**WEEK 3 NOTES — Hypothesis Testing in SPSS**

## 📌 Core Concepts

### 🔷 What is Hypothesis Testing?

It's a process to check whether a **sample result** gives enough evidence to reject a **null hypothesis** about a population.

## 🧪 Steps of Hypothesis Testing (ALWAYS follow these 5!)

1. **State the Hypotheses**
   - Null Hypothesis ($H_0$): assumes *no effect or difference*
   - Alternative Hypothesis ($H_1$ or $H_a$): assumes *a real effect or difference*
2. **Collect Data & Compute the Statistic**
   - Sample mean, standard deviation, sample size, etc.
3. **Construct the Sampling Distribution (under $H_0$)**
   - Use sample SD and sample size to estimate Standard Error:

$$SE = \frac{s}{\sqrt{n}}$$

4. **Determine the Rejection Region**
   - Use **Z or t values** (e.g. ±1.96 for 95% CI)
5. **Compare Sample Result with Rejection Region**
   - If p-value ≤ 0.05 → **reject $H_0$**
   - If p-value > 0.05 → **do not reject $H_0$**

The p-value is the probability of observing a value equal or more extreme than our sampled value, under the null hypothesis

When you perform a statistical test, a p-value helps you determine the significance of your results in relation to the null hypothesis.

The null hypothesis (H0) states no relationship exists between the two variables being studied (one variable does not affect the other). It states the results are due to chance and are not significant in supporting the idea being investigated. Thus, the null hypothesis assumes that whatever you try to prove did not happen.

The alternative hypothesis (Ha or H1) is the one you would believe if the null hypothesis is concluded to be untrue.

The alternative hypothesis states that the independent variable affected the dependent variable, and the results are significant in supporting the theory being investigated (i.e., the results are not due to random chance).

A p-value, or probability value, is a number describing how likely it is that your data would have occurred by random chance (i.e., that the null hypothesis is true).

The smaller the *p*-value, the less likely the results occurred by random chance, and the stronger the evidence that you should reject the null hypothesis.

| Hypotheses | Suitable test | Decision |
|---|---|---|
| $H_0$: means are equal<br>$H_a$: means are not equal | • *test statistic* and<br>• *degrees of freedom* | p-value>0.05 go with $H_0$<br>p-value≤0.05 go with $H_a$ |

| Hypotheses | One sample t-test | |
|---|---|---|
| $H_0$: $\mu=\mu_0$<br>$H_a$: $\mu\neq\mu_0$ | $t=\frac{\bar{x}-\mu_0}{s.e.}$, df=N-1<br>$s.e.=\sqrt{s^2/N}$ | Is the population mean ($\mu$) equal to a certain value ($\mu_0$)? |

| Hypotheses | One sample $\chi^2$-test | |
|---|---|---|
| $H_0$: $\pi=\pi_0$<br>$H_a$: $\pi\neq\pi_0$ | $\chi^2=\sum\frac{(O-E)^2}{E}$, df=c-1 | Is the population proportion ($\pi$) equal to a certain value ($\pi_0$)? |

## 📉 One-Sample t-Test (Testing a mean)

### 📋 Hypotheses:

- $H_0$: $\mu = \mu_0$
- $H_a$: $\mu \neq \mu_0$

**Example:**

> $H_0$: The average income of women = £40,000
> $H_1$: The average income ≠ £40,000

Result: $\bar{x}$ = £35,200, s = 2000, n = 100

$$SE = \frac{2000}{\sqrt{100}} = 200$$

$$CI = 40000 \pm 1.96 \times 200 \rightarrow [39600, 40400]$$

Since £35,200 is **outside** this range → **Reject $H_0$**

## 🧮 One-Sample Chi-Square Test (Testing a proportion)

### 📋 Hypotheses:

- $H_0$: $\pi = \pi_0$
- $H_1$: $\pi \neq \pi_0$

**Example:**

> Are 50% of women born in the same city they work?

If observed proportion differs **significantly** from expected 50%, you may reject $H_0$.

