Institute of Psychiatry, Psychology and Neuroscience

KING'S College LONDON

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Department: Biostatistics and Health Informatics

Module Title: Introduction to Statistics

Session Title: Equality of means: t-tests

Topic title: Comparing groups I (parametric methods)

Topic materials:
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Contributions:
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Improvements:
Nick Beckley-Hoelscher
Kim Goldsmith
Sabine Landau

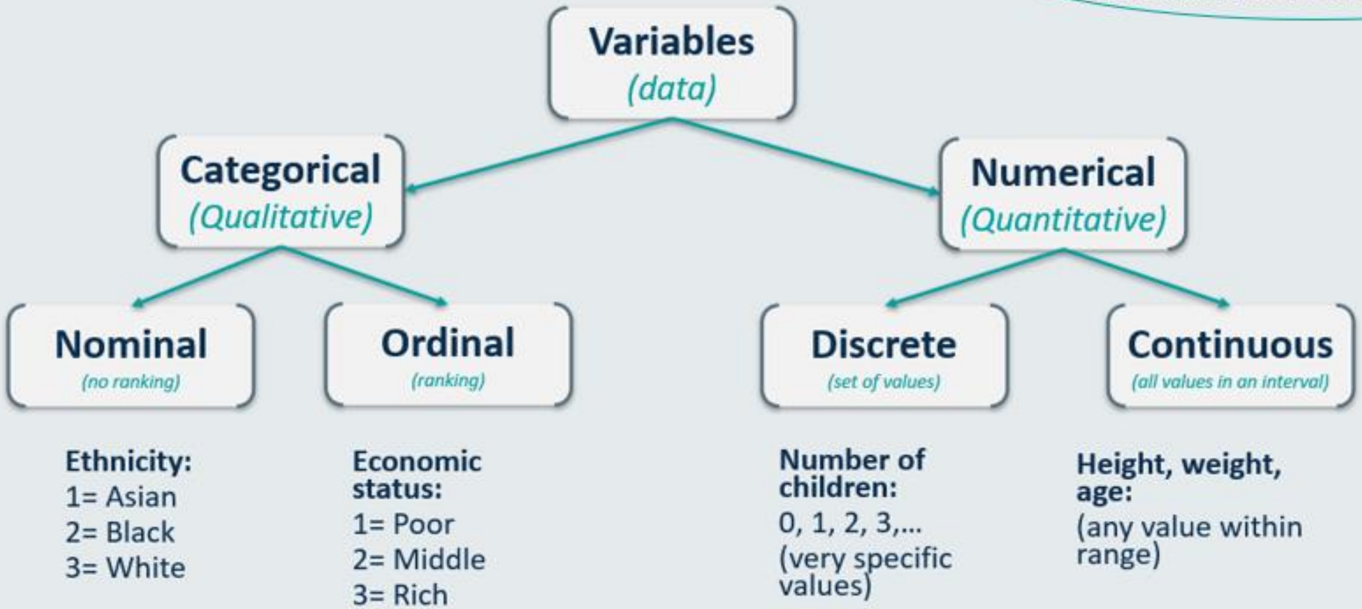
Learning Outcomes

- Learn when and how to use student t-tests for equality of means.
- Understand the assumptions of the various tests for equality of means.
- Be able to conduct these tests in a statistical software.

Previously on ‘Introduction to Statistical Methods’…

Based on the type of each variable, we use different ways to describe the data.

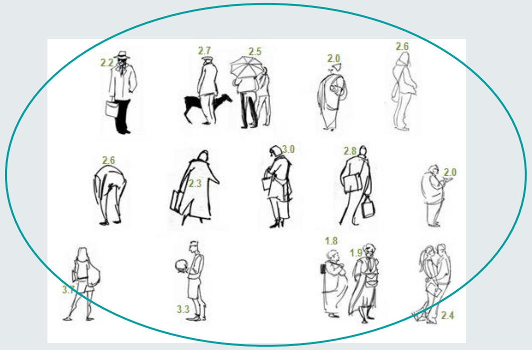
Blue for symmetrical (normal)
Purple italics for skewed or discrete data



	Categorical	Numerical
• Descriptive indices	Frequencies (Percentages %)	location: mean, *median*, mode Dispersion: SD, *min-max*, range
• Charts/plots	Bar Chart	Histogram, Box plot

Equality of Means: The Three t-tests

one sample t-test



$H_0: \mu = \mu_0$

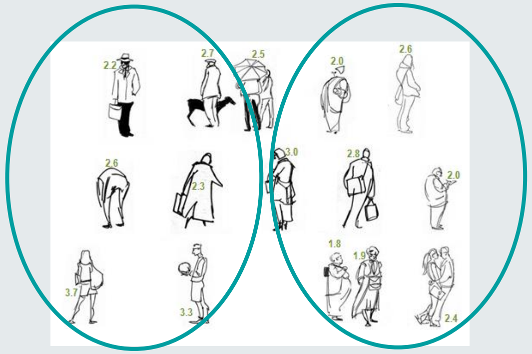
$H_a: \mu \neq \mu_0$

Examples

Difference from test value:

- age≠25yo
- height ≠ 1.60cm
- weight ≠ 80kg

independent samples t-test



$H_0: \mu_A = \mu_B$

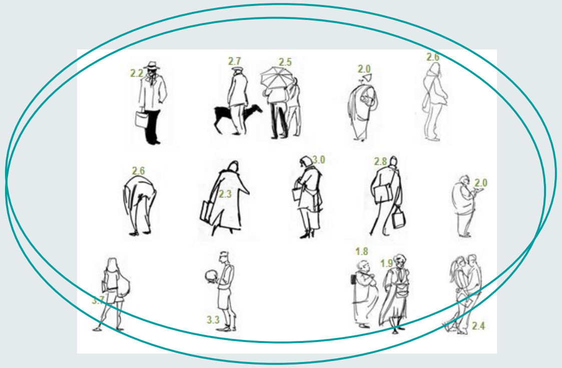
$H_a: \mu_A \neq \mu_B$

Examples

Difference in the means:

- young vs old
- males vs females
- City A vs City B

paired samples t-test



$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

Examples

Difference in the means:

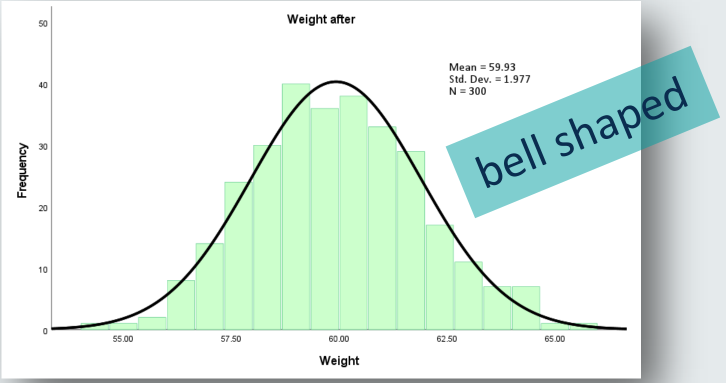
- before and after treatment
- twin studies
- matched cases vs controls

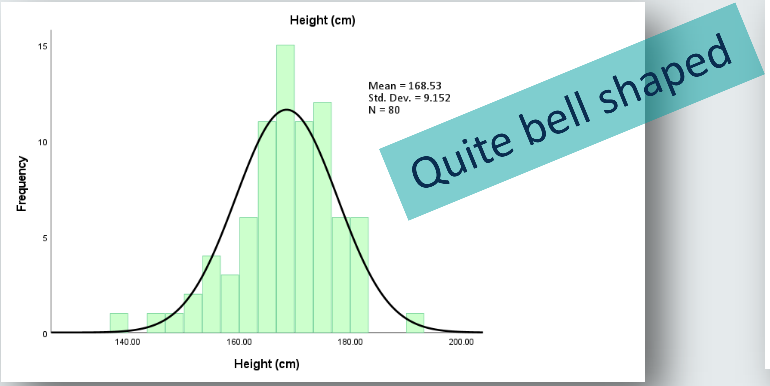
The variable whose mean is tested needs to be fairly symmetrical (bell-shaped)

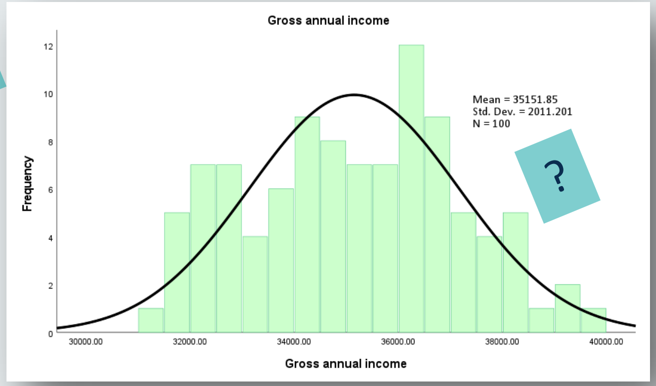
Equality of Means: The Three t-tests

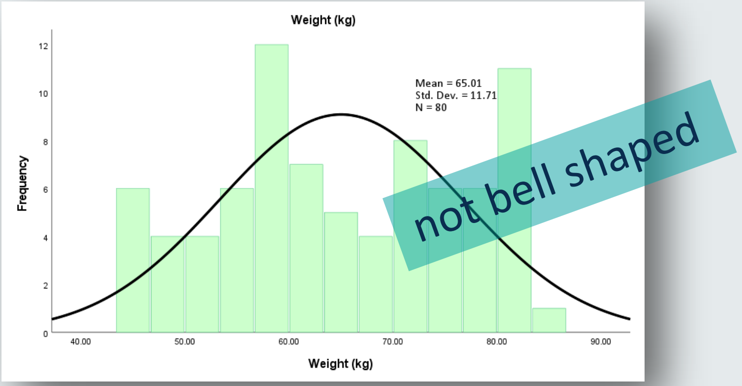
Bell shaped, symmetrical, normal data?

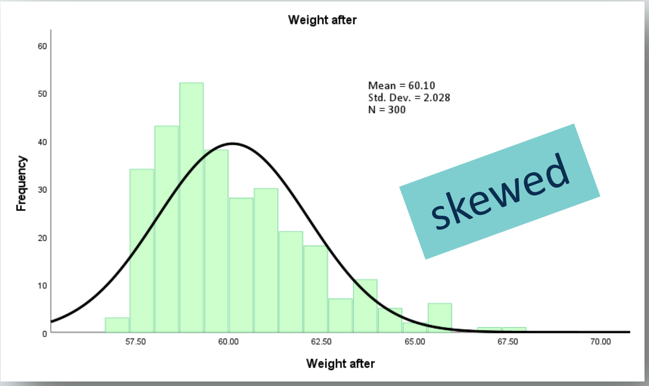
Often in real research the distributions will not be perfect bells. It can be challenging to tell the difference.









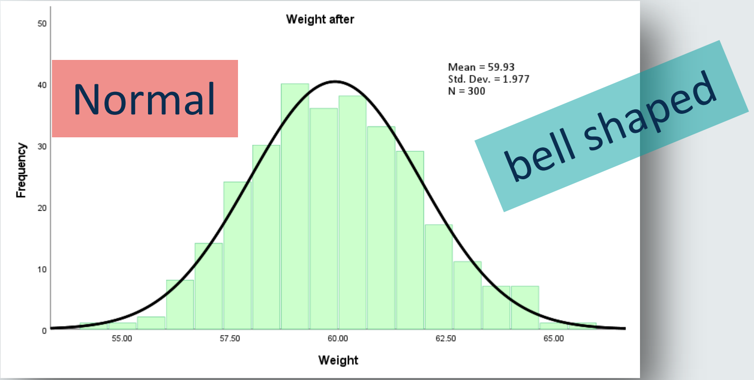


Equality of Means: The Three t-tests

Some researchers will relay to ‘normality tests’ available in SPSS and other software such as the Kolmogorov-Smirnov test. Here, we do not recommend the tests as they can be very conservative.

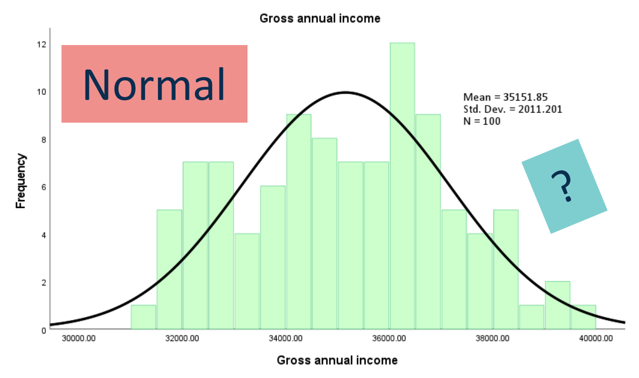
The tests essentially test if your data (green) are too far from the corresponding normal distribution (the one with the same mean and standard deviation as your data-black curve).

