The Supplementary Material of the Paper "A Stepwise Weight Adaptation Approach for MOEA/D"

I. Supplementary Tables

TABLE I
HV results (mean and SD) of MOEA/D-AWA and SWA-MOEA/D-AWA.

Problem	MOEA/D-AWA	SWA-MOEA/D-AWA
DTLZ1	9.715E-01(2.3E-03)†	**9.740E-01(4.1E-05)**
DTLZ2	7.421E+00(2.9E-04)	7.421E+00(2.6E-03)
DTLZ3	7.415E+00(7.1E-03)	7.415E+00(4.0E-03)
DTLZ5	2.188E-01(8.4E-05)†	**2.196E-01(4.1E-05)**
DTLZ7	1.318E+01(5.5E-01)†	**1.319E+01(9.6E-01)**
CDTLZ2	7.951E+00(4.8E-04)†	**7.951E+00(1.9E-04)**
IDTLZ1	6.884E-01(4.7E-04)	6.882E-01(4.3E-04)
IDTLZ2	6.723E+00(5.9E-03)†	**6.726E+00(3.9E-03)**
ZDT3	4.815E+00(1.4E-04)†	**4.815E+00(1.2E-04)**
FON1	**3.062E+00(6.2E-05)**†	3.062E+00(1.5E-03)
SCH1	**2.228E+01(6.9E-04)**†	2.227E+01(9.1E-04)
SCH2	3.822E+01(2.6E-03)†	**3.826E+01(9.0E-04)**
SDTLZ1	1.657E+02(4.5E-01)†	**1.662E+02(4.3E-03)**
SDTLZ2	**1.326E+03(1.0E-02)**†	1.326E+03(7.1E-02)
VNT2	**1.648E+03(4.6E-03)**†	1.648E+03(6.7E-03)
DTLZ1-5	9.988E-01(1.8E-04)†	**9.989E-01(3.3E-04)**
DTLZ2-5	3.165E+01(1.4E-02)†	**3.169E+01(5.0E-03)**
DTLZ5IM-5	8.575E+03(1.3E+00)†	**8.579E+03(1.2E-01)**
DTLZ7-5	7.225E+01(7.9E+00)	7.737E+01(8.6E+00)
DTLZ1-10	**9.971E-01(2.0E-03)**†	9.948E-01(3.5E-03)
DTLZ2-10	**9.588E-01(1.2E-02)**†	9.436E-01(2.1E-02)
DTLZ5IM-10	9.889E-02(8.8E-04)†	**1.007E-01(3.2E-04)**
DTLZ7-10	1.834E-01(9.0E-03)†	**2.048E-01(7.1E-03)**
IDTLZ1-10	7.355E-08(3.3E-08)†	**3.062E-07(5.0E-07)**
$+/=/-$	$14/4/6$	

'†' indicates that SWA-MOEA/D-AWA is of statistically significant difference from MOEA/D-AWA at a 0.05 level by Wilcoxon's rank sum test. The better mean for each case is highlighted in boldface. The symbols $+$, $-$, and $=$ indicate that the results of SWA-MOEA/D-AWA is significantly better than, worse than, and equivalent to MOEA/D-AWA.

Table I shows the comparative HV results of MOEA/D-AWA and SWA-MOEA/D-AWA on the 24 test problems regarding the mean and standard deviation (SD) values. To have statistically sound conclusions, Wilcoxon's rank sum test at a 0.05 significance level was adopted to test the significance of the differences between evaluation results obtained by two competing algorithms. As can be seen from the table, SWA-MOEA/D-AWA wins on 14 test problems out of 24 test problems, while MOEA/D-AWA wins on 6 test problems.

Table II shows the comparative HV results of the two algorithms on the 24 test problems. In terms of HV, SWA-MOEA/D-AWA wins on 12 test problems out of 24 test problems. In contrast, AdaW wins on 4 test problems.

Table III shows the HV results of SWA-AdaW with the other five state-of-the-art algorithms. As can be seen from the table, SWA-AdaW ranks the best overall compared with the five peer algorithms. SWA-AdaW outperforms MOEA/D, A-NSGA-III, DEA-GNG, iRVEA and RVEA-iGNG on 17, 20, 21, 21, and 13 test problems out of 24 test problems in total, respectively. In contrast, MOEA/D, A-NSGA-III, DEA-GNG, iRVEA, and RVEA-iGNG win on 5, 1, 2, 2, and 5 test problems, respectively.

TABLE II
HV results (mean and SD) of AdaW and SWA-AdaW.