# ✅ One-Sample t-Test

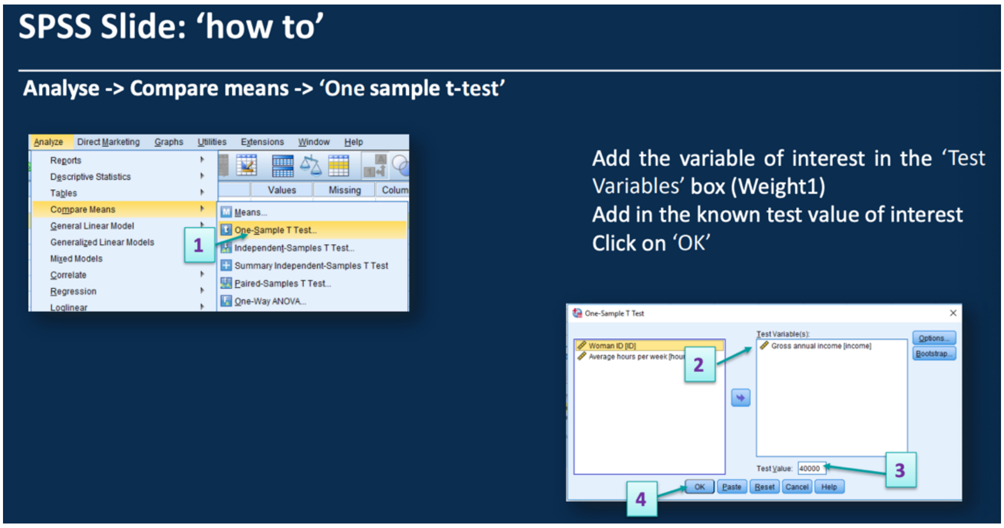
### Menu path:

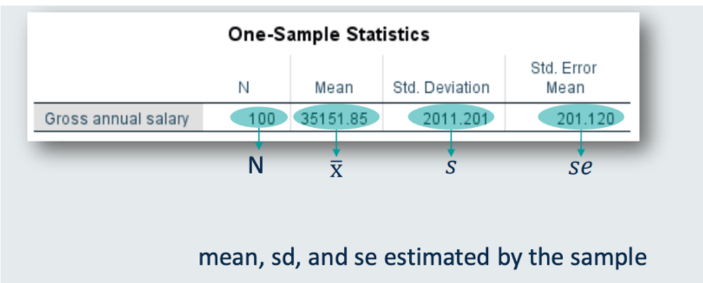
```sql
Analyze > Compare Means > One-Sample t-test
```

- Put your variable into **Test Variable(s)**
- Enter the **Test Value** (e.g. 40000)
- Click **OK**

🧠 SPSS will give you:

- Sample Mean, SD, SE
- t-statistic, df, and **p-value**



One-Sample Statistics

| | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|
| Gross annual salary | 100 | 35151.85 | 2011.201 | 201.120 |

Population mean if the null is TRUE

$\mu_0$

**One-Sample Test**

| | Test Value = 40000 | | | | | |
|---|---|---|---|---|---|---|
| | | | | | 95% Confidence Interval of the Difference | |
| | t | df | Sig. (2-tailed) | Mean Difference | Lower | Upper |
| Gross annual salary | -24.106 | 99 | .000 | -4848.150 | -5247.22 | -4449.08 |

We tested if the difference is zero, rather than if the values are equal.

Now this is for the difference.

$H_0$: $\mu = £40000 \leftrightarrow \mu - £40000 = 0$

$H_\alpha$: $\mu \neq £40000 \leftrightarrow \mu - £40000 \neq 0$

**One-Sample Test**

| | Test Value = 40000 | | | | | |
|---|---|---|---|---|---|---|
| | | | | | 95% Confidence Interval of the Difference | |
| | t | df | Sig. (2-tailed) | Mean Difference | Lower | Upper |
| Gross annual salary | -24.106 | 99 | .000 | -4848.150 | -5247.22 | -4449.08 |

Test statistic
$t = \frac{\bar{x} - \mu_0}{s.e.}$

Degrees of freedom: n-1

p-value: $p<0.001$

*If we reject the null hypothesis, there is less than 1 chance in a 1000 to be wrong.*

*If the null hypothesis is true, there would be less than 1 chance in a 1000 to sample the value we did. Thus we feel confident enough to reject the null hypothesis based on our data.*

**One-Sample Test**

| | Test Value = 40000 | | | | | |
|---|---|---|---|---|---|---|
| | | | | | 95% Confidence Interval of the Difference | |
| | t | df | Sig. (2-tailed) | Mean Difference | Lower | Upper |
| Gross annual salary | -24.106 | 99 | .000 | -4848.150 | -5247.22 | -4449.08 |

We infer, the difference between men's and women's incomes is ‘statistically’ significant.