The null hypothesis is ‘there is no difference between your data and normality’. Therefore the data are normal if the p-value turns out to be p>0.05.



One-Sample Kolmogorov-Smirnov Test

		Weight after
N		300
Normal Parameters[a,b]	Mean	59.9268
	Std. Deviation	1.97707
Most Extreme Differences	Absolute	.026
	Positive	.026
	Negative	-.018
Test Statistic		.026
Asymp. Sig. (2-tailed)[c]		.200[d]



One-Sample Kolmogorov-Smirnov Test

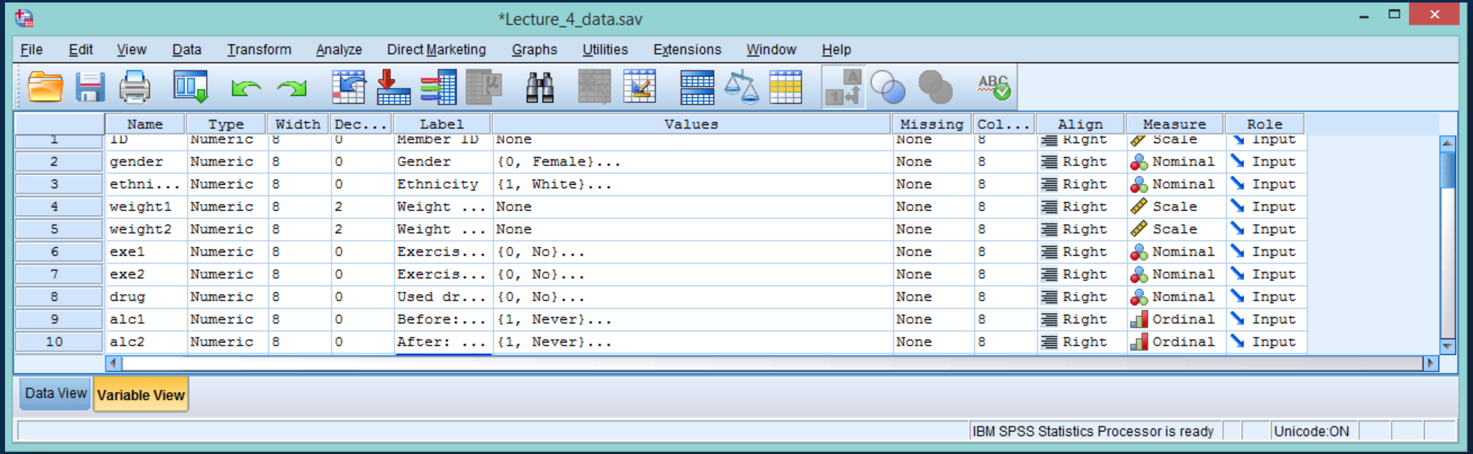
		Gross annual income
N		100
Normal Parameters[a,b]	Mean	35151.85
	Std. Deviation	2011.201
Most Extreme Differences	Absolute	.059
	Positive	.059
	Negative	-.056
Test Statistic		.059
Asymp. Sig. (2-tailed)[c]		.200[d]

One-Sample Kolmogorov-Smirnov Test

		Years living in London
N		80
Normal Parameters[a,b]	Mean	10.9000
	Std. Deviation	10.58133
Most Extreme Differences	Absolute	.335
	Positive	.335
	Negative	-.216
Test Statistic		.335
Asymp. Sig. (2-tailed)[c]		.000

SPSS Slide

Download the data that we are going to use during the lecture. The dataset is the **lecture_4_data.sav**. We have data for 300 individuals.



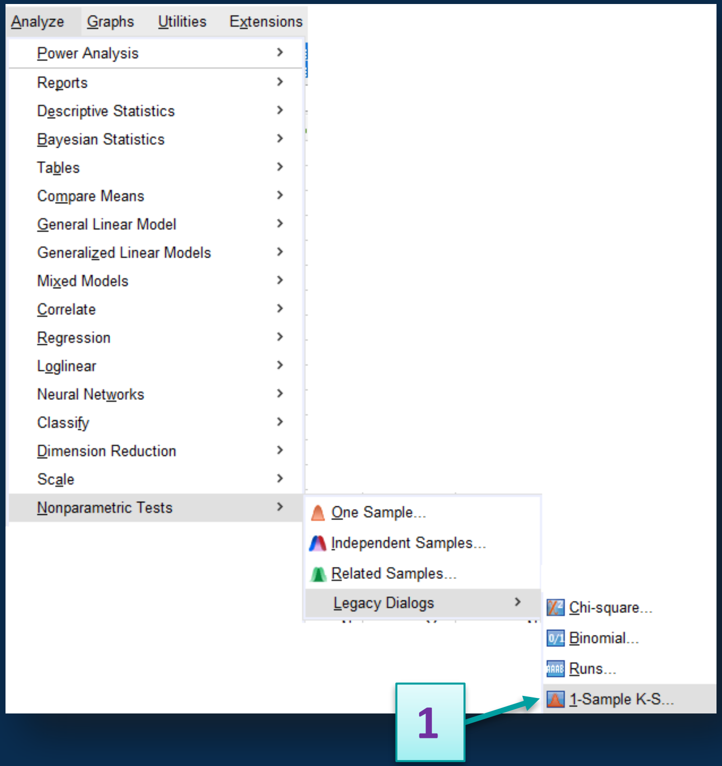
	Name	Type	Width	Dec...	Label	Values	Missing	Col...	Align	Measure	Role
1	ID	Numeric	8	0	Member ID	None	None	8	Right	Scale	Input
2	gender	Numeric	8	0	Gender	{0, Female}...	None	8	Right	Nominal	Input
3	ethni...	Numeric	8	0	Ethnicity	{1, White}...	None	8	Right	Nominal	Input
4	weight1	Numeric	8	2	Weight ...	None	None	8	Right	Scale	Input
5	weight2	Numeric	8	2	Weight ...	None	None	8	Right	Scale	Input
6	exe1	Numeric	8	0	Exercis...	{0, No}...	None	8	Right	Nominal	Input
7	exe2	Numeric	8	0	Exercis...	{0, No}...	None	8	Right	Nominal	Input
8	drug	Numeric	8	0	Used dr...	{0, No}...	None	8	Right	Nominal	Input
9	alc1	Numeric	8	0	Before:...	{1, Never}...	None	8	Right	Ordinal	Input
10	alc2	Numeric	8	0	After: ...	{1, Never}...	None	8	Right	Ordinal	Input

