Appendix A: The expression of p^{c*}, $\check{p}^{c*}$and e^{c*}

$$\begin{aligned}
p^{c*} = & \frac{1}{4(\beta+1)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta^2-1)\gamma} \\
& \times \{-N(\eta+\tilde{e}-\beta\tilde{e})\left[(2\theta-1)\eta-(3+2\theta\beta-2\theta+\beta)\tilde{e}\right] \\
& +2N(-1+\theta-\theta\beta)k+2(\beta+1)\tilde{e}\left[2\eta(\eta+\tilde{e}-\beta\tilde{e})+(\beta-1)\gamma\right]\check{e}\}
\end{aligned}$$

$$\begin{aligned}
\check{p}^{c*} = & \frac{1}{4(\beta+1)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta^2-1)\gamma} \\
& \times \{N(\eta+\tilde{e}-\beta\tilde{e})\left[(2\theta-1)\eta+\tilde{e}-2\theta(\beta-1)\tilde{e}+3\beta\tilde{e}\right] \\
& +2N\left[\theta(\beta-1)-\beta\right]\gamma+2(\beta+1)\tilde{e}\left[2\eta(\eta+\tilde{e}-\beta\tilde{e})+(\beta-1)\gamma\right]\check{e}\}
\end{aligned}$$

$$e^{c*} = -\frac{(\eta+\tilde{e}-\beta\tilde{e})[N+2(\beta-1)f\check{e}]}{2[(\eta+\tilde{e}-\beta\tilde{e})^2+(\beta-1)\gamma]}$$

Appendix B: The expression of $\check{p}^{d}$, $\mathring{p}^{d}$, e^{d} and H_2

$$q^{c^*} = -\frac{N(2\theta-1)(\eta+\tilde{e}-\beta\tilde{e})^2+2N(\theta-1)(\beta-1)\gamma-2(\beta-1)^2\tilde{e}\gamma\check{e}}{4\left[(\eta+\tilde{e}-\beta\tilde{e})^2+(\beta-1)\gamma\right]}$$

$$\tilde{q}^{c^*} = \frac{N(2\theta-1)(\eta+\tilde{e}-\beta\tilde{e})^2+2N\theta(\beta-1)\gamma-2(\beta-1)^2\tilde{e}\gamma\check{e}}{4\left[(\eta+\tilde{e}-\beta\tilde{e})^2+(\beta-1)\gamma\right]}$$

$$\begin{aligned}
\pi^{c^*} = & \frac{1}{8(\beta+1)\left[(\eta+\tilde{e}-\beta\tilde{e})^2+(\beta-1)\gamma\right]} \\
& \times \left\{N^2\left[(1-2\theta)^2(\eta+\tilde{e}-\beta\tilde{e})^2-2\gamma+4(\theta-1)\theta(\beta-1)\gamma\right]\right. \\
& \left. -4N\left(\beta^2-1\right)\tilde{e}\gamma\check{e}-4\left(\beta^2-1\right)(\beta+1)\tilde{e}^2\gamma\check{e}^2\right\}
\end{aligned}$$

$$H_2 = \begin{bmatrix}
2\left(\frac{\beta^2}{2}-1\right) & \beta & \eta+\frac{\beta\eta}{2}+\left(1+\frac{\beta}{2}\right)(\beta-1)\tilde{e} \\
\beta & -1 & \frac{\eta}{2}+\frac{1}{2}(\beta-1)\tilde{e} \\
\eta+\frac{\beta\eta}{2}+\left(1+\frac{\beta}{2}\right)(\beta-1)\tilde{e} & \frac{\eta}{2}+\frac{1}{2}(\beta-1)\tilde{e} & -2\left[-2\eta-\frac{1}{2}(\beta-1)\eta\right]\tilde{e}-\gamma
\end{bmatrix}$$

