8	19150	9
1700	7	4200	8	6700	8	9200	7	11700	8	14200	9	16700	9	19200	9
1750	7	4250	7	6750	7	9250	8	11750	8	14250	9	16750	8	19250	9
1800	7	4300	7	6800	8	9300	8	11800	8	14300	8	16800	8	19300	9
1850	7	4350	7	6850	8	9350	8	11850	8	14350	8	16850	9	19350	9
1900	7	4400	7	6900	8	9400	8	11900	8	14400	8	16900	9	19400	9
1950	7	4450	7	6950	8	9450	8	11950	8	14450	8	16950	8	19450	9
2000	7	4500	7	7000	7	9500	8	12000	8	14500	8	17000	9	19500	8
2050	7	4550	8	7050	7	9550	8	12050	8	14550	8	17050	9	19550	8
2100	7	4600	7	7100	8	9600	8	12100	8	14600	9	17100	8	19600	9
2150	7	4650	8	7150	7	9650	8	12150	8	14650	9	17150	9	19650	8
2200	7	4700	8	7200	8	9700	8	12200	8	14700	8	17200	9	19700	9
2250	7	4750	8	7250	8	9750	8	12250	8	14750	9	17250	8	19750	9
2300	7	4800	8	7300	7	9800	8	12300	8	14800	8	17300	8	19800	9
2350	7	4850	8	7350	8	9850	8	12350	8	14850	8	17350	9	19850	10
2400	7	4900	7	7400	8	9900	8	12400	8	14900	8	17400	9	19900	8
2450	7	4950	8	7450	8	9950	8	12450	8	14950	8	17450	8	19950	9
2500	7	5000	8	7500	8	10000	9	12500	8	15000	8	17500	9	20000	9

From this table, it's evident that the number of generated views changes with different sample sizes but generally increases as the sample size grows. To further investigate this relationship, we performed a statistical analysis on the data. The results of this analysis are presented below.

K	$\lvert U\rvert$	$\log_{10}\lvert U\rvert$	$(\log_{10} K)/\lvert U\rvert$
5	325.0000	2.5119	0.5024
6	931.2500	2.9691	0.4948
7	3621.5190	3.5589	0.5084
8	11294.0928	4.0529	0.5066

In this table, the first and second columns show the number of generated views and the average number of samples corresponding to each number of views, respectively. The third and fourth columns display the logarithmic values of the average sample size and the ratio of the logarithmic value of the number of generated views to the average sample size. Note that results are presented only for view numbers 3, 4, 9, and 10, as data for other numbers of views was insufficient to reveal a clear trend. By examining the results in the last column, it is observed that the ratio values are approximately equal and converge to a constant (approximately 0.5), suggesting that the number of views and the number of samples follow the relationship:

$$
\begin{aligned}
&\frac{\log_{10} K}{\lvert U\rvert} \approx 0.5, \\
\Longleftrightarrow\ &\lvert U\rvert \approx 2\log_{10} K, \\
\Longleftrightarrow\ &\lvert U\rvert \approx 2\frac{\log_{\sqrt{10}} K}{\log_{\sqrt{10}} 10}, \\
\Longleftrightarrow\ &\lvert U\rvert \approx \log_{\sqrt{10}} K, \\
\Longleftrightarrow\ &\lvert U\rvert \approx -\log_{\frac{1}{\sqrt{10}}} K.
\end{aligned}
$$

These results are consistent with the conclusion. In such cases, the constant q is equal to $\frac{1}{\sqrt{10}}$.

III. Time complexity

The proposed method is structured into three stages: generating multi-scale views, computing sample outlier scores, and training a weighted SVM. In the initial draft, we analyzed the time complexity of computing sample outlier scores and training the weighted SVM, both of which have time costs of $O(|A||U|^2)$, where $|A|$ is the number of attributes and $|U|$ is the number of samples. We also noted that generating single-scale granular balls costs less than $O(|U|\log|U|)$, as detailed in subsection 3.3. Consequently, it is important to evaluate the time cost associated with generating multi-scale views.

For the k-th scale view, the time cost to generate granular balls is $O(|GBS_k|\log|GBS_k|)$, where $|GBS_k|$ represents the number of granular balls in the k-th scale view. Thus, the total time cost for generating multi-scale views is $O(\sum_{k=1}^{K}|GBS_k|\log|GBS_k|)$, with $K=|GBSV|$ denoting the number of generated views. As discussed, the number of views K approximately follows $K\sim\log|U|$. Therefore, the time cost for view generation can be computed as follows:

$$\begin{aligned}
&O(\sum_{k=1}^{K}|GBS_k|\log|GBS_k|)\\
\leq &O(\sum_{k=1}^{K}|U|\log|U|)\\
= &O(K|U|\log|U|)\\
\leq &O(|U|(\log|U|)^2).
\end{aligned}$$

Similarly, the time cost for anomaly detection across all views can be computed as follows:

$$\begin{aligned}
&O(\sum_{k=1}^{K}|A||GBS_k|^2)\\
\leq &O(\sum_{k=1}^{K}|A||U|^2)\\
= &O(K|A||U|^2)\\
\leq &O(|A||U|^2\log|U|)
\end{aligned}$$

Hence, the overall time cost is calculated as

$$\begin{aligned}
&O(|U|(\log|U|)^2+|A||U|^2\log|U|+|A||U|^2)\\
= &O(|A||U|^2\log|U|).
\end{aligned}$$

The results indicate that the proposed multiple-view method introduces an additional cost that scales as $\log|U|$ compared to single-view methods. However, both theoretical analysis and experimental results demonstrate that this additional cost is relatively minor.