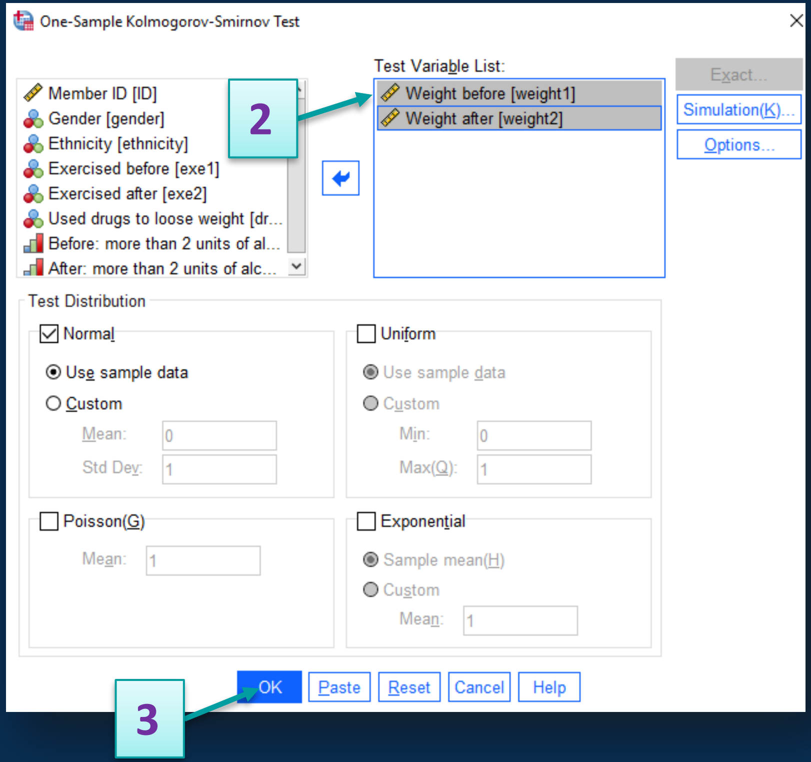
- **gender**: 1-male, 0-female and **ethnicity** : 1-white, 2-black, 3-Asian, 4-other
- **weight1**: their weight when they entered the programme (in kg)
- **weight2**: their weight by the end of the programme (in kg)
- **exe1**: info if they regularly exercised (1-yes, 0-no) when they entered the programme
- **exe2**: info if they regularly exercised (1-yes, 0-no) by the end of the programme
- **drug**: if they have ever used drugs to lose weight (1-yes, 0-no)
- **alc1**: more than 2 units of alcohol, before (1:never, 2: sometimes, 3:always)
- **alc2**: more than 2 units of alcohol, after (1:never, 2: sometimes, 3:always)

SPSS Slide

Two of the variables are numerical continuous and we wish to know if they are normally distributed.

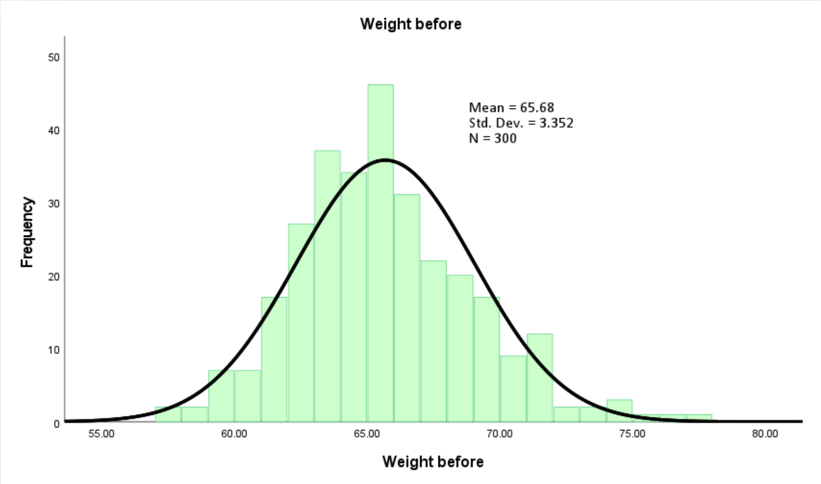
- **weight1**: their weight when they entered the programme (in kg)
- **weight2**: their weight by the end of the programme (in kg)

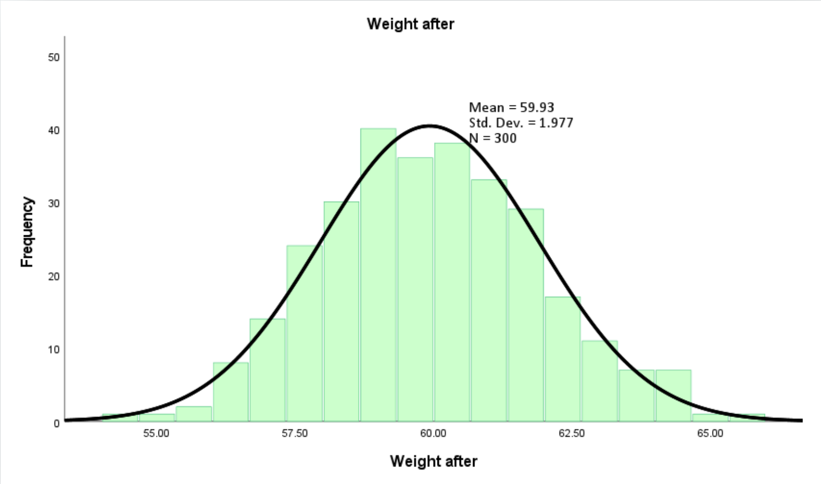




Output & Interpretation Slide

Are the data normal?





H_0: The data do not differ from normal

One-Sample Kolmogorov-Smirnov Test

			Weight before	Weight after
N			300	300
Normal Parameters[a,b]	Mean		65.6768	59.9268
	Std. Deviation		3.35230	1.97707
Most Extreme Differences	Absolute		.059	.026
	Positive		.059	.026
	Negative		-.042	-.018
Test Statistic			.059	.026
Asymp. Sig. (2-tailed)[c]			.014	.200[e]
Monte Carlo Sig. (2-tailed)[d]	Sig.		.011	.893
	99% Confidence Interval	Lower Bound	.008	.885
		Upper Bound	.014	.901

a. Test distribution is Normal.

b. Calculated from data.

c. Lilliefors Significance Correction.

d. Lilliefors' method based on 10000 Monte Carlo samples with starting seed 1455697065.

e. This is a lower bound of the true significance.