Appendix C: The expression of p^{d*}, $\check{p}^{d*}$, e^{d*}, $\mathring{p}^{d*}$, q^{d*}, $\tilde{q}^{d*}$, $\mathring{\pi}^{d*}$, $\hat{\pi}^{d*}$, p_x and $\mathring{p}x^*$

$$\begin{aligned}
p^{d*} = & \frac{1}{2(\beta+1)\left[(\beta+3)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta-1)\gamma\right]} \\
& \times \left\{-N(\eta+\tilde{e}-\beta\tilde{e})\left[(2\theta-1)(2\beta+3)\eta-(5+3\beta-4\theta+2\theta\beta+2\theta\beta^2)\tilde{e}\right]\right. \\
& +2N\left[-3+\beta^2-\theta(\beta-1)(\beta+3)\right]\gamma \\
& \left. +2(\beta+1)\tilde{e}\left[(\beta+3)\eta(\eta+\tilde{e}-\beta\tilde{e})+(\beta^2-1)\gamma\right]\check{e}\right\}
\end{aligned}$$

$$\begin{aligned}
\check{p}^{d*} = & \frac{1}{2(\beta+1)\left[(\beta+3)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta-1)\gamma\right]} \\
& \times \left\{N(\eta+\tilde{e}-\beta\tilde{e})\left[(2\theta-1)\eta+\tilde{e}+\beta(2\beta+5)\tilde{e}-2\theta(-2+\beta+\beta^2)\tilde{e}\right]\right. \\
& +4N\left[\theta(\beta-1)-\beta\right]\gamma \\
& \left. +2(\beta+1)\tilde{e}\left[(\beta+3)\eta(\eta+\tilde{e}-\beta\tilde{e})+2(\beta-1)\gamma\right]\check{e}\right\}
\end{aligned}$$

$$e^{d*} = -\frac{(\eta+\tilde{e}-\beta\tilde{e})\left[N(1+\theta+\beta-\theta\beta)+(\beta-1)(\beta+3)\tilde{e}\check{e}\right]}{(\beta+3)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta-1)\gamma}$$

$$\begin{aligned}
\mathring{p}^{d*} = & \frac{1}{2(\beta+1)\left[(\beta+3)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta-1)\gamma\right]} \\
& \times \left\{-N(\eta+\tilde{e}-\beta\tilde{e})\left[(2\theta-1)(\beta+2)\eta-(4+2\theta\beta-2\theta+\beta^2+3\beta)\tilde{e}\right]\right. \\
& +4N(-1+\theta-\theta\beta)\gamma \\
& \left. +2(\beta+1)\tilde{e}\left[(\beta+3)\eta(\eta+\tilde{e}-\beta\tilde{e})+2(\beta-1)\gamma\right]\check{e}\right\}
\end{aligned}$$

$$
\begin{aligned}
q^{d*} &= \frac{-N\left(2\theta-1\right)\left(\eta+\tilde{e}-\beta\tilde{e}\right)^2-2N\left(\theta-1\right)\left(\beta-1\right)\gamma+2(\beta-1)^2\tilde{e}\gamma\check{e}}{2\left(\beta+3\right)\left(\eta+\tilde{e}-\beta\tilde{e}\right)^2+8\left(\beta-1\right)\gamma}\\
\tilde{q}^{d*} &= \frac{N\left(2\theta-1\right)\left(\eta+\tilde{e}-\beta\tilde{e}\right)^2-2N\left[\theta\left(\beta-2\right)-\beta\right]\left(\beta-1\right)\gamma+2\left(2-3\beta+\beta^3\right)\tilde{e}\gamma\check{e}}{2\left(\beta+3\right)\left(\eta+\tilde{e}-\beta\tilde{e}\right)^2+8\left(\beta-1\right)\gamma}\\
\mathring{\pi}^{d*} &= \frac{1}{4\left(\beta+1\right)\left[\left(\beta+3\right)\left(\eta+\tilde{e}-\beta\tilde{e}\right)^2+4\left(\beta-1\right)\gamma\right]}\\
&\quad\times\left\{N^2\left[(1-2\theta)^2(\eta+\tilde{e}-\beta\tilde{e})^2-2\gamma-2\gamma\theta^2\left(\beta-3\right)\left(\beta-1\right)+4\gamma\theta(\beta-1)^2-2\gamma\beta^2\right]\right.\\
&\quad+4N\left[-1+\theta\left(\beta-1\right)-\beta\right]\left(\beta^2-1\right)\tilde{e}\gamma\check{e}\\
&\quad\left.-2(\beta-1)^2\left(\beta+1\right)\left(\beta+3\right)\tilde{e}^2\gamma\check{e}^2\right\}\\
\hat{\pi}^{d*} &= \frac{\left[N(2\theta-1)(\eta+\tilde{e}-\beta\tilde{e})^2+2N(\theta-1)(\beta-1)\gamma-2(\beta-1)^2\tilde{e}\gamma\check{e}\right]^2}{4\left[(\beta+3)(\eta+\tilde{e}-\beta\tilde{e})^2+4(\beta-1)\gamma\right]^2} \qquad (1)\\
p_x &= \frac{\eta e(-1+\mu)+N(-1+\theta+\mu-\theta\mu)-\mathring{p}x}{+}\beta(-1+\mu)p_{dx}2(-1+\mu)
\end{aligned}
$$