That is, womens' income is statistically different than that of men.

***Based on our sample, the expected mean difference in the income between women and men is -£4848.2 (95% CI: [-5247.2,-4449.1]). This difference is statistically significant (t=-24.106, df=99, p<0.001).***

## ✅ One-Sample Chi-Square Test

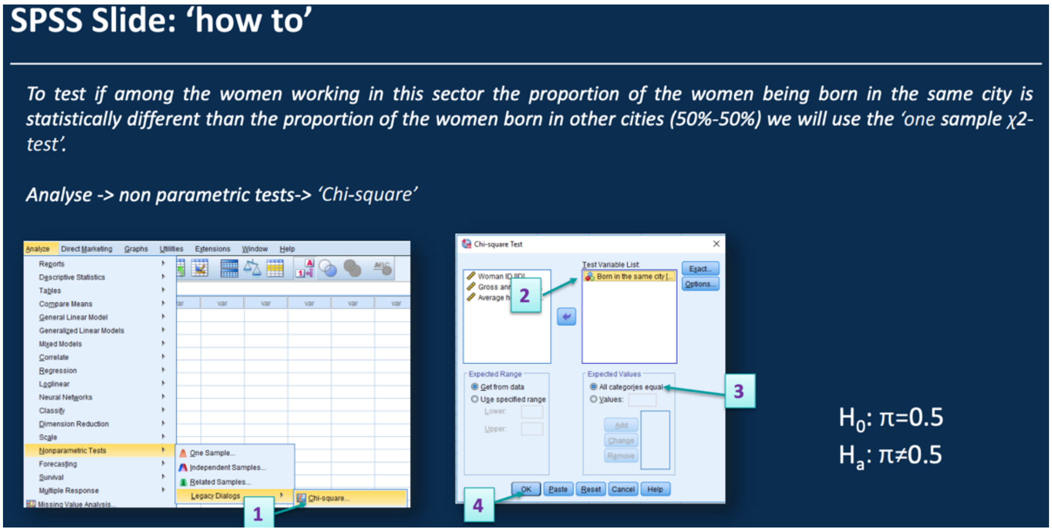
### Menu path:

```scss
Analyze > Nonparametric Tests > Chi-Square
```

- Choose your **categorical variable** (e.g. place of birth)
- Click **Expected Values**, enter proportion (e.g. 50%-50% or 80%-20%)
- Run the test

🧠 SPSS will give you:

- Observed vs expected counts
- Chi-square value, df
- **p-value**



# Equality of Proportions: The One Sample $\chi^2$–test

*SPSS prints a table with descriptive statistics and one with the one sample $\chi^2$ -test*

**Born in the same city**

| | Observed N | Expected N | Residual |
|---|---|---|---|
| No | 81 | 49.5 | 31.5 |
| Yes | 18 | 49.5 | -31.5 |
| Total | 99 | | |

How things are in our sample

How things should be if the null was correct ($\pi$=0.5)

Difference $(O - E)$

$H_0$: $\pi$=0.5
$H_a$: $\pi \neq$0.5

**Test Statistics**

| | Born in the same city |
|---|---|
| Chi-Square | 40.091[a] |
| df | 1 |
| Asymp. Sig. | .000 |

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 49.5.

Test statistic $\sum \frac{(O-E)^2}{E}$

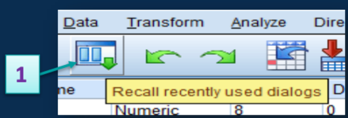
Degrees of freedom: c-1

p-value: p<0.001

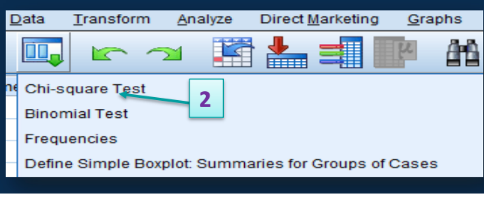
***Based on our sample, among the women working in this industry, the proportion of the women being born in the same city is statistically different than the proportion of the women born in other cities ($\chi^2$=40.091, df=1, p<0.001).***

*Say now that the proportion of women who work in companies in cities different than those they were born ($\pi$) is believed to be 0.2 (20%). Do our data provide evidence against this null hypothesis?*