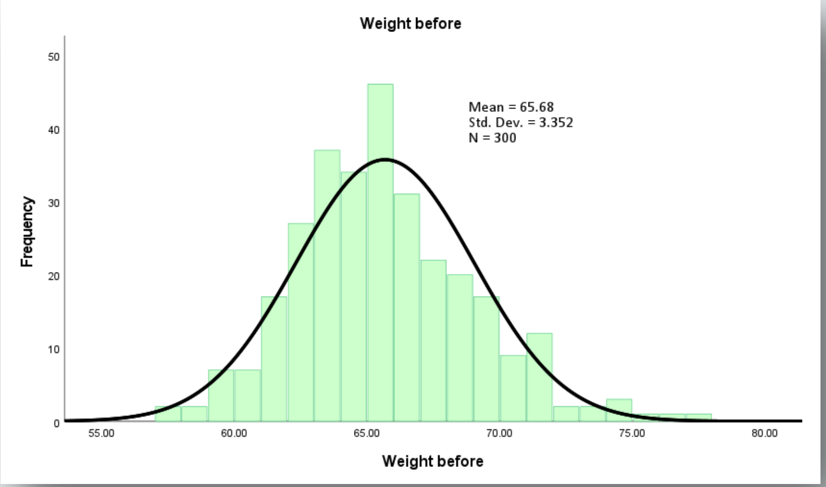
The test rejects the null for 'weight before' but not for 'weight after'

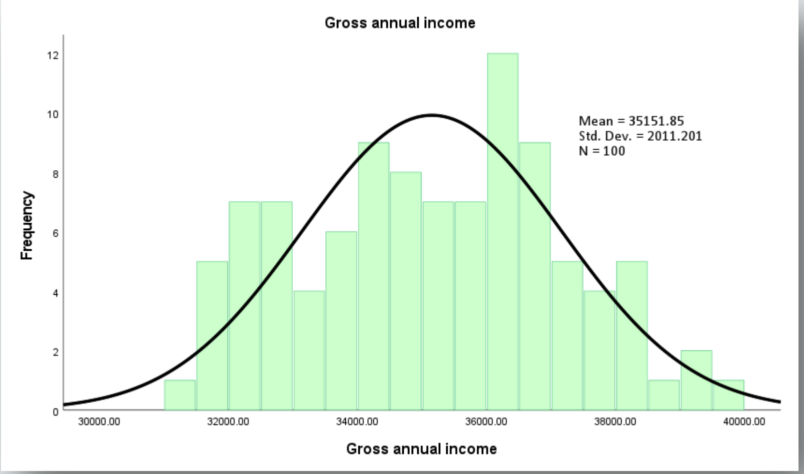
The data in both variables look symmetrical and bell shaped and we are going to accept normality

Output & Interpretation Slide

Are the data normal?

H_0: The data do not differ from normal





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Most Extreme Differences	Absolute	.059
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Test Statistic		.059
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Equality of Means: The Three t-tests

Hypotheses	Suitable test	Decision
H_0: is equal H_a: not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses	One sample t-test
H_0: $\mu=\mu_0$ H_a: $\mu\neq\mu_0$	$t = \frac{\bar{x}-\mu_0}{s.e.}$, df=n-1 $s.e. = \sqrt{s^2/n}$

Is the population mean (μ) equal to a certain value (μ_0)?

One Sample t-test

When to use

To test if, according to the current data, the mean in the population differs from a pre-specified test value.

Hypotheses:

H_0: Mean equals a certain pre-specified value **$\mu = \mu_0$**

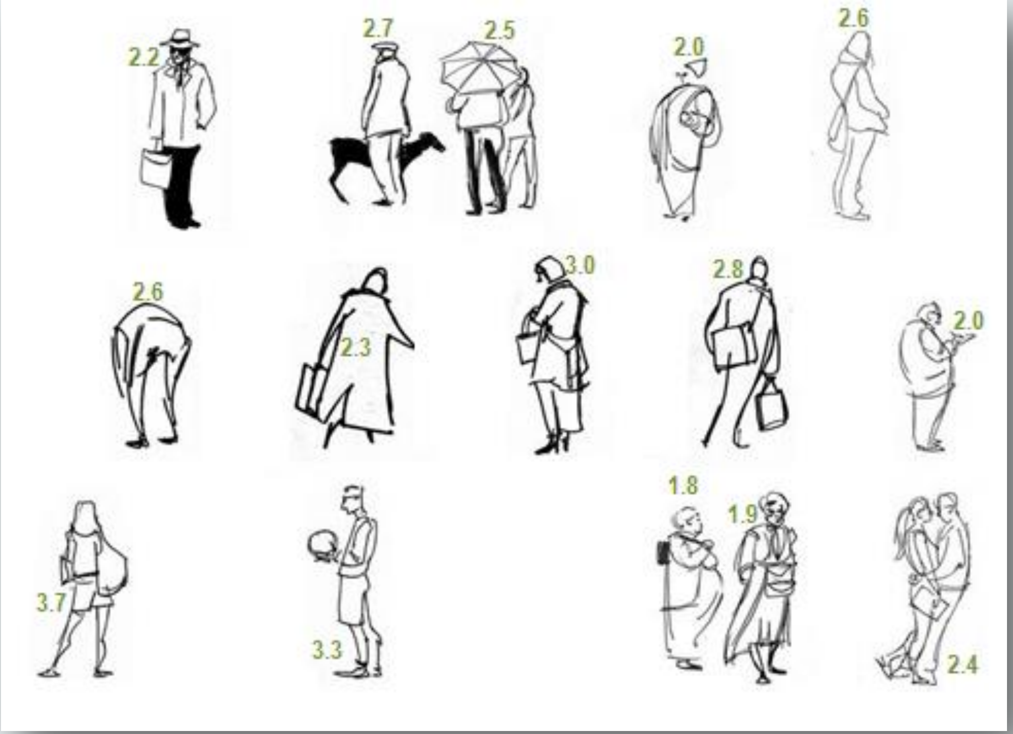
H_a: Mean is different than a certain pre-specified value **$\mu \neq \mu_0$**

Assumptions:

- The observations are randomly and independently drawn
- Symmetrical, bell shaped data (approximately normally distributed)
- There are no outliers

One Sample t-test

Do the people in the population exercise 2hrs/week?