$$
\begin{aligned}
\mathring{p}x^* &= \frac{(-1+\mu)}{4(\beta+1)(\eta+\tilde{e}-\beta\tilde{e})^2+4(-1+\beta^2)\gamma}\\
&\quad\times\left\{-N(\eta+\tilde{e}-\beta\tilde{e})\left[(-1+2\theta)\beta\eta+2\tilde{e}+\beta\tilde{e}-2\theta\beta\tilde{e}(-1+\beta)+\beta^2\tilde{e}\right]\right.\\
&\quad\left.+2N\beta(\eta+\tilde{e}-\beta\tilde{e})\gamma+2(1+\beta)\tilde{e}\left[-2\eta(\eta+\tilde{e}-\beta\tilde{e})+(-2+\beta)(-1+\beta)\gamma\right]\check{e}\right\}
\end{aligned}
$$

Appendix D: Proof from Proposition 1 to Proposition 6

Proposition 1: In the case of decentralized decision-making, product carbon emission reductions are positively correlated with consumer preference for the manufacturer's online direct sales channel, consumer preference for online channel of traditional retailers has no effect on optimal carbon emission reduction. In contrast, in the centralized decision-making scenario, consumers' channel preferences have no impact on carbon emission reductions.

Proof: Find the first order derivative with respect to θ and it is obtained by derivation: $\frac{\partial e^{d*}}{\partial\theta} = -\frac{N(-1+2\beta)(\eta+\tilde{e}-2\beta\tilde{e})}{2(-2+\beta)(\eta+\tilde{e}-2\beta\tilde{e})^2+4(-1+\beta)(-1+2\beta)\gamma}$, when $\gamma > -\frac{(-2+\beta)(\eta+\tilde{e}-2\beta\tilde{e})^2}{2-6\beta+4\beta^2}$, unit carbon emission reductions under decentralized decision-making increase with the coefficient of consumer preference for the manufacturer's online direct sales channel, independent of the coefficient of consumer preference for the retailer's online channel. Carbon reduction under centralized decision making is not related to consumer channel preference.

Proposition 1 suggests that increasing consumer preference for a manufacturer's online channel when a traditional retailer opens an online channel can reduce product carbon emissions. In other words, when consumers are more likely to choose to buy from a manufacturer's online channel, the manufacturer is willing to increase its carbon reduction efforts and the carbon emissions of the product will be lower, whereas the consumer's purchasing preference for a traditional retailer's online channel does not affect the carbon emissions of the product.

Proposition 2: In both centralized and decentralized decision-making scenarios, the carbon emission reductions of a product are related to the consumer low-carbon preference coefficient and are affected by the initial carbon emissions $\check{e}$.

In the case of centralized decision-making, when $0 < \check{e} < \frac{N}{3\tilde{e}-6\beta\tilde{e}}$, carbon emission reductions are positively correlated with the coefficient of consumer low-carbon preference.

In the case of decentralized decision-making, when $0 < \check{e} < \frac{N+N\theta-2N\theta\beta}{4\tilde{e}-10\beta\tilde{e}+4\beta^2\tilde{e}}$, carbon emission reductions are positively correlated with the coefficient of consumer low-carbon preference.

