The Brain as Quantum-like Machine Operating on Subcognitive and Cognitive Time Scales

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Abstract

We propose a *quantum-like* (QL) model of the functioning of the brain. It should be sharply distinguished from the reductionist *quantum* model. By the latter cognition is created by *physical quantum processes* in the brain. The crucial point of our modelling is that discovery of the mathematical formalism of quantum mechanics (QM) was in fact discovery of a very general formalism describing *consistent processing of incomplete information* about contexts (physical, mental, economic, social). The brain is an advanced device which developed the ability to create a QL representation of contexts. Therefore its functioning can also be described by the mathematical formalism of QM. The possibility of such a description has nothing to do with composing of the brain of quantum systems (photons, electrons, protons,...). Moreover, we shall propose a model in that the QL representation is based on conventional neurophysiological model of the functioning of the brain. The brain uses the QL rule (given by von Neumann trace formula) for calculation of *approximative averages* for mental functions, but the physical basis of mental functions is given by neural networks in the brain. The QL representation has a *temporal basis*. Any cognitive process is based on (at least) two time scales:

subcognitive time scale (which is very fine) and cognitive time scale (which is essentially coarser).

1   Introduction

We start with formulation of a number of questions which will be considered in this paper. We shall try to find proper answers to those questions. From the very beginning we recognize that the present level of experimental research in quantum physics, neurophysiology, cognitive science, and psychology is not high enough to support (or to reject) our quantum-like (QL) model of cognition.

Although basic theoretical tools behind our model are mathematics and physics, we would like to address this paper to really multi-disciplinary auditorium: philosophers, neurophysiologists, psychologists, mathematicians, physicists, people working in cognitive and social sciences, economy (our model can be easily generalized from a single brain to a collection of brains), and even abnormal phenomena. Therefore we practically excluded mathematical technique from this paper. A reader can find corresponding details in references on author's papers, e.g., Khrennikov (2005a,b, 2006b–d).

This paper has a detailed introduction which is in principle sufficiently complete to create a general picture of our QL cognitive model. The last part of this paper is more complicated. It can be interesting for neurophysiologists and psychologists, since we couple our QL model with the temporal structure of processes in the functioning of the brain. A number of time scales will be discussed as possible candidates for QL and "sub-QL" (vs. cognitive and subcognitive) time scales.

1.1   Questions and answers

1). Should one distinguish the mathematical formalism of QM from quantum physics? Yes.

2). Can be this formalism be applied outside physics, e.g. in cognitive science or sociology? Yes. But corresponding QL models should be tested experimentally.

3). Can one escape in the QL framework difficulties of existing "really quantum" models of cognition (based on reduction of cognition to quantum

physical processes in the brain)? Yes.

4). Can the present neurophysiological model of the functioning of the brain be combined with a QL model? Yes, see also question 9.

5). Can quantum formalism coexist with hidden variables? It seems – yes, but the problem is extremely complicated. Further theoretical and experimental investigations should be performed.

6). Did Bell's arguments (and corresponding experiments, e.g., Aspect et al (1982) and Weihs et al (1998), Weihs (2007)) as well as other "NO-GO" theorems (e.g. von Neumann, Kochen-Specker) really imply that local realism is incompatible with the formalism of QM? Not at all, but the problem is extremely complicated. Further theoretical and experimental investigations should be performed.

7). What (Who?) is quantum (QL): the consciousness or the unconsciousness? By our model the consciousness is QL and the unconsciousness is classical (the latter operates via classical neural networks). We remark that our viewpoint is *opposite* to the conventional viewpoint of quantum reductionist approach. By the latter the unconsciousness is quantum and the consciousness is classical.

8). Are QL cognitive models consistent with ideas of Leibniz, Freud, Nietzsche, see also Näätänen (1992) for modern neurophysioloical considerations of this problem, that consciousness is a projection of huge ocean of unconscious information? Yes, by our interpretation the mathematical formalism of QM describes *operation with flows of incomplete information.*

9). What is a neural mechanism of the QL conscious representation of information? We propose a model in that the QL representation is based on the presence of variety of time scales in processing of mental information in the brain.

1.2 Short description of quantum-like model of cognition

UN). *Unconsciousness.* Each brain operates with huge amounts of unconscious information which is produced via neural activity. Some basic cognitive processes (at low levels of mental organization) are performed in this unconscious neural representation.

PR). *Projection.* The brain projects this unconscious information ocean on the QL representation. It is a probabilistic representation. Information about neural activity is represented by a complex probability amplitude, *a mental wave function.*

CON). *Consciousness.* We identify such a QL projection with the *conscious representation.* The consciousness operates with complex probability amplitudes. Its decision making is based on probabilities and averages described by the mathematical formalism of QM.

COM). *The brain as QL computer.* Hence, the consciousness operates with algorithms which can be described by the mathematical formalism of QM. These are so called quantum algorithms. Since the brain realizes them without appealing to "really quantum physical processes in microworld," we can consider it as QL (and not as quantum) computer, cf. Penrose's model of the brain as quantum computer.

HV). *Hidden mental variables.* Our QL model of cognition has hidden variables – states of neurons. Thus advantages of QL computation, in contrast to really quantum computation, are not due to superposition of states for individual systems (in our case neurons), but due to parallel working of billions of neurons. The consciousness does not operate in the neural representation. It operates in the QL representation. Therefore it proceeds essentially faster than classical computer[1]. However, the QL representation is created by activity of a huge number of neurons working parallel . This is a purely classical explanation of the process of QL computation.

G). *The temporal origin of the QL representation.* We suppose that any conscious QL representation (and the brain can operate with a number of such representations) is based on two time scales: sub-QL (subcognitive) and QL (cognitive). Probabilities and averages with respect to classical stochastic processes for pre-QL time scale are represented in the QL way. Then consciousness operates with such QL probabilistic objects on the QL (cognitive) time scale.

E). The QL representation of averages is based on approximation of classical averages by using the Taylor approximation of psychological functions. Operation with approximations (instead of complete classical averages) tremendously increases the speed of processing of probabilistic information.

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[1]In particular, I completely agree with Penrose's critique of the famous in 50s-80s project of artificial intelligence which was an attempt to explain cognition in the framework of classical computations.

1.3 On the level of mathematics and physics used in this paper

We remark that the only mathematical things which will be used in the last (more technical) part are three formulas, two from probability theory (one from classical and another from quantum) and third from mathematical analysis:

a) The classical average (also called mean value or mathematical expectation) of a random variable is given by integral. In the case of a discrete random variable it is simply a normalized sum.

b) The quantum average is given by the operator-trace in Hilbert space.[2]

c) Taylor's formula: any smooth function can be approximated by using its derivatives (by Taylor's polynomial).

However, one can proceed rather far (in any event through an extended introduction) before she will meet mathematics at all. Even in the last part we escaped long mathematical formulas. There will be used just a few mathematical symbols.

Regarding quantum physics we only assume that a reader is familiar with such fundamental problems of quantum foundations as Eistein-Polodosky-Rosen paradox, completeness/incompleteness of QM, Bell's inequality, "death of reality", quantum nonlocality. One need not know mathematical and physical details, but just the general picture of 80 years of debates on quantum foundations.