Problem	AdaW	SWA-AdaW
DTLZ1	**9.739E-01(1.4E-04)**†	9.737E-01(2.8E-04)
DTLZ2	7.410E+00(6.4E-03)†	**7.414E+00(6.9E-03)**
DTLZ3	7.393E+00(2.7E-02)†	**7.414E+00(4.3E-03)**
DTLZ5	2.197E-01(5.5E-05)	2.197E-01(1.2E-04)
DTLZ7	1.349E+01(3.2E-02)†	**1.351E+01(1.5E-02)**
CDTLZ2	**7.952E+00(2.1E-04)**†	7.951E+00(2.2E-04)
IDTLZ1	6.864E-01(2.2E-03)	6.870E-01(1.1E-03)
IDTLZ2	6.729E+00(4.0E-03)	6.728E+00(3.3E-03)
ZDT3	4.815E+00(1.4E-03)†	**4.815E+00(4.9E-04)**
FON1	**3.061E+00(4.2E-03)**†	3.053E+00(7.7E-03)
SCH1	**2.227E+01(7.6E-04)**†	2.227E+01(8.9E-04)
SCH2	3.825E+01(4.7E-03)†	**3.826E+01(3.3E-03)**
SDTLZ1	1.402E+02(9.8E-02)†	**1.430E+02(1.3E-02)**
SDTLZ2	7.481E+02(8.3E-01)†	**7.557E+02(6.9E-01)**
VNT2	1.648E+03(6.0E-03)	1.648E+03(7.1E-03)
DTLZ1-5	9.990E-01(4.0E-05)	9.990E-01(4.2E-05)
DTLZ2-5	3.167E+01(1.1E-02)†	**3.169E+01(1.1E-02)**
DTLZ5IM-5	8.579E+03(6.4E-01)	8.578E+03(2.0E+00)
DTLZ7-5	8.497E+01(1.5E+00)†	**8.588E+01(3.6E+00)**
DTLZ1-10	9.985E-01(7.7E-04)	9.978E-01(1.8E-03)
DTLZ2-10	9.611E-01(1.2E-03)†	**9.643E-01(1.1E-03)**
DTLZ5IM-10	1.009E-01(2.9E-04)	1.010E-01(2.7E-04)
DTLZ7-10	1.749E-01(7.4E-03)†	**1.831E-01(9.0E-03)**
IDTLZ1-10	3.893E-07(5.7E-07)†	**5.948E-07(4.6E-07)**
$+/=/-$	$12/8/4$	

'†' indicates that SWA-AdaW is of statistically significant difference from AdaW at a 0.05 level by Wilcoxon's rank sum test. The best mean for each case is highlighted in boldface. The symbols $+$, $-$, and $=$ indicate that the results of SWA-AdaW is significantly better than, worse than, and equivalent to AdaW.

TABLE III
HV RESULTS (MEAN AND SD) OF THE SIX ALGORITHMS.