Proof: For the centralized decision-making scenario, the first-order derivative of carbon emission reduction e^{c*} with respect to the low-carbon preference coefficient η is found for equation (16):

$$
\frac{\partial e^{c^*}}{\partial\eta} = \frac{\left[3(\eta+\tilde{e}-2\beta\tilde{e})^2+2\gamma-4\beta\gamma\right]\left[N+3(-1+2\beta)\tilde{e}\check{e}\right]}{\left[3(\eta+\tilde{e}-\beta\tilde{e})^2-2\gamma+4\beta\gamma\right]^2}
$$

, when $\gamma > -\frac{3(\eta+\tilde{e}-2\beta\tilde{e})^2}{-2+4\beta}$ and $0 < \check{e} < \frac{N}{3\tilde{e}-6\beta\tilde{e}}$, the unit carbon emission reduction increases with the consumer's low-carbon preference.

For the decentralized decision-making scenario, the first-order derivative of carbon emission reduction e^{d*} with respect to the low-carbon preference coefficient η is found for equation (15):

$$\frac{\partial e^{d^*}}{\partial \eta}=\frac{\left[2\left(-2+\beta\right)\left(\eta+\tilde{e}-2\beta\tilde{e}\right)^2-4\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right]\left[N\left(-1-\theta+2\theta\beta\right)+2\left(-2+\beta\right)\left(-1+2\beta\right)\tilde{e}\check{e}\right]}{\left[2\left(-2+\beta\right)\left(\eta+\tilde{e}-2\beta\tilde{e}\right)^2+4\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right]^2}$$

, when $\gamma>-\frac{(-2+\beta)(\eta+\tilde{e}-2\beta\tilde{e})^2}{2-6\beta+4\beta^2}$ and $0<\check{e}<\frac{N+N\theta-2N\theta\beta}{4\tilde{e}-10\beta\tilde{e}+4\beta^2\tilde{e}}$, it can be shown that there is $\frac{\partial e^{d*}}{\partial \eta}>0$, and carbon emission reduction increases with the low carbon preference coefficient.

Proposition 2 suggests that unit carbon emission reductions under centralized versus decentralized decision-making increase with the increase in consumer low-carbon preference coefficients when traditional retailers open online channels.

Proposition 3: In both the centralized and decentralized decision-making scenarios, carbon emission reductions are related to the carbon tax rate e levied by the government and are affected by the initial carbon emissions $\check{e}$. Specific correlations are as follows:

In centralized decision-making situations, when $0<\check{e}<\frac{-3N(\eta+\tilde{e}-2\beta\tilde{e})^2+2N(-1+2\beta)\gamma}{9\eta(\eta+\tilde{e}-2\beta\tilde{e})^2+6(-1+2\beta)[\eta+(2-4\beta)\tilde{e}]\gamma}$, carbon emission reductions are positively related to the carbon tax rate; when $\check{e}>\frac{-3N(\eta+\tilde{e}-2\beta\tilde{e})^2+2N(-1+2\beta)\gamma}{9\eta(\eta+\tilde{e}-2\beta\tilde{e})^2+6(-1+2\beta)[\eta+(2-4\beta)\tilde{e}]\gamma}$, carbon emission reduction is negatively correlated with the carbon tax rate.

When $0<\check{e}<\frac{\sqrt{-\frac{N^2(-1+\beta)(-1+2\beta)(1+\theta-2\theta\beta)^2\gamma}{(-2+\beta)[(-2+\beta)\eta^2+2(-1+\beta)(-1+2\beta)\gamma]^2}}}{\sqrt{2}}+\frac{N(1+\theta-2\theta\beta)\eta}{2(-2+\beta)\eta^2+4(-1+\beta)(-1+2\beta)\gamma}$, carbon emission reduction is positively correlated with the carbon tax rate.