1.4 Quantum vs. quantum-like cognitive models

The idea that the description of brain functioning, cognition, and consciousness could not be reduced to the theory of neural networks and dynamical systems (cf. Ashby (1952), Hopfield (1982), Amit (1989), Bechtel and Abrahamsen (1991), Strogatz (1994), van Gelder (1995), van Gelder and Port (1995), Eliasmith (1996)) and that quantum theory may play an important role in such a description has been discussed in a huge variety of forms, see e.g. Whitehead (1929, 1933, 1939), Orlov (1982), Healey (1984), Albert and Loewer (1988, 1992), Lockwood (1989, 1996), Penrose (1989, 1994), Donald (1990, 1995, 1996), Jibu and Yasue (1992, 1994), Bohm and Hiley (1993), Stapp (1993), Aerts, D. and Aerts, S. (1995, 2007), Hameroff (1994, 1998),

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[2]In the case of discrete spectrum the operator trace is simply the sum of diagonal elements of the corresponding matrix.

Loewer (1996), Hiley and Pylkkänen (1997), Deutsch (1997), Barrett (1999), Khrennikov (1999a, 2000, 2002b, 2003b, 2004a, 2006a), Hiley (2000), Vitiello (2001), Choustova (2001, 2004), Behera et al (2005), Haven (2006), Conte et al. (2007), Atmanspacher (2007) and literature thereby.

This idea that QM might have some consequences for cognitive science and psychology was discussed at many occasions already by fathers of quantum theory. We can mention, for example, attempts of Niels Bohr to apply the quantum principle of complementarity to psychology (see A. Plotnitsky 2001, 2002, 2007 for discussions). We can also mention the correspondence between Pauli and Jung about analogy between quantum and mental processes.

During the last 30 years it was done a lot for the realization of the very ambitious program of quantum reductionism. There were various attempts to reduce mental processes to quantum physical processes in the brain. Here we point out to fundamental works Hameroff (1994, 1998) and Penrose (1989, 1994, 2005).

However, the quantum formalism provides essentially more possibilities for modelling of physical, biological, and social processes. One should distinguish QM as physical theory and its formalism. In principle, there is nothing surprising that a formalism which was originally developed for serving for one special physical theory can be used in other domains of science. For example, we are not surprised that differential calculus which was developed to serve for classical Newtonian mechanics was later used in field theory, QM, biology, economics. Nobody protests against applying the classical probability calculus (the Kolmogorov measure-theoretic model) to modelling of financial processes and so on. In the same way we import the mathematical formalism of quantum mechanics to cognitive science and psychology, even without trying to perform a reduction of mental processes to quantum physical processes.

To escape misunderstanding, we shall reserve notations classical and quantum for physics. And in applications outside physics we shall use notations *classical-like* (CL) and *quantum-like* (QL).

In this article we shall argue that the functioning of the human brain, in particular that responsible for and engaging with consciousness, may be QL, without, as against the arguments advanced by recent quantum theoretical approaches to consciousness, necessary being physically *quantum.* Specifically the QL character of this functioning means that it may be described by

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the same mathematical formalism as that used in QM (say, in von Neumann's version of the formalism), even though the physical dynamics underlying this functioning may not be the same as that of the physical processes considered in QM (as physical theory). In other words, the article offers a new *mathematical model* of the dynamics of the brain, the model based on the same mathematics as that used in QM, without viewing this dynamics itself as physically quantum. In opposite to views of quantum (physical) reductionists of brain's functioning – e.g. Hameroff (1994, 1998) and Penrose (1989, 1994, 2005), we are not interested in brain's physical structure on the micro level. We do not expect to extract cognitive features of the brain from quantum physical processes.

1.5   On quantum reduction of cognition

By using QL-models we might escape some problems arising in the quantum reductionist approach, e.g., the presence of the huge gap between the quantum (physical) and neurophysiological scales.

Roughly speaking at the neuronal level the brain is too hot and too noisy to be able to operate as a physical quantum system. Of course, one could try to save the quantum reductionist model by rejecting the commonly accepted neuronal model of the functioning of the brain. One might speculate that "real mental life" is going on the micro level and that the macro neuronal activity plays a subsidiary mental role.

Moreover, we could not reject even the hypothetical possibility that "real mental life" is going on the scales of space and time which are even finer than those which are coupled to QM (of e.g. electrons, protons or neutrons): Penrose speculated that consciousness is created on the scales of *quantum gravity.*

However, such a revolutionary approach would lead to a number of problems. First it is not easy to accept that neurons are not at all basic units of information processing in the brain. Both theoretical and experimental neurophysiology give a huge number of evidences in favor of neural processing of cognitive information. Then even if neural activity plays a subsidiary role in cognitive processes comparing with basic quantum activity in the crowed, the brain could not completely ignore the former. Therefore we should be able to explain how "quantum cognition" could be lifted from micro world to the neural level. At the moment such "lifting models" do not exist.

1.6 Can neuronal and quantum-like models be combined?

As was pointed out the QL approach provides just a special representation of information which is described by the mathematical apparatus of QM – the calculus of averages which are calculated not by using classical integration (summation in the case of discrete random variables), but by taking the operator trace in Hilbert space.

Therefore information underlying the QL representation can be produced by micro as well as macro systems. In contrast to "really quantum" reductionist theories of cognition, a QL model can be based on the QL representation of (purely classical) information produced by neurons, see Khrennikov (2002b, 2004a) and Behera et al. (2005).

One can see huge difference between our QL approach and the "really quantum" approach. By the latter a neuron could not process quantum information, because it (as a hot and noisy macroscopic system) could not be being in superposition of two macroscopically distinguishable states: firing and nonfiring.[3] We do not need to consider such "neuronal Schrödinger cats." In a coming QL model of cognition internal states of neurons are "mental hidden variables" for the QL representation.

1.7 Conflict with the Copenhagen interpretation of quantum mechanics

Of course, any reader who is little bit aware about 80 years of debates on the problem of so called *completeness of QM* immediately understands that we would be in trouble: the majority of the modern quantum community believes that QM is complete. It seems that hidden variables and in particular *mental hidden variables* could not be introduced (even in principle).

The discussion on completeness of QM was initiated by the famous paper of Einstein, Podolsky and Rosen (1935). In fact, Albert Einstein was strongly against the so called *Copengahen interpretation of QM.* Such a rejection of one special interpretation of QM is often misinterpreted as rejection of QM by itself. Of course, Einstein was not against QM. However, he considered QM as only an *approximative description* of physical processes in the micro

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[3]Penrose (1994): "It is hard to see how one could usefully consider a quantum superposition consisting of one neuron *firing,* and simultaneously *nonfiring.*"

world. In particular, he was sure that one will create (sooner or later) a more fundamental theory which would explain so called *"quantum randomness"* in terms of classical stochastic processes.

By the Copenhagen interpretation quantum randomness differs crucially from classical randomness. The latter is reducible to statistical variation of features of elements of a huge ensemble of systems under consideration. The former is fundamentally irreducible. It is "individual randomness" of e.g. electron of photon.

Einstein was sure that quantum randomness can be reduced to classical one. In particular, the wave function should be associated not with a single quantum system, but with an ensemble of equally prepared quantum systems. This is so called Einsteinian or *ensemble interpretation* of QM. Thus for Einstein it was totally meaningless to speak about "the wave function of electron" or about "collapse of such a wave function". We remind that in the late 20s and early 30s Schrödinger was very sympathetic to the ensemble interpretation, see his correspondence with Einstein in Schrödinger (1982).[4] However, finally Scrödinger was about to reject not only the Copenhagen, but also the ensemble interpretation, since *Einstein was not able to explain why probabilistic formalisms of classical and quantum physics differ so much*: on the one hand, the integral calculus and, on the other hand, the operator algebra. This important problem of *coupling of the quantum probability calculus with the classical probability calculus was solved* in Khrennikov (2005a,b).