Sample mean $\bar{x}$=2.5
Sample stand. dev. s=0.53
Sample size n=15

Standard error: $s.e. = {}^{s}/_{\sqrt{n}} = {}^{0.53}/_{\sqrt{15}} = 0.14$

H_0: $\mu=\mu_0=\mathbf{2}$
H_a: $\mu\neq\mu_0=\mathbf{2}$

$$t = \frac{\boxed{2.5} - \boxed{2}}{\boxed{0.14}}, \text{ Df} = \boxed{15} - 1$$

P – value

p > 0.05 Fail to reject the null hypothesis and conclude μ is not significantly different to μ_0=2

p ≤ 0.05 Reject the null hypothesis and conclude μ is significantly different to μ_0=2

SPSS Slide: ‘how to’

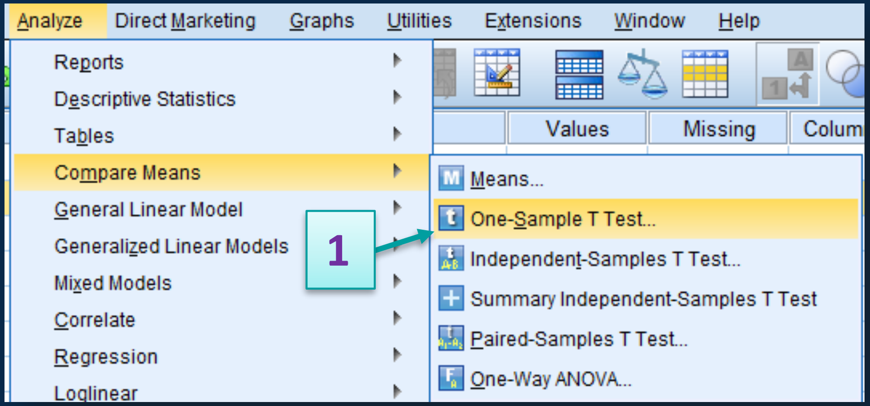
Step 1: Check the suitability of the data, here: what type of variable is ‘weight1’?

Step 2: Use the appropriate test, here: ‘one sample t-test’.

H_0: $\mu=66$

H_a: $\mu\neq66$

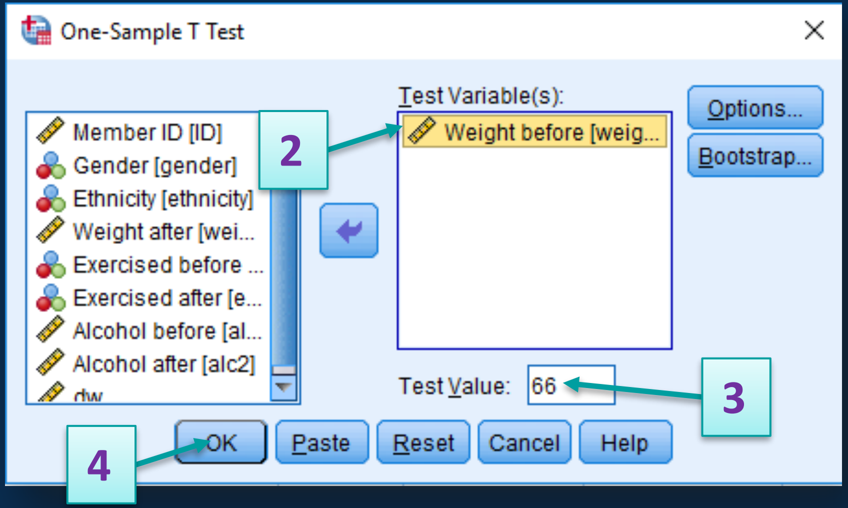
Analyse -> Compare means -> ‘One sample t-test’



Add the variable of interest in the ‘Test Variables’ box (Weight1)

Add in the known test value of interest

Click on ‘OK’



Output & Interpretation Slide

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Weight before	300	65.6768	3.35230	.19355

One-Sample Test

	Test Value = 66					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Weight before	-1.670	299	.096	-.32323	-.7041	.0577

Based on our sample, the expected mean weight was 0.32kg lower than 66kg (95% CI for the difference: [-0.70, 0.06]). This difference was not statistically significant ($t=-1.670$, $df=299$, $p=0.096$).

Equality of Means: The Three t-tests

Hypotheses	Suitable test	Decision
H_0: is equal H_a: not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses

H_0: $\mu=\mu_0$

H_a: $\mu\neq\mu_0$

One sample t-test

$t = \frac{\bar{x}-\mu_0}{s.e.}$, df=n-1

$s.e. = \sqrt{s^2/n}$

In the population, is the population mean (μ) equal to a certain value (μ_0)?

Hypotheses

H_0: $\mu_A=\mu_B$

H_a: $\mu_A\neq\mu_B$

Independent samples t-test

$t = \frac{\bar{x}_A-\bar{x}_B}{\sqrt{s_A^2/n_A+s_B^2/n_B}}$, df= $n_A + n_B$-2

In the population, is the mean of group A (μ_A) equal to the mean of group B (μ_B)?

Two Independent Samples t-tests

When to use

To test if according to the current data, the mean in the population differs across two groups.

Hypotheses:

H_0: the mean in group A equals the mean in group B $\quad \mu_A = \mu_B$
H_a: the mean in group A is different than the mean in group B $\quad \mu_A \neq \mu_B$

Assumptions:

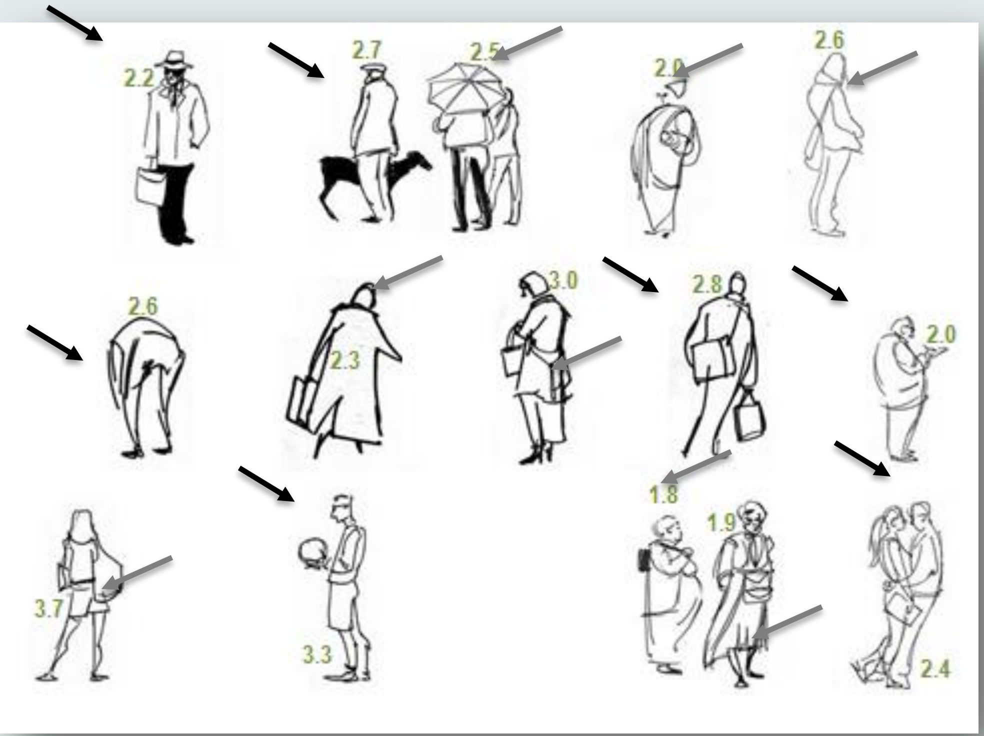
- The observations are randomly and independently drawn
- Symmetrical data, within each group
- There are no outliers, within each group

Two Independent Samples t-tests

In the population, do **women** exercise more than **men**?

H_0: $\mu_{males}=\mu_{females}$

H_a: $\mu_{males}\neq\mu_{females}$



$$t = \frac{\boxed{2.6} - \boxed{2.5}}{\sqrt{\frac{\boxed{0.4^2}}{\boxed{7}} + \frac{\boxed{0.6^2}}{\boxed{8}}}}, \qquad df = \boxed{7} + \boxed{8} - 2$$

Sample mean	$\bar{x}_A=2.6$	$\bar{x}_B=2.5$
Sample stand. dev.	$s_A=0.4$	$s_B=0.6$
Sample size	$n_A=7$	$n_B=8$

P – value

P>0.05 Fail to reject the null hypothesis and conclude μ_{males} is not significantly different to $\mu_{females}$

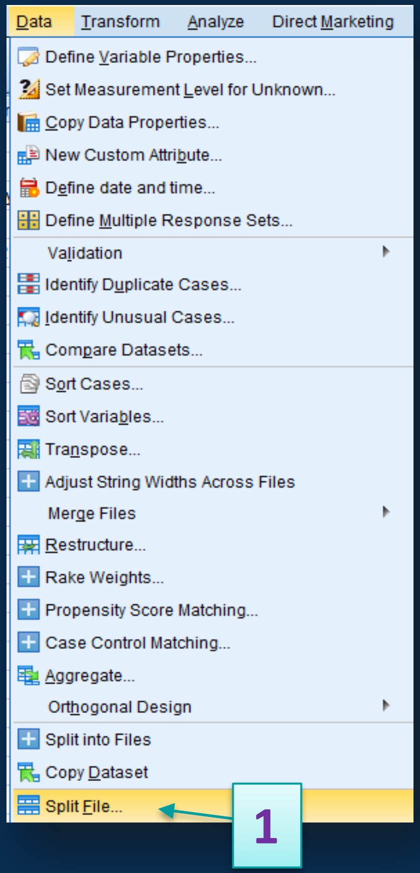
$p \leq 0.05$ Reject the null hypothesis as true and conclude μ_{males} is significantly different to $\mu_{females}$

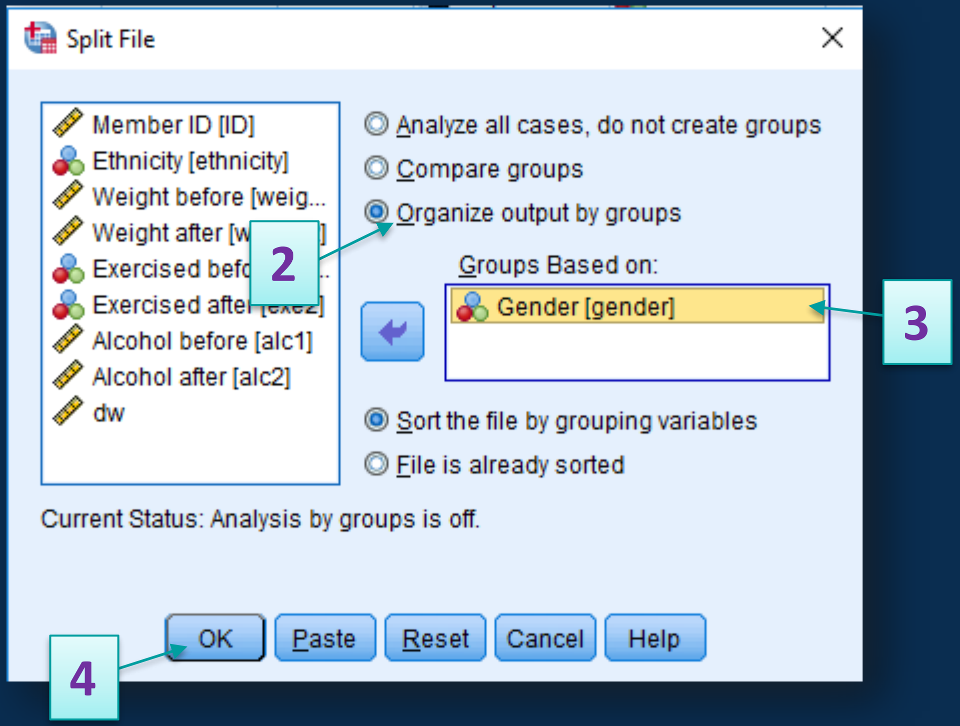
SPSS Slide: 'how to'

The next question is whether the 'weight before' was different across genders.

Step 1: Check the suitability of the data, here: what type of variable is 'weight1', for each gender ?