Proof: Under centralized decision making, find the first-order derivative with respect to the carbon tax rate e for the carbon emission reductions e^{c*}:

$$\frac{\partial e^{c*}}{\partial \tilde{e}}=\frac{(-1+2\beta)\left\{N\left[-3(\eta+\tilde{e}-2\beta\tilde{e})^2-2\gamma+4\beta\gamma\right]-9\eta(\eta+\tilde{e}-2\beta\tilde{e})^2\check{e}-6\left(-1+2\beta\right)\left[\eta+\left(2-4\beta\right)\tilde{e}\right]\gamma\check{e}\right\}}{\left[3(\eta+\tilde{e}-2\beta\tilde{e})^2-2\gamma+4\beta\gamma\right]^2}$$

, when $\gamma>-\frac{3(\eta+\tilde{e}-2\beta\tilde{e})^2}{-2+4\beta}$ and $0<\check{e}<\frac{-3N(\eta+\tilde{e}-2\beta\tilde{e})^2+2N(-1+2\beta)\gamma}{9\eta(\eta+\tilde{e}-2\beta\tilde{e})^2+6(-1+2\beta)[\eta+(2-4\beta)\tilde{e}]\gamma}$, $\frac{\partial e^{c*}}{\partial \tilde{e}}>0$, this suggests that carbon emission reductions increase as the carbon tax rate increases.

When $\check{e}>\frac{-3N(\eta+\tilde{e}-2\beta\tilde{e})^2+2N(-1+2\beta)\gamma}{9\eta(\eta+\tilde{e}-2\beta\tilde{e})^2+6(-1+2\beta)[\eta+(2-4\beta)\tilde{e}]\gamma}$, $\frac{\partial e^{c*}}{\partial \tilde{e}}<0$, this suggests that carbon emission reductions decrease as the carbon tax rate increases.

Under decentralized decision making, find the first-order derivative with respect to the carbon tax rate e for carbon emission reductions e^{d*}:

$$\begin{aligned}\frac{\partial e^{d*}}{\partial \tilde{e}}=&-\frac{1}{\left[2\left(-1+\beta\right)\left(\eta+\tilde{e}-2\beta\tilde{e}\right)^2+4\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right]^2}\\&\times\left\{2\left(-1+2\beta\right)\left\{N\left[-4+\theta\left(-1+2\beta\right)\right]\left[-\left(-2+\beta\right)\left(\eta+\tilde{e}-2\beta\tilde{e}\right)^2+2\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right]\right.\right.\\&\left.\left.-2\left(-2+\beta\right)\left\{\left(-2+\beta\right)\eta(\eta+\tilde{e}-2\beta\tilde{e})^2+2\left(-1+\beta\right)\left(-1+2\beta\right)\left(\eta+2\tilde{e}-4\beta\tilde{e}\right)\gamma\right\}\check{e}\right\}\right\}\end{aligned}$$

When $\gamma>-\frac{(-2+\beta)\eta^2}{2-6\beta+4\beta^2}$ and $\check{e}>\frac{\sqrt{-\frac{N^2(-1+\beta)(-1+2\beta)(1+\theta-2\theta\beta)^2\gamma}{(-2+\beta)[(-2+\beta)\eta^2+2(-1+\beta)(-1+2\beta)\gamma]^2}}}{\sqrt{2}}+\frac{N(1+\theta-2\theta\beta)\eta}{2(-2+\beta)\eta^2+4(-1+\beta)(-1+2\beta)\gamma}$, the discriminant of

$$\begin{aligned}&\left\{4(1-2\beta)^2\left(-2+\beta\right)\left\{N\left[-1+\theta\left(-1+2\beta\right)\right]\eta+2\left[\left(-2+\beta\right)\eta^2+2\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right]e_0\right\}\right\}^2\\&-4\left\{-2\left(-2+\beta\right)\left(-1+2\beta\right)^3\left\{N\left[-1+\theta\left(-1+2\beta\right)\right]+2\left(-2+\beta\right)\eta\check{e}\right\}\right\}\times\left\{2\left(-1+2\beta\right)\left\{N\left[-1+\theta\left(-1+2\beta\right)\right]\right.\right.\\&\left.\left.\times\left[-\left(-2+\beta\right)\eta^2+2\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right]-2\left(-2+\beta\right)\eta\left\{\left(-2+\beta\right)\eta^2+2\left(-1+\beta\right)\left(-1+2\beta\right)\gamma\right\}\check{e}\right\}\right\}\end{aligned}$$

is greater than 0. So the equation has two points of intersection with the horizontal coordinates, which are:

Proposition 3 suggests that when a traditional retailer opens an online channel, the unit carbon emission reductions increase as the carbon tax increases when the carbon tax rate and initial unit carbon emissions are in a specific range. However, when the carbon tax rate and initial unit carbon emissions are in other ranges, the unit carbon emission reductions gradually decrease as the carbon tax increases.