**On the command bar press the recall button**

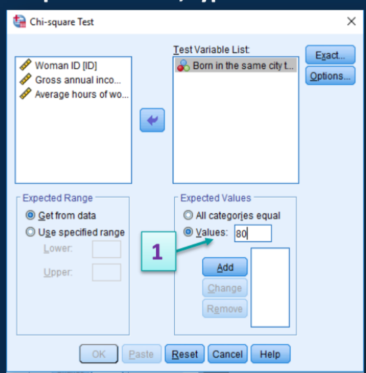


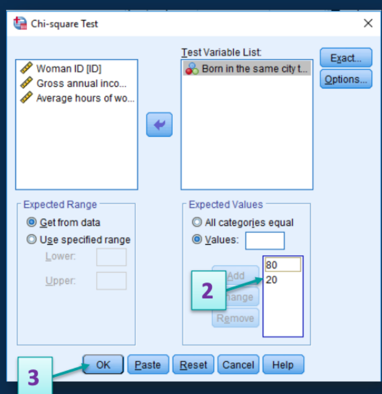

$H_0$: $\pi$=0.2
$H_a$: $\pi \neq$0.2




*Say now that the proportion of women who work in companies in cities different than those they were born ($\pi$) is believed to be 0.2 (20%). Do our data provide evidence against this null hypothesis?*

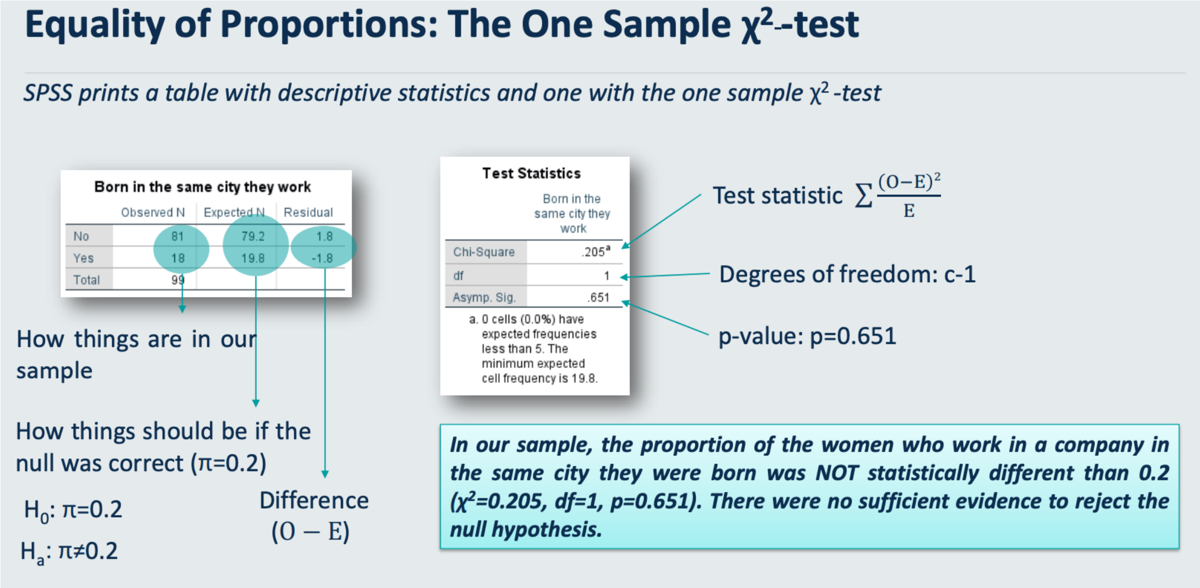
**In the expected values, type 80% and then 20%**




$H_0$: $\pi$=0.2
$H_a$: $\pi \neq$0.2

*SPSS wants you to add test values for all categories, in order (0, 1, 2, 3.....)*




## 🧠 Analogy Used in Class (Treasure Hunt!)

- $H_0$ = No treasure
- $H_1$ = There is treasure
- Finding it = rejecting $H_0$
- Not finding it = doesn't mean it's not there → **do not accept $H_0$**

### ⚠️ Important Terms to Know

| Term | Meaning |
|---|---|
| **Null Hypothesis ($H_0$)** | No difference/effect |
| **Alternative Hypothesis ($H_1$)** | There is a difference/effect |
| **Type I Error (α)** | Rejecting $H_0$ when it's actually true |
| **Type II Error (β)** | Failing to reject $H_0$ when it's false |
| **Power (1–β)** | Probability of correctly rejecting a false $H_0$ |