Problem	MOEA/D	A-NSGA-III	DEA-GNG	iRVEA	RVEA-iGNG	SWU-AdaW
DTLZ1	**9.741E-01(4.0E-05)**†	9.712E-01(2.6E-03)†	9.467E-01(1.8E-02)†	9.734E-01(5.4E-04)†	9.737E-01(1.2E-04)	9.737E-01(2.8E-04)
DTLZ2	**7.418E+00(4.3E-06)**	7.415E+00(3.2E-03)	7.349E+00(2.6E-02)†	7.411E+00(1.4E-03)†	7.414E+00(3.5E-03)	7.414E+00(6.9E-03)
DTLZ3	7.412E+00(5.3E-03)†	7.383E+00(1.9E-02)†	7.412E+00(5.3E-03)†	7.402E+00(2.3E-02)†	7.413E+00(4.3E-03)	**7.414E+00(4.3E-03)**
DTLZ5	1.993E-01(6.0E-06)†	2.134E-01(1.8E-03)†	2.194E-01(2.1E-04)†	2.196E-01(7.9E-05)†	2.196E-01(6.0E-05)†	**2.197E-01(1.2E-04)**
DTLZ7	1.313E+01(1.1E+00)†	1.334E+01(4.4E-02)†	1.307E+01(5.6E-01)†	1.343E+01(6.2E-02)†	1.317E+01(9.5E-01)†	**1.351E+01(1.5E-02)**
CDTLZ2	7.907E+00(3.6E-03)†	7.936E+00(7.5E-03)†	7.909E+00(2.2E-02)†	7.949E+00(5.7E-04)†	7.950E+00(9.4E-04)†	**7.951E+00(2.2E-04)**
IDTLZ1	6.678E-01(6.3E-05)†	6.676E-01(1.5E-03)†	6.637E-01(1.5E-02)†	6.814E-01(4.9E-03)†	**6.878E-01(4.6E-04)**†	6.870E-01(1.1E-03)
IDTLZ2	6.638E+00(2.7E-03)†	6.614E+00(4.0E-02)†	6.584E+00(4.5E-02)†	6.696E+00(1.2E-02)†	6.691E+00(1.6E-02)†	**6.728E+00(3.3E-03)**
ZDT3	4.731E+00(1.1E-01)†	4.814E+00(1.9E-04)†	4.812E+00(8.5E-04)†	4.763E+00(1.3E-01)†	4.791E+00(9.3E-02)†	**4.815E+00(4.9E-04)**
FON1	3.062E+00(7.1E-05)†	3.058E+00(4.3E-03)	3.062E+00(1.3E-03)†	3.062E+00(1.4E-04)†	**3.063E+00(5.7E-05)**†	3.053E+00(7.7E-03)
SCH1	2.197E+01(7.6E-03)†	2.225E+01(6.1E-03)†	2.227E+01(2.8E-03)†	2.227E+01(1.9E-03)†	**2.228E+01(7.9E-04)**†	2.227E+01(8.9E-04)
SCH2	3.365E+01(5.4E-04)†	3.465E+01(9.3E-03)†	3.462E+01(5.4E-02)†	3.463E+01(2.0E-02)†	3.396E+01(4.3E-01)†	**3.826E+01(3.3E-03)**
SDTLZ1	9.687E+01(3.5E-02)†	1.399E+02(5.6E-01)	1.265E+02(1.3E+01)†	1.367E+02(4.8E+00)†	1.392E+02(1.4E-01)†	**1.430E+02(1.3E-02)**
SDTLZ2	4.858E+02(5.4E-02)†	7.392E+02(2.3E+00)†	7.333E+02(2.6E+00)†	7.404E+02(1.2E+00)†	7.410E+02(2.0E+00)†	**7.557E+02(6.9E-01)**
VNT2	1.609E+03(2.1E-02)†	1.638E+03(1.0E+01)†	1.647E+03(1.2E+00)†	1.647E+03(1.2E+00)†	1.647E+03(3.3E-01)†	**1.648E+03(7.1E-03)**
DTLZ1-5	**9.990E-01(1.6E-05)**	9.989E-01(1.1E-04)†	8.420E-01(1.1E-01)†	9.989E-01(3.8E-05)†	9.989E-01(3.6E-05)†	9.990E-01(4.2E-05)
DTLZ2-5	**3.170E+01(4.5E-05)**†	3.170E+01(2.3E-03)†	3.085E+01(2.0E-01)†	3.168E+01(2.4E-03)†	3.168E+01(5.0E-03)†	3.169E+01(1.1E-02)
DTLZ5IM-5	8.256E+03(7.1E+02)†	8.544E+03(3.4E+01)†	**8.579E+03(2.0E-02)**†	8.679E+03(2.4E-02)†	8.579E+03(1.9E-01)†	8.578E+03(2.0E+00)
DTLZ7-5	8.034E+01(4.0E-01)†	7.889E+01(1.8E+00)†	7.510E+01(2.5E+00)†	8.351E+01(1.0E+00)†	8.523E+01(3.3E+00)†	**8.588E+01(3.6E+00)**
DTLZ1-10	**9.995E-01(3.4E-05)**†	9.936E-01(2.1E-02)†	7.510E-01(9.7E-02)†	3.309E-01(3.3E-01)†	9.986E-01(3.3E-04)	9.978E-01(1.8E-03)
DTLZ2-10	**9.679E-01(1.9E-04)**†	9.475E-01(4.1E-02)†	8.399E-01(2.9E-02)†	9.591E-01(1.4E-03)†	9.588E-01(1.2E-02)	9.642E-01(1.1E-03)
DTLZ5IM-10	8.583E-02(3.4E-02)†	3.387E-02(3.1E-02)†	5.035E-03(1.9E-02)†	1.002E-01(3.3E-03)	1.008E-01(3.0E-04)	**1.010E-01(2.7E-04)**
DTLZ7-10	7.477E-04(3.1E-03)†	1.768E-01(9.8E-03)†	1.886E-01(1.7E-02)	1.217E-01(3.0E-02)†	**1.914E-01(5.9E-03)**†	1.831E-01(9.0E-03)
IDTLZ1-10	8.797E-09(4.5E-09)†	4.793E-07(5.8E-08)†	4.425E-07(1.5E-07)†	4.965E-07(7.3E-07)†	3.384E-07(2.0E-07)†	**5.948E-07(4.6E-07)**
$+/=/-$	$17/2/5$	$20/3/1$	$21/1/2$	$21/1/2$	$13/6/5$	

$'\dagger'$ indicates that SWA-AdaW is of statistically significant difference from the corresponding peer algorithm at a 0.05 level by Wilcoxon's rank sum test. The best mean for each case is highlighted in boldface. The symbols $+$, $-$, and $=$ indicate that the results of SWA-AdaW is significantly better than, worse than, and equivalent to the corresponding peer algorithm.