Proposition 4: Centralized decision-making situations: $\frac{\partial {p_1}^{c*}}{\partial \theta}<0$; $\frac{\partial {p_2}^{c*}}{\partial \theta}<0$; $\frac{\partial \tilde{p}^{c*}}{\partial \theta}>0$; $\frac{\partial {p_1}^{c*}}{\partial \phi}>0$; $\frac{\partial {p_2}^{c*}}{\partial \phi}<0$. Decentralized decision-making situations: $\frac{\partial {p_1}^{d*}}{\partial \theta}<0$; $\frac{\partial {p_2}^{d*}}{\partial \theta}<0$; $\frac{\partial \tilde{p}^{d*}}{\partial \theta}>0$; $\frac{\partial \check{p}^{d*}}{\partial \theta}>0$; $\frac{\partial {p_1}^{d*}}{\partial \phi}>0$; $\frac{\partial {p_2}^{d*}}{\partial \phi}<0$.

Proof: Find the first-order partial derivatives of the offline retail prices of traditional retailers in the centralized decision-making scenario ${p_1}^{c*}$ with respect to θ and ϕ, respectively: $\frac{\partial {p_1}^{c*}}{\partial \theta} = -\frac{N\phi}{2+2\beta} < 0$, $\frac{\partial {p_1}^{c*}}{\partial \phi} = \frac{N-N\theta}{2+2\beta} > 0$, likewise, $\frac{\partial {p_2}^{c*}}{\partial \theta} = \frac{N(-1+\phi)}{2(1+\beta)} < 0$, $\frac{\partial {p_2}^{c*}}{\partial \phi} = \frac{N(-1+\theta)}{2(1+\beta)} < 0$, $\frac{\partial \check{p}^{c*}}{\partial \theta} = \frac{N}{2+2\beta} > 0$. Decentralized decision-making is the same. Proposition 4 shows that after a traditional retailer runs an online channel, the manufacturer's online direct selling price is positively related to the coefficient of consumer preference for the manufacturer's online channel, and the retail price of the traditional retailer's two channels is negatively related to the coefficient of consumer preference for the manufacturer's online channel. The retail price of traditional retailers' online channel is positively related to the coefficient of consumer preference for retailers' online channel, and the retail price of traditional retailers' offline channel is negatively related to the coefficient of consumer preference for retailers' online channel. Manufacturers' wholesale prices under decentralized decision-making are negatively correlated with the coefficient of consumer preference for manufacturers' network channels.

Proposition 5: In the case of centralized decision-making: $\frac{\partial {p_2}^{c*}}{\partial \eta} > 0$; $\frac{\partial {p_1}^{c*}}{\partial \eta} > 0$; $\frac{\partial \check{p}^{c*}}{\partial \eta} > 0$; $\frac{\partial \pi^{c*}}{\partial \eta} > 0$; In the case of decentralized decision-making: $\frac{\partial {p_2}^{d*}}{\partial \eta} > 0$; $\frac{\partial {p_1}^{d*}}{\partial \eta} > 0$; $\frac{\partial \check{p}^{d*}}{\partial \eta} > 0$; $\frac{\partial \mathring{p}^{d*}}{\partial \eta} > 0$; $\frac{\partial \hat{\pi}^{d*}}{\partial \eta} > 0$; $\frac{\partial \mathring{\pi}^{d*}}{\partial \eta} > 0$; $\frac{\partial \pi^{d*}}{\partial \eta} > 0$.