1.8   Bell's inequality

In spite of many years of debates, there is still no definite answer to the question which was asked in Einstein et al (1935). Although arguments based on Bell's inequality, see e.g. Bell (1964, 1987) and Clauser et al (1982), and subsequent experimental studies, see Aspect et al (1982) and Weihs et al (1998), Weihs (2007), induced rather common opinion that if QM were incomplete (i.e., a deeper "hidden variable" description is possible), then any subquantum classical statistical model is nonlocal. Thus one could choose between "death of reality" (impossibility to introduce hidden variables) and nonlocality. It is interesting that not so many quantum physicist are ready to accept a classical nonlocal model of e.g. Bohmian type as a subquantum

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[4]It may be not so well known that even the example with *Schrödinger's cat* was created under the influence of Einstein to show absurdness of the Copenhagen interpretation. Originally Einstein considered bomb.

model. Although the "official conclusion" from Bell's arguments is incompatibility of QM and local realism, in reality it is completeness of QM (i.e., impossibility of a deeper "subquantum" description of micro phenomena) that is questioned.[5]

At the first sight such a problem as "death of reality" might be interesting only for philosophers. However, last 10 years quantum information was intensively developed. And nowadays purely philosophic problems of quantum foundations play an important role in nanotechnological projects for billions of dollars. Take quantum cryptography as an example. Definitely completeness and not contradiction between local realism and QM plays the crucial role in justification of projects in quantum cryptography. Postulated incredibly security of quantum cryptography is based on the impossibility for Eva to create a classical probabilistic model with hidden variables underlying quantum communication between Alice and Bob. If such a model were possible, Eva would enjoy reading of quantum communications between Alice and Bob. As was pointed out, people ignore the possibility of classical, but nonlocal attacks on quantum cryptographic schemes.

1.9 Can a mathematical theorem be used as a crucial physical argument?

In spite of the evident fact that the majority believes in Bell's arguments, a number of experts actively criticise these arguments, see e.g. Pearle (1970), Accardi and Fedullo (1982), Pitowsky (1982), Fine (1982), De Baere (1984), Gisin, N. and Gisin, B. (1999), Klyshko (1993a,b, 1995, 1996a,b, 1998a,b), Larsson (2000), Hess and Philipp (2001, 2002, 2006), Volovich (2001), Khrennikov (1999b), Khrennikov and Volovich (2005), Adenier and Khrennikov (2006), Accardi and Khrennikov (2007). We are not able to discuss those anti-Bell arguments in this paper. We just point out that Bell's inequality as any mathematical theorem is based on a number of mathematical assumptions. Some assumptions can be questioned either from mathematical or physical viewpoints.[6]

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[5]We remark that this conclusion totally contradicts to the initial plan of J. Bell to show that QM is incomplete, but subquantum classical reality is nonlocal. One of my colleagues and a former student of David Bohm pointed out at many occasions to the real abuse of original Bell's arguments in modern quantum physics.

[6]For example, Bell assumed that ranges of values of classical "subquantum variables" coincide with ranges of values of corresponding quantum observables (for example, the

Personally I am very surprised that serious people hope that it might be possible to prove something about physical reality on the basis of a mathematical theorem.[7]

Can one say that non-Euclidean geometries do not exist in nature, because Pythagorean theorem is mathematically rigorous? Of course, one could also check that geometry of space is Euclidean by measurements of angles in triangles. Suppose that measurements show that geometry is Euclidean. But for what scale? Can one exclude that that geometry might be non-Euclidean at a finer scale?

Finally, suppose that measurement devices work so badly that about 80% of triangles are simply destroyed. One should base her conclusions only on a sub-sample containing 20% of triangles. Moreover, one could not even exclude that sampling is unfair, i.e., that the measurement devices just select Euclidean triangles and destroy non-Euclidean, cf. QM – violation of Bell's inequality: Pearle (1970), Gisin, N. and Gisin, B. (1999), Larsson (2000), Adenier and Khrennikov (2006).

1.10 Quantum-like processing of incomplete information

We decide not to wait for the end of the exciting Einstein-Bohr debate and the complete clarification of the theoretical and experimental situation for the EPR experiment and Bell's inequality. We proceed already now by creating a QL model in that the mathematical structure of QM is reproduced. Such a model is based on an incomplete representation of classical information.

What are advantages of the incomplete-information interpretation of the mathematical formalism of QM?

In such an approach the essence of the quantum mathematical formalism is not the description of a special class of physical systems, so called quantum systems, having rather exotic and even mystical properties, but the

classical spin variable should take the same values as the quantum spin observable, i.e., ± 1). If we proceed without this assumption (so we consider a more complicated picture of correspondence between classical and quantum models) then we can reproduce quantum correlations as classical ensemble correlations, see Accardi and Khrennikov (2007).

[7]I recall that Einstein completely ignored the first known "NO-GO" theorem which was proved by von Neumann in early 30s. It is especially curious, since von Neumann's book, see e.g. von Neumann (1955), was being in Einstein's office during many years.

possibility to operate with *incomplete information* about contexts.[8] Thus one can apply the formalism of QM in any situation which can be characterized by the incomplete description of contexts (physical, biological, mental). This (mathematical) formalism could be used in any domain of science, cf. Aerts, D. and Aerts, S. (1995, 2007), Khrennikov (1999a, 2000, 2002b, 2003b, 2004a, 2006a), Choustova (2001, 2004), Haven (2006), Busemeyer et al (2006, 2007 a,b), Franco (2007): in cognitive and social sciences, economy, information theory. Contexts under consideration need not be exotic. However, the complete description of them should be not available or ignored (by some reasons).

We shall use the incomplete-information interpretation of the formalism of QM. By our interpretation it is a special mathematical formalism working in the absence of complete information about context. By using this formalism a cognitive system permanently ignores huge amount of information. However, such an information processing does not induce chaos. It is extremely *consistent.* Thus the QL information cut off is done in a clever way. This is the main advantage of the QL processing of information.

Suppose that a biological organism is not able to collect or/and to process the complete set of information about some context: either because of some restrictions for observations or because it is in hurry – to decide immediately or to die! Such an organism can create a model of phenomena which is based on permanent and consistent ignorance of a part of information. Evolution can take a large number of generations. Consistency of ignorance plays the crucial role. Organisms in a population should elaborate the same system of rules for ignorance of information, otherwise they would not be able to communicate. By our interpretation the QL formalism provides the consistent rules for such a modelling of reality. We assume that higher level biological organisms and especially human beings developed in the process of evolution the ability of QL processing of information. Thus QL processing is incorporated in our brains.

By our approach cognitive evolution of biological systems can be considered as evolution from purely classical cognition (neural processing of complete data) to QL cognition. From this viewpoint human beings are essentially more quantum than e.g. dogs and dogs than crocodiles. Essential number of algorithms which are performed by the human brain are QL (of course, classical algorithms and networks also play essential roles). One might

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[8]In principle, context may be represented by an ensemble of systems, but may be not.

speculate that crocodiles operate merely with classical algorithms. Reading of e. g. Fabre (2006) gives the strong impression that insects' activity has no QL-elements.

1.11 Time scales of the functioning of brain and QL model

By our model, see section 4.2, the QL brain works on two time scales: subcognitive (pre-QL) and cognitive (QL). At the first (fine) time scale information process is described by classical stochastic dynamics which is transformed into QL stochastic dynamics at the second (rough) time scale.