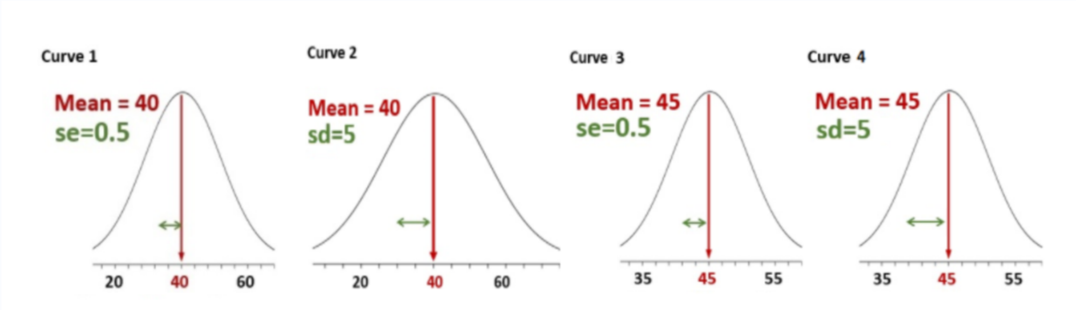
The type 1 error = Probability of rejecting a true null hypothesis (p-value)

The type 2 error = Probability of not rejecting a false null

The Power = Probability of rejecting a false null

A researcher tests if the mean hours of work in a population is $\mu_0$=40 hours per week. He samples N=100 individuals and finds a sample mean of 45h/week and a standard deviation of 5 hours per week. The estimated sample error is therefore 0.5 hours per week.

Which of the following normal curves represents the sampling distribution under the null hypothesis?




Normal Curve 1

The mean of the sampling distribution under the null hypothesis equals the test value µ0 (here 40h/week) and not the sample mean (here 45h/week).

The sd of the sampling distribution equals the sd of the population, divided by the square root of the sample size (and is called the standard error). To estimate it we use the sd from our sample (here se=5/10=0.5).

Curve 1

The correct answer is: Curve 1

## ✅ When can we *reject* the null hypothesis ($H_0$)?

You can **reject $H_0$** when:

- The **p-value** is **less than or equal to your significance level (α)**, usually:

$$\text{If } p \leq 0.05 \Rightarrow \text{Reject } H_0$$

- Your test statistic falls into the **rejection region** (e.g., outside the 95% confidence interval under $H_0$).
- Your observed sample result is **very unlikely to occur by chance** if $H_0$ were true.

📌 *This means there's **enough evidence** to support the alternative hypothesis ($H_1$).*

### 🚫 When can we *not reject* the null hypothesis?

You **do not reject $H_0$** when:

- The **p-value is greater than 0.05**
- Your result falls **within** the confidence interval expected under $H_0$

📌 *This means you **don't have enough evidence** to support $H_1$ — **but it doesn't prove $H_0$ is true**!*

Institute of Psychiatry, Psychology and Neuroscience

**Module Title:** Introduction to Statistics

**Session Title:** Lecture Knowledge Quiz 3 - Solutions

# Topic title: Confidence and significance (II)

# Lecture Progress Quiz 3

Welcome to the **Topic 3 Knowledge Quiz**

This quiz is made up of 5 multiple choice questions and a bonus question.

You have are given 10 mins to complete the quiz.  The quiz is timed and you will not be able to answer a question after the time has ended.

The quiz has free navigation and you can move between the questions using the navigation pane to your left.

# Question 1

A researcher wishes to test if the mean hours of work in a population is 40h per week. She samples N=100 individuals and finds a sample mean of 45h per week. Which of the below is the correct null hypothesis.

Select one:

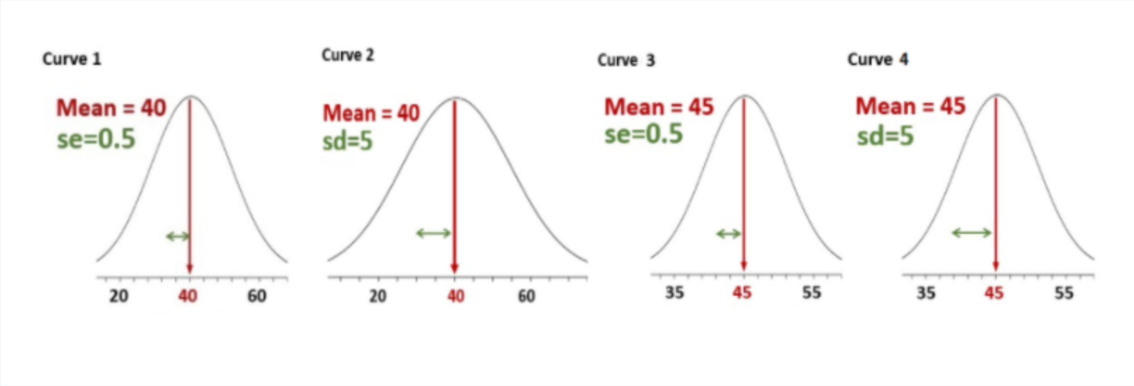
- a. $H_0$: The population mean is different than $\mu_0$= 45 hours/week
- b. $H_0$: The population mean is equal to $\mu_0$= 40 hours/week
- c. $H_0$: The population mean is equal to $\mu_0$= 45 hours/week
- d. $H_0$: The population mean is different than $\mu_0$= 40 hours/week

The null hypothesis is $H_0$: $\mu=\mu_0=40$ versus the alternative $H_a$: $\mu\neq\mu_0=40$

# Question 2

A researcher tests if the mean hours of work in a population is $\mu_0$=40 hours per week. He samples N=100 individuals and finds a sample mean of 45h/week and a standard deviation of 5 hours per week. The estimated sample error is therefore 0.5 hours per week.

Which of the following normal curves represents the sampling distribution under the null hypothesis?




Select one:

1. Curve 1
2. Curve 2
3. Curve 3
4. Curve 4

The mean of the sampling distribution under the null hypothesis equals the test value $\mu_0$ (here 40h/week) and not the sample mean (here 45h/week).

The sd of the sampling distribution equals the sd of the population, divided by the square root of the sample size (and is called the standard error). To estimate it we use the sd from our sample (here se=5/10=0.5).

# Question 3

A researcher uses the 'one sample t-test' to test the null hypothesis 'Ho: the population mean equals 40h/week'.

The result shows that the p-value = 0.089. What does the researcher need to state in his report?

Select one:

- a. We reject the null hypothesis that the population mean is 40h/week
- b. We accept the null hypothesis that the population mean is 40h/week
- c. We accept the alternative hypothesis that the population mean is different to 40h/week
- d. We do not reject the null hypothesis that the population mean is 40h/week

The p-value is larger than 0.05, therefore there is no significant difference. That is, we do NOT have evidence to reject the null hypothesis (but we can never use the term 'accept the null').

$H_0$: $\mu$=40h/week

$H_a$: $\mu \neq$40h/week

# Question 4

The p-value is

Select one:

1. the probability of observing a value
2. the probability of observing a value equal to our sampled value
3. the probability of observing a value equal or more extreme than our sampled value
4. the probability of observing a value equal or more extreme than our sampled value, under the null hypothesis

The p-value is a conditional probability:
P-value=P($\bar{x} \geq$ sampled value | $H_o$ is true)

# Question 5

Rejecting a false null hypothesis is:

Select one:

- a. the effect size
- b. the Type II error
- c. the Type I error
- d. the Power

| | | Sample | |
|---|---|---|---|
| | | $H_0$ rejected | $H_0$ not rejected |
| **Population** | $H_0$ was true | Type I error **$\alpha$** | Correct inference **$1-\alpha$** |
| | $H_0$ was false | Correct inference **$1-\beta$** | Type II error **$\beta$** |

# Question 6

She obtains a random sample of 49 post graduate students and administers a conceptual problem-solving test to them. It is known that post graduate students in traditional lecture have an average score of 82 on the test. The students in this study score an average of 86 with a standard deviation of 5. Compute the 95% CI under the null hypothesis to be able to choose the correct one among the statements below.

Select one:

- a. Reject the null hypothesis and claim that flipped sessions affect problem-solving skills.
- b. Accept the research hypothesis and claim type of session does not affect problem-solving skills.
- c. Do not reject the null hypothesis and claim that flipped sessions affect problem-solving skills.
- d. There wasn't enough statistical power for you to find an effect.

The 95% confidence interval under the null hypothesis is given by

$$\boldsymbol{\mu_0} \pm \mathbf{1.96}\frac{s}{\sqrt{n}} = 82 \pm \mathbf{1.96}\frac{\mathbf{5}}{\sqrt{49}} = [80.6, 83.4]$$

As the estimated population mean is 86 and outside of this range, we reject the null hypothesis and conclude that flipped sessions affect problem solving skills.