Proof: Find a first-order partial derivative of the offline retail price ${p_2}^{c*}$ about η for a traditional retailer under centralized decision making: $\frac{\partial {p_2}^{c*}}{\partial \eta} = -\frac{\left[3\tilde{e}(\eta+\tilde{e}-2\beta\tilde{e})^2-2\eta\gamma\right][N+3(-1+2\beta)\tilde{e}\check{e}]}{9(\eta+\tilde{e}-2\beta\tilde{e})^4+12(-1+2\beta)(\eta+\tilde{e}-2\beta\tilde{e})^2\gamma+4(1-2\beta)^2\gamma^2} > 0$, similarly, it can be proven that $\frac{\partial {p_1}^{c*}}{\partial \eta} > 0$; $\frac{\partial \check{p}^{c*}}{\partial \eta} > 0$; $\frac{\partial \pi^{c*}}{\partial \eta} > 0$; $\frac{\partial {p_2}^{d*}}{\partial \eta} > 0$; $\frac{\partial {p_1}^{d*}}{\partial \eta} > 0$; $\frac{\partial \check{p}^{d*}}{\partial \eta} > 0$; $\frac{\partial \mathring{p}^{d*}}{\partial \eta} > 0$; $\frac{\partial \hat{\pi}^{d*}}{\partial \eta} > 0$; $\frac{\partial \mathring{\pi}^{d*}}{\partial \eta} > 0$; $\frac{\partial \pi^{d*}}{\partial \eta} > 0$.

Proposition 5 suggests that traditional retailers' online and offline retail prices, manufacturers' online direct prices, wholesale prices, and supply chain margins are positively related to low-carbon preferences.

Proposition 6: In the case of centralized decision-making: $\frac{\partial {p_1}^{c*}}{\partial \tilde{e}} > 0$;$\frac{\partial {p_2}^{c*}}{\partial \tilde{e}} > 0$; $\frac{\partial \check{p}^{c*}}{\partial \tilde{e}} > 0$; $\frac{\partial \pi^{c*}}{\partial \tilde{e}} < 0$. In the case of decentralized decision-making: $\frac{\partial {p_1}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial {p_2}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial \check{p}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial \mathring{p}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial \hat{\pi}^{d*}}{\partial \tilde{e}} < 0$; $\frac{\partial \mathring{\pi}^{d*}}{\partial \tilde{e}} < 0$; $\frac{\partial \pi^{d*}}{\partial \tilde{e}} < 0$.

Proof: Taking a first-order partial derivative of the retail price of a traditional retailer under centralized decision-making $\partial {p_1}^{c*}$ with respect to e:

$$\begin{aligned}\frac{\partial {p_1}^{\mathrm{c}*}}{\partial \tilde{e}} = &\frac{1}{9(\eta+\tilde{e}-2\beta\tilde{e})^4+12(-1+2\beta)(\eta+\tilde{e}-2\beta\tilde{e})^2\gamma+4(1-2\beta)^2\gamma^2} \\ &\times \left\{3N\eta(\eta+\tilde{e}-2\beta\tilde{e})^2-2N(1-2\beta)^2\tilde{e}\gamma\right. \\ &\quad \left. + \left[9\eta^2(\eta+\tilde{e}-2\beta\tilde{e})^2-3(-1+2\beta)\left(-3\eta^2+4(-1+2\beta)\eta\tilde{e}+(1-2\beta)^2\tilde{e}^2\right)\gamma+2(1-2\beta)^2\gamma^2\right]\check{e}\right\}\end{aligned}$$

, by solving for this we get $\frac{\partial {p_1}^{c*}}{\partial \tilde{e}} > 0$. Similarly, it can be proven that $\frac{\partial {p_1}^{c*}}{\partial \tilde{e}} > 0$; $\frac{\partial {p_2}^{c*}}{\partial \tilde{e}} > 0$; $\frac{\partial \check{p}^{c*}}{\partial \tilde{e}} > 0$; $\frac{\partial \pi^{c*}}{\partial \tilde{e}} < 0$; $\frac{\partial {p_1}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial {p_2}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial \check{p}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial \mathring{p}^{d*}}{\partial \tilde{e}} > 0$; $\frac{\partial \hat{\pi}^{d*}}{\partial \tilde{e}} < 0$; $\frac{\partial \mathring{\pi}^{d*}}{\partial \tilde{e}} < 0$; $\frac{\partial \pi^{d*}}{\partial \tilde{e}} < 0$.

Proposition 6 shows that traditional retailers' online and offline retail prices, manufacturers' online direct prices, and wholesale prices are all increasing functions of the carbon tax rate, and supply chain members' profits are all decreasing functions of the carbon tax rate.