To couple our model to physiology, behavioral science, and psychology, we consider a number of known fundamental time scales in the brain. Although the elaboration of those scales was based on advanced experimental research, there are still many controversial approaches and results. The temporal structure of the brain functioning is very complex. As *the physiological and psychological experimental basis* of our QL-model we chosen results of investigations on one special quantal temporal model of mental processes in the brain, namely, *Taxonomic Quantum Model* –TQM, see Geissler et al (1978), Geissler and Puffe (1982), Geissler (1983, 85, 87,92), Geissler and Kompass (1999, 2001), Geissler, Schebera, and Kompass (1999). The TQM is closely related with various experimental studies on the temporal structure of mental processes, see also Klix and van der Meer (1978), Kristofferson (1972, 80, 90), Bredenkamp (1993), Teghtsoonian (1971). We also couple our QL-model with well known experimental studies, see, e.g., Brazier (1970), which demonstrated that there are well established time scales corresponding to the alpha, beta, gamma, delta, and theta waves; especially important for us are results of Aftanas and Golosheykin (2005), Buzsaki (2005).

The presence of fine scale structure of firing patterns which was found in Luczak et al (2007) in experiments which demonstrated self-activation of neuronal patterns in the brain is extremely supporting for our QL-model.[9]

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[9]"Even in the absence of sensory stimulation, cortex shows complex spontaneous activity patterns, often consisting of alternating "DOWN" states of generalized neural silence and "UP" states of massive, persistent network activity. To investigate how this spontaneous activity propagates through neuronal assemblies in vivo, we recorded simultaneously from populations of 50-200 cells in neocortical layer V of anesthetized and awake rats. Each neuron displayed a virtually unique spike pattern during UP states, with diversity seen among both putative pyramidal cells and interneurons, reflecting a complex but stereo-

Of course, not yet everything is clear in neurophysiological experimental research, see Luczak et al (2007): "The way spontaneous activity propagates through cortical populations is currently unclear: while in vivo optical imaging results suggest a random and unstructured process Kerr et al (2005), in vitro models suggest a more complex picture involving local sequential organization and/or travelling waves, Cossart et al (2003), Mao (2001), Ikegaya (2004), Sanchez-Vives and McCormick (2000), Shu, Hasenstaub, and McCormick(2003), MacLean (2005)."

In any event our QL-model for brain functioning *operates on time scales which are used in neurophysiology, psychology and behavioral science. This provides an interesting opportunity to connect the mathematical formalism of QM with theoretical and experimental research in mentioned domains of biology.* We hope that our approach could attract the attention of neurophysiologists, psychologists and people working in behavioral science to quantum modelling of the brain functioning. On the other hand, our QL-model might stimulate theoretical and experimental research on temporal structures of the brain functioning.

2 QL averages as the basis of advanced cognition

2.1 Classical neural averages as cognitive images

We consider the following model of the functioning of the brain. Brain's state x is combined of states of individual neurons. Any mental function f is realized as a function of the state: $f(x)$. We consider x as classical random vector (so mathematician would write $x = x(\omega)$, where ω is a random parameter) describing fluctuations of neural activity.[10] By our model the brain does not "feel" fluctuations of the mental function $f(x)$ induced by fluctuations of the neural state x. Brain's images are purely statistical. The brain "feels" the average $\langle f \rangle$ of the mental function. Distributions of neural

typically organized sequential spread of activation through local cortical networks. The timescale of this spread was 100ms, with spike timing precision decaying as UP states progressed," see Luczak et al (2007).

[10]In a more advanced model the brain is split in a number of networks and each network determines its own state-variable. Different mental functions are associated with different networks.

activity produce probabilistic cognitive images.

This is a purely classical *probabilistic model of cognition,* see Khrennikov (2004a,b) and for its neural realization. Of course, to elaborate it one should discover "statistical neural code": to make correspondence between averages of a mental function and cognitive images corresponding to this functions. One could not exclude the possibility that different mental functions may have different coding systems. This is a problem of the greatest complexity, cf. problem of neural code.

However, we do not claim that an advanced brain works on the basis of such a classical statistical cognition. The main problem is the huge dimension of the neural-state vector (at least for advanced mental functions). To find the statistical average $\langle f \rangle$, the brain should perform integration over the space having the dimension of a few billions. It is clear that it is totally impossible to proceed in this way.

2.2   Production of QL averages-images

If cognitive systems really use statistical encoding of images they should find some approximative representation of averages. We shall see that one of simplest approximations of the classical average coincides with the quantum average. The latter is given by the operator-trace formula, see von Neumann (1955).

By our approach the QL approximation was developed by cognitive systems to speed the process of calculation of averages with respect to fluctuations of neural activity.

2.3   Quantum-like processing of information as a characteristic feature of advanced brains

If the brain does not have so many neurons and neural networks it can proceed purely classically. It can use the probabilistic representation of information which is given by classical averages: normalized sums of values of a mental function f. Therefore QL cognitive processing of information is profitable only for "advanced brains." As was pointed out in introduction, primitive cognitive systems need not process information by using QL algorithms, because they need not even represent information in the QL way. It is clear that QL evolution of biological organisms should produce a *smooth scale* of the

QL ability for information processing. In the processes of evolution brains became more complicated on the neural level. To speed information processing, brains should look for some approximative representation. In this way they develop the QL ability. However, they still preserve a number of mental functions which are based on classical probabilistic algorithms. In particular, in the human brain classical mental functions peacefully coexist with QL mental functions (which were created at the latter stage of cognitive evolution).[11]

At some moment in the process of cognitive evolution the crucial step toward QL computing was done. Thus our QL cognitive model induces an interesting problem of cognitive evolution:

"What was the first biological organism with QL information processing?"

Since our QL model has not yet been sufficiently elaborated (neither theoretically nor experimentally), it is too early to try to give a definite answer to this question. We shall speculate little bit. It seems that insects are purely classical, see again e.g. Fabre (2006), as well fishes. What is about mammals, e.g. dolphin? The brain of dolphin is very advanced as a neural system. However, it may still be, although very advanced, purely classically designed. My conjecture is that the crucial step toward QL processing of cognitive information was done by human beings. And this "QL discovery" is the main reason for rather special cognitive evolution of human beings.

Shortly, evolution of the brain can be characterized by three stages:

1) deterministic neural network processing – images are states of single neurons;

2) probabilistic processing – not individual states of neurons, but their probability distributions determine cognitive images;

3) discovery and development of the QL representation.

At the first stage, the brain was fine with a relatively small number of neurons. At the second stage the brain was interested to expand as much as possible (more neurons, more neural networks) – by a very simple reason: a larger number of probability distributions can be realized on a larger state

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[11]One should not think that all classically realized mental functions are "old" and only QL realized mental functions are "new" (relatively). If a mental function f need not so much neural resources (its state space is rather small), then there are no reason to realize f by using the QL representation. Thus even "new" but simple mental function can have the classical realization.

space. Hence such a classically more complicated brain is able to create more cognitive images. However, richness of the cognitive representation was approached by the cost of calculation of averages over spaces of huge dimension. Discovery of the QL representation solved this computational problem. Now cognitive images are processed essentially faster than classical cognitive images.

The QL brain does not demand so much computational resources as it was at the pre-QL stage of development. Moreover, QL images are less detailed than the classical ones. We recall that QL representation is an approximative representation. Thus some "details of the classical representation" disappear in the process of transition to the QL representation. We shall see in section 3 that different classical probability distributions of neural activity can be mapped into the same QL probability distribution. In contrast to the classical (probabilistic) brain, the QL brain need not be extremely precise in determining of probabilistic distributions for neural activity. Such unsharpness of cognitive images also implied liberation of computational resources. Moreover, the QL advanced brain can proceed with a smaller neural state space. It can reduce the number of neurons comparing with the second stage of brain's evolution. The advanced QL brain can be smaller than the advanced classical (probabilistic) brain.

The homo-brains really evolved in this way: the brain of homo was 10% greater than the modern human average. By our model homo neanderthalensis was the highest stage of extensional development based on the classical probabilistic model. Of course, we can not exclude that such a brain already had some elements of the QL representation. However, homo sapiens developed quicker in the QL way. Therefore homo sapiens was able to proceed better with a smaller brain. Our QL model might explain the evolutionary superiority of homo sapiens comparing with homo neanderthalensis. Otherwise it is not easy to explain extinction of the latter who was physically stronger and who had more extended neural state space.

3 The basic mathematical formula for the QL approximation of classical averages

This is the only section in our paper in that we really can not escape some mathematics.

3.1 Covariance matrix

We consider the average of the random vector x. If this vector has co-ordinates $x = (x_1, ..., x_N)$ then its average is the vector with coordinates $m_x = (m_{x_1}, ..., m_{x_N})$, where $m_{x_i} = Ex_i$ is the average of the coordinate x_i.

We recall that the covariance matrix of a random vector is defined as the matrix which elements are covariances between coordinates of this random vector.

$$\rho = (\rho_{ij}), \ \rho_{ij} = E(x_i - m_{x_i})(x_j - m_{x_j}).$$

We recall that any covariance matrix is symmetric: $\rho_{ij} = \rho_{ji}$ and it is positively defined:

$$(\rho x, x) = \sum_{ij} \rho_{ij} x_i x_j \geq 0.$$

We also remark that its trace

$$\mathrm{Tr}\rho = \rho_{11} + ... + \rho_{NN}$$

is equal to *dispersion* of the random vector $x : Dx = \sigma_x^2$, where σ_x is so called standard quadratic deviation of x.

3.2 Taylor formula

Let $f(x)$ be a smooth function. We recall that (in the multidimensional case) its derivative at some fixed point $a = (a_1, ..., a_N)$ is given by its gradient consisting of the derivatives with respect to coordinates (partial derivatives)

$$f'(a) = \Big(\frac{\partial f}{\partial x_1}(a), ..., \frac{\partial f}{\partial x_N}(a) \Big).$$

The right geometric interpretation of the gradient is that it is a so called covector. There is well defined pairing with vectors:

$$(f'(a), x) = \frac{\partial f}{\partial x_1}(a) \, x_1 + ... + \frac{\partial f}{\partial x_N}(a) \, x_N.$$

The second derivative is given by the matrix of partial derivatives of the second order which is called Hessian:

$$f''(a) = \Big(\frac{\partial^2 f}{\partial x_i \partial x_j}(a) \Big).$$

We point out that this matrix is symmetric.

By using the Taylor formula of the second order in the point a we get:

$$f(x) \approx f(a) + (f'(a), x - a) + \frac{1}{2}(f''(a)(x - a), (x - a)). \qquad (1)$$

If we now take the average of both parts of this approximative equality by choosing $a = m_x$ we obtain:

$$\langle f \rangle \equiv E f(x) \approx f(m_x) + \frac{1}{2}E(f''(m_x)(x - m_x), (x - m_x)). \qquad (2)$$

The term with the first derivative disappeared, because $E(x - m_x) = 0$ (the operation of averaging is linear). Thus only the second derivative is important for such a probabilistic approximation. By using little bit of linear algebra we can represent the last term as the matrix-trace: Tr ρ A, where the matrix $A = \frac{1}{2}f''(m_x)$ and ρ is the covariance matrix of the random vector x.

3.3   Normalization of random variable

In probability theory it is convenient to operate with normalized random variables. We set
$$y = \frac{x - m_x}{\sigma_x}$$
Then its average is equal 0 and dispersion is equal 1. In particular, for its covariance matrix the trace is equal 1.

We now consider this normalization of the random state-vector in the approximation formula (2). By assuming that a mental function is chosen in such a way that $f(0) = 0$ (if it was not so from the very beginning, we just set $f(x) \rightarrow f(x) - f(0)$) we get the following simple approximation formula:

$$\langle f \rangle \approx \text{Tr } \rho \text{ A}, \qquad (3)$$

where $A = \frac{1}{2}f''(0)$.

3.4 Quantum averaging

The quantum averaging procedure is based on linear algebra.[12] To simplify considerations, we discuss QM with finite-dimensional state space. Such a quantum model is basic for e.g. quantum information: one qubit has the two dimensional state space, N qubits have the 2^N dimensional state space.

Quantum observables are given by symmetric matrices. Quantum states are given by von Neumann density matrices[13]: symmetric, positively defined and having the unit trace.

The quantum average of an observable A with respect to a state ρ is given by the von Neumann trace formula:

$$\langle A \rangle = \text{Tr } \rho \, A, \qquad (4)$$

3.5 Wave function or density matrix?

Historically quantum states are associated with wave functions. These are so called pure states. So called mixed states are given by von Neumann density matrices. First we remark that any pure state ψ also can be represented by a density matrix, namely, by the orthogonal projector ρ_ψ onto the vector ψ. Thus formally one can proceed by using only density matrices. However, the use of pure states has a deep interpretational basis. By the Copenhagen interpretation ψ describes the state of *an individual quantum system.* For example, one speaks about the wave function of electron. On the other hand, by the ensemble interpretation there is no meaning to distinguish "pure states" and "mixed states." The ψ-function (in fact, the corresponding projector) as well as an arbitrary density matrix ρ describes an ensemble of specially prepared systems. Thus ρ_ψ describes a mixture as well. Of course, this is a mixture with respect to hidden variables. By the Copenhagen interpretation such variables do not exist at all.[14]

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[12]In fact, from the mathematical viewpoint it is even simpler than the classical averaging procedure. The latter is heavily based on theory of Lebesgue integration which is not so simple. The common opinion that quantum probability is more complicated than classical probability is not a consequence of the use of more advanced mathematics, but of counter-intuitiveness of quantum probability.

[13]We remind that Landau introduced them even earlier than von Neumann.

[14]At the last Växjö conference, "Quantum Theory: Reconsideration of Foundations – 4" Arcady Plotnitsky pointed out that the Copenhagen interpretation should not be directly identified with Bohr's interpretation. In fact, Bohr discussed only measurements and not

Since we follow the ensemble interpretation, we would not speak about "the wave function of the brain" (hence, for us "collapse of brain's wave function" is totally meaningless phrase). Instead of this, we shall consider QL states of brain given by density matrices. As we shall see, such mental states arise as special representations of classical neural statistical states.

This is just the question of simplicity. For the brain it is easier to operate with QL mental states (density matrices) than with classical probability distributions of neural activity.

3.6 Quantum-like approximation of classical averages

If we compare the formulas (3) and (4) we see that quantum averaging coincides with the approximation of classical averages which was obtained with the aid of the Taylor formula. We suppose that the brain uses the QL representation in this way. It proceeds with quantum averages, because it is simpler to calculate them. But it does not completely lose the "neural ground", because it uses approximations of real neural averages.

Thus in the QL representation the probability distribution of neural activity is represented by its covariance operator (we recall that we operate with normalized random state vector). The mental function f is represented by its second derivative (Hessian).

The QL representation is extremely incomplete. A huge class of probability distributions is mapped onto the same "covariance-density matrix" ρ. In our model abstract cognitive images are created due to the QL representation by identification of a class of classical probability distributions of neural activity.

the possibility to introduce hypothetical hidden variables. For him QM was complete from the experimental viewpoint. It was impossible to perform "better measurements" than those described by QM. The more orthodox interpretation that even in principle it is impossible to assume existence of a deeper level of the description of nature was elaborated by Fock ("the Leningrad version of the Copenhagen interpretation"). In cognitive modelling such a difference in the interpretations plays the crucial role. If mental hidden variables can exist, then they even can be measured. Bohr might be not strongly against this possibility, because mental hidden variables are macroscopic.

4 Precision of quantum-like approximation of classical averages

4.1 Mental and neural realities

One can ask: "What is the precision of the QL approximation, see (3), of classical averages?" This is not simply a mathematical problem of the precision of some approximation algorithm. This problem plays the fundamental role in our QL cognitive modelling.

If approximation is very good, then the QL approximation (given by the von Neumann trace formula) does not differ so much from the "real average" corresponding to neural activity. In such a case the use of the QL representation just make faster operation with averages-images (because it is easier to calculate trace than the Lebesgue integral). However, the QL representation would not deform essentially the classical (neural) picture of reality.

If the Taylor approximation is rather rough, then the QL approximation differs essentially from the "real average." In such a case the use of the QL representation not only make faster operation with averages-images, but it also deforms essentially the classical (neural) picture of reality. The brain created new QL "mental reality" which does not have direct relation to neural reality (although "mental reality" is still not independent from neural reality).

4.2 Time scales and the precision of the quantum-like approximation

In Khrennikov (2006d) it was shown that one can couple the precision of the QL approximation with time scales of classical neural processing (subcognitive) and QL processing (cognitive). We would not like to use too much mathematics in this paper which is oriented to a multi-disciplinary auditorium. Therefore we just recall the main idea of Khrennikov (2006d) considerations.

We start with the basic stochastic process $x(s, \omega)$ (where s is the subcognitive time and ω is the chance variable) generated by neural activity. This process proceeds on its own time scale. We call it subcognitive (or sub-QL) time scale: s_{pcogn}. Classical neural averages are created as the result of

fluctuations at this time scale.

Cognition is "not interested" in individual fluctuations which take place at this time scale. It is not interested in corresponding fluctuations of a mental function: $f = f(x(s, \omega))$. It is interested only in the average of the mental function and, moreover, QL cognition uses only the QL approximation of the average. Therefore cognition has its own time scale, *cognitive time scale* (or QL-scale): t_{cogn}. The interval s_{pcogn} is small with respect to the cognitive time scale (it is an "instant of time" in the cognitive scale). Thus cognitive images-averages are processed on the cognitive time scale – t_{cogn} and neural images on the subcognitive time scale s_{pcogn}.

Under some assumptions on the underlying neural stochastic process (e.g. for the process of Brownian motion or more general Gaussian processes) we can prove that the QL approximation (the trace) deviates from the real average (integral) by the term of the order κ, where the latter parameter determines the relative size of the subcognitive and cognitive scales:

$$\kappa = \frac{s_{\text{pcogn}}}{t_{\text{cogn}}}. \tag{5}$$

It provides a numerical measure of deviation of the QL (fuzzy, unsharp) representation of information from the CL (complete, sharp) representation.

Under the assumption that the subcognitive time scale s_{pcogn} is fixed, we find that for small periods of fluctuations t_{cogn} the parameter κ is very large. Thus *higher frequencies (at the cognitive time scale) induce larger deviations from the (complete) CL-processing of information.* Huge amounts of information which are processed at the subcognitive time scale are neglected, but not arbitrary (randomly). There is the QL-consistency in the information processing.

Consequently, for low frequencies (oscillations with long periods) this coefficient is small. Therefore the QL-processing does not imply large deviations from the CL-computational regime.

One of the fundamental consequences of our QL model for neurophysiology and cognitive science is that

Conscious (QL) processing of mental information is associated with relatively high frequencies.[15]

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[15]In this context "relatively" is with respect to a subcognitive scale. Thus the crucial role is played not by absolute values of frequencies, but by the gap between sub-QL and QL time scales.

The crucial problem is to find those biological time scales which induce the QL-representation of information. We split the problem into the two parts:

1) to find the subcognitive time scale;
2) to find the cognitive time scale.

And in reality the problem is even more complicated. Our fundamental assumption is that *there exist various pairs of scales inducing various QL-representations of information.* We could not hope to find once and for ever defined pair of time scales. Various mental functions can be based on various pairs of subcognitive and cognitive time scales.

4.3 Cognitive time scale: neurophysiological and cognitive data

It seems that (as in physics, see section 6) the first problem is more complicated. First we consider the second one. We start the discussion on the choice of the cognitive time scale by considering experimental evidences, that a moment in *psychological time* correlates with ≈ 100 msec of physical time for neural activity. In such a model the basic assumption is that the physical time required for the transmission of information over synapses is somehow neglected in the psychological time. The time (≈ 100 msec) required for the transmission of information from retina to the inferiotemporal cortex (IT) through the primary visual cortex (V1) is mapped to a moment of psychological time. It might be that by using $t_{\text{cogn}} = 100$ msec, we shall get the right cognitive time scale.

However, the situation is not so simple even for the second problem. There are experimental evidences that the temporal structure of neural functioning is not homogeneous. The time required for completion of color information in V4 (≈ 60 msec) is shorter that the time for the completion of shape analysis in IT (≈ 100 msec). In particular it is predicted that there will be under certain conditions a rivalry between color and form perception. This rivalry in time is one of manifestations of complex level temporal structure of brain.

Many cognitive architecture models, e.g., John Anderson's ACT-R model, Anderson (2007), assume that each computation step (production rule firing) takes ≈ 50 msec. This accounts for data well. It might be that by using

$$t_{\text{cogn}} \approx 50 - 100 \ \ \text{msec}$$

we shall get the right scale of the QL-coding.

5 Quantum-like and classical-like regimes of brain's functioning

As was already mentioned, if the scaling parameter κ is very small the brain does not lose too much classical information. This is practically the CL-computation. But if κ is rather large, then the brain works in a nonclassical regime. One may say (cf. Birkhoff and von Neumann (1936)) that in such a regime the brain uses *nonclassical logic.* However, in our approach the brain "produces QL logic" on the basis of purely classical (Boolean) logic of neural processing.

In such a QL process huge amounts of information are permanently neglected. But this does not generate a kind of chaos. Information is neglected in the consistent QL way.

As was pointed out a few times, such a QL-processing of information save a lot of computational resources. It might be an important factor of the natural selection of biological organisms.

6 Quantum and subquantum physical models

We now come back to physics (which was the starting point of our QL modelling). Here we can proceed in the same way and consider two time scales, Khrennikov (2006d). One scale, we call it *subquantum,* is a fine time scale, another, we call it *quantum,* is a coarser time scale. Oscillations at the subquantum time scale are averaged. However, the conventional QM is not about such subquantum classical averages. It is a calculus of approximative averages given by (3). Such an approximation works well at the quantum time scale. The latter time scale is considered as *an observational time scale.* The scale of observations in modern laboratories. Therefore it is natural to choose it as *the atom time scale:*

$$t_{\mathrm{q}} \approx 10^{-21} \text{ sec.}$$

The problem of the choice of a subquantum scale is more complicated. One of possible choices of a subquantum time scale is the *Planck time scale:*

$$s_{\mathrm{subq}} \approx 10^{-44} \text{ sec.}$$

We remark that for such a choice of time scales the scaling parameter

$$\kappa \approx 10^{-23}$$

is negligibly small. On the one hand, such a good approximation explains the huge predictive power of QM. On the other hand, it makes practically impossible experimental tests which would show the deviation of the QM-predictions from the predictions of the subquantum model: namely, the deviation of the trace average given by the QM formalism from the real average obtained on the basis of experimental data.

Thus if the above choice of the time scales was done properly, then our model has only theoretical value for physics. However, in cognitive science the gap between scales is not so huge. Therefore we can expect visible effects of QL behavior.

7 Variety of time scales in brain and quantum-like cognitive representations

The main lesson from the experimental and theoretical investigations on the temporal structure of processes in brain is that there are various time scales. They correspond to (or least they are coupled with) various aspects of cognition. Therefore we are not able to determine once and for ever the cognitive time scale t_{cogn} ("psychological time"). There are few such scales. We shall discuss some evident possibilities.

7.1 On variety of quantum and subquantum time scales in physics

Before to go deeper in the temporal structure of mental processes, we shall analyze in more detail the multi-scale temporal aspects of QM. Such aspects have never been discussed. On the one hand, it was commonly assumed that QM is complete (this is the Copenhagen interpretation). On the other hand, the quantum formalism is used by only one class of observers – human beings who discovered the mathematical formalism of QM for one special observational (quantum) time scale.

However, we can consider a possibility that there exits a class of observers ("super-clocks civilization") which use a time scale t'_{q} which is essentially finer

than our time scale t_{q} :

$$t'_{\mathrm{q}} << t_{\mathrm{q}}.$$

Thus such a civilization approached another time scale in its laboratories which is essentially better than atomic time scale.

Suppose that the super-clocks civilization has also created QM – a special probabilistic representation of information about measurements. Of course, its time scale should not be extremely fine comparing with the subquantum time scale s_{subq} :

$$s_{\mathrm{subq}} << t'_{\mathrm{q}}$$

(we assume that both civilizations – our and super-clocks – are interested in processes at the same subquantum time scale). The super-clocks civilization would discover the same mathematical formalism of QM. But the presence of deviation from subquantum reality would be more evident with respect to their time scale (since s_{subq} is the same, but t'_{q} is smaller than t_{q}, the coefficient κ' for the super-clocks civilization is larger than the coefficient κ for our civilization).

On the one hand, the super-clocks civilization has a better possibility to find deviations of the incomplete quantum description from the complete classical . However, there might be chosen a strategy to ignore such deviations and still use the quantum picture of the world. Even if it does not match precisely with the complete set of information about external world, it might be, nevertheless, convenient (by computational and consistency reasons) to proceed with the quantum pictures of reality.

7.2   Temporal structure of the brain

By our QL model a similar functioning with a few time scales is present in the brain. How can we find those scales?

It is well known, see, e.g., Brazier (1970), that there are well established time scales corresponding to the alpha, beta, gamma, delta, and theta waves. Let us consider these time scales as different cognitive scales. There is one technical deviation from the QL-scheme which was discussed above. We cannot determine precisely definite cognitive times corresponding to these scales. The scales are defined by ranges of frequencies and hence ranges of scaling times.

For the alpha waves we choose its upper limit frequency, 12 Hz, and hence the $t_{\mathrm{c},\alpha} \approx 0.083$ sec. For the beta waves we consider (by taking upper bounds

of frequency ranges) three different time scales: 15 Hz, $t_{c,\beta,\text{low}} \approx 0.067$ sec. – low beta waves, 18Hz, $t_{c,\beta} \approx 0.056$ sec. – beta waves, 23 Hz $t_{c,\beta,\text{high}} \approx 0.043$ sec. – high beta waves. For gamma waves we take the characteristic frequency 40 Hz and hence the time scale $t_{c,\gamma} \approx 0.025$ sec.

The gamma scale is the finest and hence processes represented at this scale has the highest degree of QL-ness. On the other hand, we know that gamma waves patterns in the brain are associated with perception and consciousness. The beta scale is coarser than the gamma scale and it has less degree of QL-ness in processing of information. We know that beta states are associated with normal waking of consciousness.

The theta waves are even less QL than the alpha waves. They are commonly found to originate from occipital lobe during periods of relaxation, with eyes closed but still awake. They are involved into a representation of information with a high degree of classicality. And these rhythms are observed during some sleep states, and in states of quiet focus, for example, meditation, Aftanas and Golosheykin (2005). However, there are also experimental evidences that the theta rhythms are very strong in rodent hippocampi and entorhinal cortex during learning and memory retrieval. We can just speculate that learning needs using of an essentially more detailed information representation. Thus learning (or at least a part of it) is less QL and hence more CL. The same we can say about memory retrieval. It also needs a more complete, CL-representation of information. Large body of evidence, Buzsaki (2005), indicates that theta-rhythms are used in spatial learning and navigation. Here we present the same reasons: such tasks are based on the CL-representation of information.

Finally, we consider delta waves. Comparing with the highest scale – the gamma scale, the delta time scale is extremely rough. This induces a low degree of QL-ness. This is the state of deep sleep.[16]

Although we still did not come to the difficult problem, namely, determination of the subcognitive time scale, we can, nevertheless, compare the degree of QL-ness of various time scales.

Our choice of the subcognitive time scale will be motivated by so called *Taxonomic Quantum Model*, see Geissler et al (1978), Geissler and Puffe (1982), Geissler (1983, 85, 87,92), Geissler and Kompass (1999, 2001), Geissler, Schebera, and Kompass (1999), for the representation of cognitive processes

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[16]The phenomena of sleep and dreaming are extremely complicated. We do not plan to study them in this paper.

in the brain (which was developed on the basis of the huge experimental research on time-mind relation, see also Klix and van der Meer (1978), Kristofferson (1972, 80, 90), Bredenkamp (1993), Teghtsoonian (1971). In the following section we recall briefly the main features of this model.

8 Taxonomic quantum model

There could be presented a portion of good criticism against starting from EEG bands. Indeed, this band structure is one of the few indications that directly point to behaviorally relevant physiological properties. Physiologists suggesting the definitions had a good intuition. However, that these definitions depend on behavioral information is shown by enormous individual differences in the band structures that can be defined only on a behavioral basis. To some degree this concerns also the general band structure. Because of individual differences, alpha is often restricted to the common range which is too short to be theoretically fully relevant. Definitions often go only from 9 to 12 Hz. Most careful investigators (earliest Livanov) defined the band by the range 7.5 to 13.5 Hz.

Therefore we propose to start with Taxonomic Quantum Model (TQM), Geissler et al (1978), Geissler and Puffe (1982), Geissler (1983, 85, 87,92), Geissler and Kompass (1999, 2001), Geissler, Schebera, and Kompass (1999). Why do we propose to use TQM for start of theory instead of, say, some characteristic physiological parameters such as neuronal refractoriness, transmission times, coupling strength etc.? In my view, the reason is that *the only basis for interpreting physiological facts of brain processes are psychophysical (behavioral) observations, either based on motor reactions of conscious beings or verbal reports on conscious events.* This was the main way of thinking of von Bekesy (1936). Of course, many of the functional statements of physiologists have the same basis. For our purpose, this statement is absolutely essential, because a coherent account of temporal properties of brain activity must not only be related to behavioral observations, but it must be based on temporal invariants extracted by a coherent theoretical account of behavioral observations, and only these can provide the guideline to find the proper physiological correspondences.

The best short cut to the approach is through the history of its emergence: The first impulse towards a taxonomic turn arose in the early 1970s from the discontent of Geissler, see, e.g., Geissler et al (1978), with the fact

that in simple psychophysical tasks data could indistinguishably be fitted to models resorting to widely differing, often enough even contradicting, assumptions. In his research in visual recognition, to circumvent this difficulty, Geissler introduced a technique of chronometric cross-task comparison. The main idea was to disambiguate models by temporal parametrization, thereby postulating invariance of time parameters under variation of stimulus parameters and task constraints (see e.g. Geissler et al. (1978) and Geissler and Puffe (1982)). At that time another research group at the same institute did something similar by fitting latencies in standardized reasoning tasks to predicted numbers of operations, e.g., Klix and van der Meer (1978). The estimates from the two lines of studies yielded a surprising picture: There seemed to exist small bands of operation times centering at around 55, 110 and 220 ms, thus exhibiting near-doubling relations. As a datum from the literature which fitted into this regularity the asymptotic value of 36.5 ms determined by Kristofferson (1972), see also Kristofferson (1980, 90), came to mind which up to the first decimal is 1/3 of 110 ms. Taken together, these four values suggested a system of magic numbers. Herein a period of 110 ms represents something like a prototype duration from which the rest of periods derives by either integer division or multiplication. From various fit procedures for step lengths, Buffart and Geissler came up with an largest common denominator (l.c.d.) of 9.13 ms (see Geissler, 1985) showing a standard deviation of 0.86 ms across individuals. It turned out that the four above-mentioned periods, although partly many times larger than this small period, can be represented as integer multiples of it, with nearly absolute precision: $4 \times 9.13 = 36.5; 6 \times 9.13 = 54.8; 12 \times 9.13 = 109.6; 24 \times 9.13 = 219.1$. Of course, this might have been some strange coincidence. Yet, later, chronometric analysis seemed to support a modular unit of some 9 ms (see Geissler (1985); Puffe (1990); Bredenkamp (1993). Further investigations justify a modified assumption about quantal graining:

Regression yields the largest common denominator (l.c.d.) 4.6 ms, which is nearly exactly one half of 9.13 ms.

Note that, in terms of hypothetical quanta, a period of such duration represents the next smaller candidate of a true elementary time quantum which is compatible with the recognition data. In the following, let us adopt provisionally the (ideal) value of

$$Q_0 = 4.565 \text{ms}$$

for this time quantum hypothesis.

The solution TQM offers to these seeming contradictions, see Geissler (1987, 92, 85) can be considered as a generalization or at least an analogue of the psychophysical principle of relative-range constancy. According to Teghtsoonian (1971), this principle expresses itself in the fact that for all sensory continua, in terms of output magnitudes, the ratio of the largest to the smallest quantity is a constant of around 30. About the same value is obtained from the so-called Subjective Weber Law.

The generalization of the principle in the realm of quantal timing is the quantal-range constraint. To see how this analogue reads, consider first the assumed smallest period Q_0. For integer multiples $n \times Q_0$, consistency with the relative range constraint implies $n \leq M$, with M being a constant of the hypothetical value 30. It follows that periods of durations in excess of $30 \times Q_0 \approx 137$ ms cannot be represented within this smallest possible range. To account for such periods, we have to assume larger ranges with correspondingly larger admissible smallest quantal periods to be operative. To retain consistency with the time quantum assumption, these periods must be integer multiples of Q_0 or, formally,

$$Q_q = q \times Q_0 \tag{6}$$

with integer q must hold. Thus, in general, the maximum extension of any quantal of periods T_i belonging to it is given by $q \times Q_0 \leq T_i \leq M \times q \times Q_0$. Note that the lower bound $q \times Q_0$ also defines the smallest possible distance between admissible periods within a range. For this reason we will speak of it as the quantal resolution within a given range. Of course, in the actual development, this abstract definition resulted from a variety of empirical relationships suggesting a range ordering of quantal periods with upper bounds maximally at 30 times the value of quantal resolution.

TQM does not exclude the possibility that there can be found smaller characteristic time scales, e.g., $Q_0/30$.

9   On the choice of subcognitive time scale

We choose Q_0 as the unit of the subcognitive time:

$$s_{\mathrm{pcogn}} = Q_0 = 4.6ms \tag{7}$$

This corresponds to frequencies ≈ 220 Hz. Under such an assumption about the subcognitive scale we can find the measure of QL-ness for different EEG bands. For the alpha scale, we have

$$\kappa_\alpha = \frac{Q_0}{t_{c,\alpha}} \approx 0.055.$$

For the beta scales, we have:

$$\kappa_{c,\beta,\text{low}} = \frac{Q_0}{t_{c,\beta,\text{low}}} \approx 0.069; \ \kappa_{c,\beta} = \frac{Q_0}{t_{c,\beta}} \approx 0.082; \ \kappa_{c,\beta,\text{high}} = \frac{Q_0}{t_{c,\beta,\text{high}}} \approx 0.107.$$

For the gamma scale we have:

$$\kappa_\gamma = \frac{Q_0}{t_{c,\gamma}} \approx 1.84.$$

Thus QL-ness of processing of information increases. "Thinking through the alpha waves" is more likely processing of information by ordinary computer. Not so much information is neglected. Therefore the information processing is not so tricky: there is no need to manipulate with extremely incomplete information in the consistent way. "Thinking through the gamma waves" is similar to processing of information by an analogue of quantum computer – QL-computer, see Khrennikov (2006a). Such an information processing is very tricky: permanent informational cuts, but in the consistent QL-way. Finally, we come to the theta and delta scales. For the theta scale $t_{c,\theta} = 0.125$ sec. Thus

$$\kappa_\theta = \frac{Q_0}{t_{c,\theta}} \approx 0.037.$$

And for the delta scale $t_{c,\delta} = 0.5$ sec and hence:

$$\kappa_\delta = \frac{Q_0}{t_{c,\delta}} \approx 0.009.$$

Here the difference between the biological QL-processing of information in the brain and the CL-processing (as in models of artificial intelligence) is practically negligible.

We now compare our QL-scales of time with the "quantum scales" which were chosen in Khrennikov (2006d):

$$s_{\text{pcogn}} \approx 10^{-3} \text{ sec}, \quad s_{\text{pq}} \approx 10^{-44} \text{ sec}. \tag{8}$$

$$t_{\text{cogn}} = 30Q_0 \approx 10^{-1} \text{ sec}, \quad t_q \approx 10^{-21} \text{ sec}. \tag{9}$$

Thus our model is based on macroscopic time scales, in the opposition to really quantum reductionist models.

If we follow TQM in more detail then we should consider a possibility that in the brain there exist a *hierarchy of subcognitive times,* i.e., the above model with one fixed subcognitive time given by (7) was oversimplified. From the point of view of TQM each Q_q given by (6) could serve as the basis of a subcognitive time scale. We obtain a picture of extremely complex QL-processing of information in the brain which is based of the huge multiplicity of various subcognitive/cognitive scales.

In this framework the notion "subcognitive" loses its absolute meaning. The notions "subcognitive"/"cognitive" become relative with respect to a concrete psychological function (cognitive task). Moreover, a time scale which is subcognitive for one psychological function can be at the same time cognitive for another.

But the crucial point is that the same cognitive time scale, say t_{cogn}, can have a number of different subcognitive scales:

$$Q_{q_1} \leq ... \leq Q_{q_m}.$$

Each pair of scales

$$(Q_{q_1}, t_{\text{cogn}}), ..., (Q_{q_m}, t_{\text{cogn}})$$

induces its own QL-representation of information. Therefore the same t_{cogn}-rhythm can be involved in the performance of a few different psychological functions.

The final message from TQM is that the cognitive time t_{cogn} scale should be based on an integer multiplier of the time quant Q_0 :

$$t_{\text{cogn}} = NQ_0. \tag{10}$$

In such a model we can totally escape coupling with directly defined different EEG bands, alpha, beta, gamma,... We shall use only behaviorally defined time scales. The Weber law gives us the restriction to the value of the multiplier: $N \leq 30$.

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