Go to 'Data' to use the 'Split File' function -> Split the file by groups (gender). Click on 'OK'

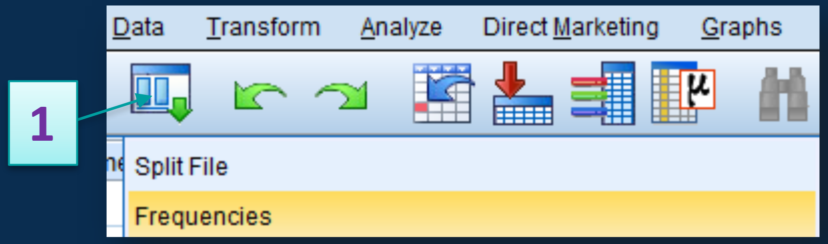


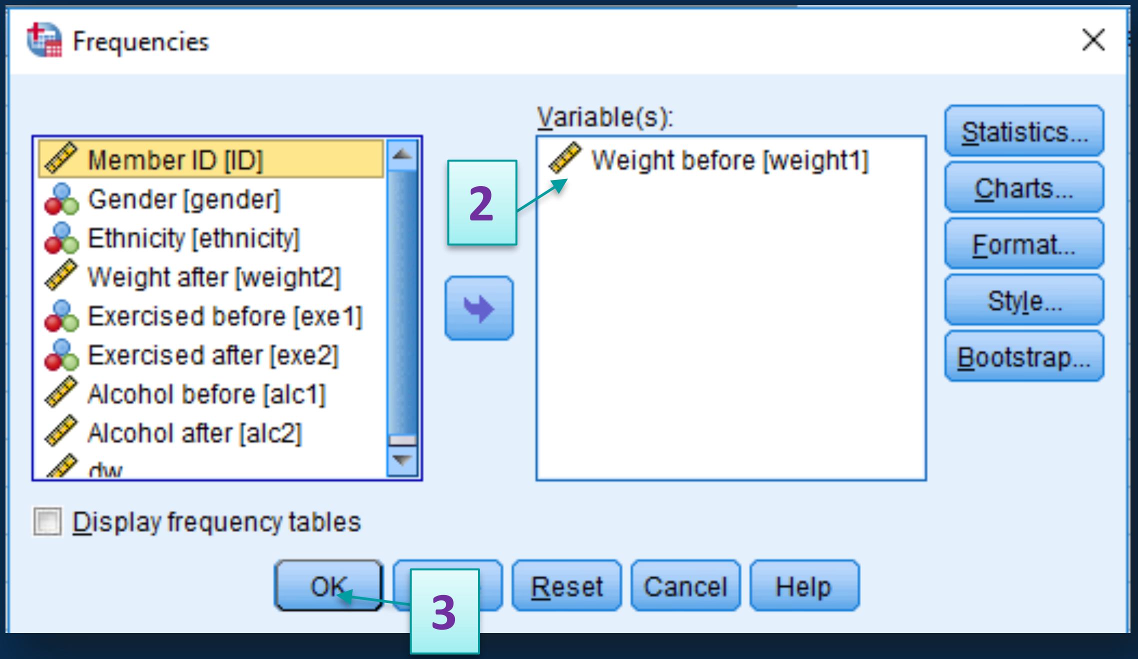


SPSS Slide: ‘how to’

Step 1: Check the suitability of the data, here what type of variable is ‘weight1’, for each gender ?

SPSS is now ready to show us the frequencies for each gender separately. You can use the ‘**recall button**’.





Or click on the ‘Analyse Tab’→ ‘Descriptive Statistics’ → ‘Frequencies’

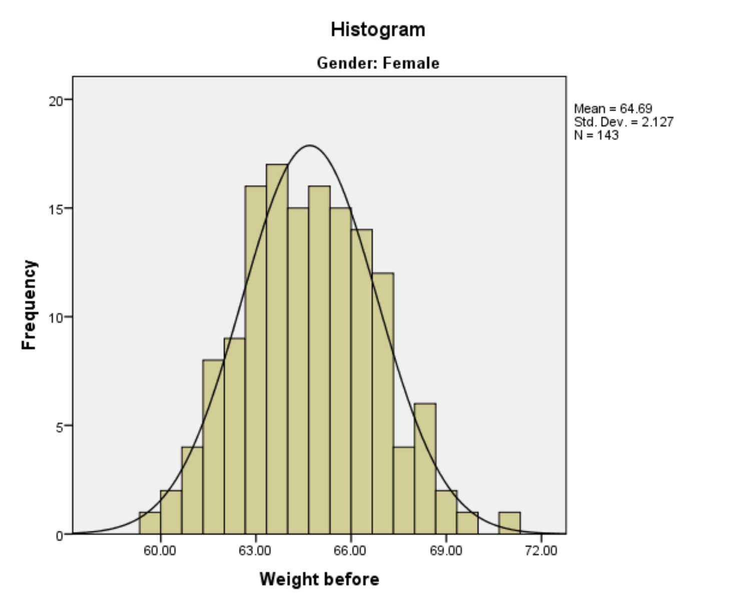
Add the variable of interest (weight1) into the ‘Variable(s)’ box.

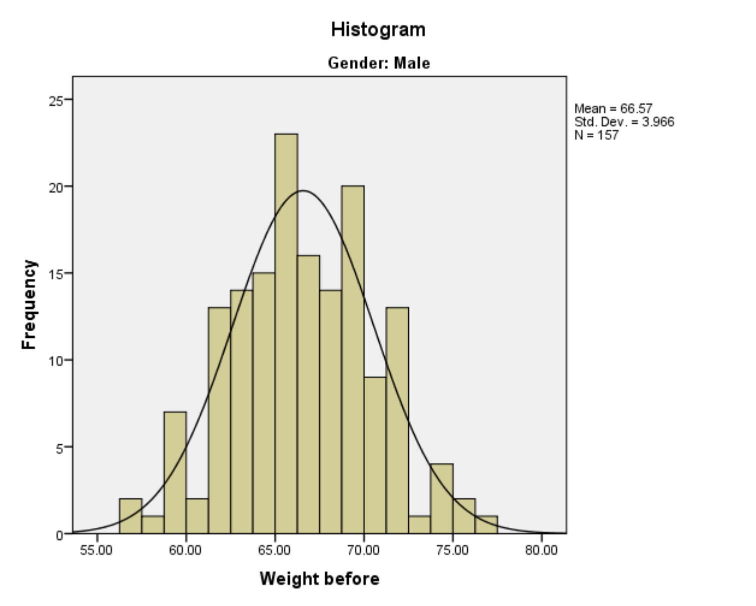
In ‘Charts’ choose to display histograms

Click on ‘OK.

Output & Interpretation Slide

Step 1: Check the suitability of the data, here: what type of variable is ‘weight1’, for each gender?





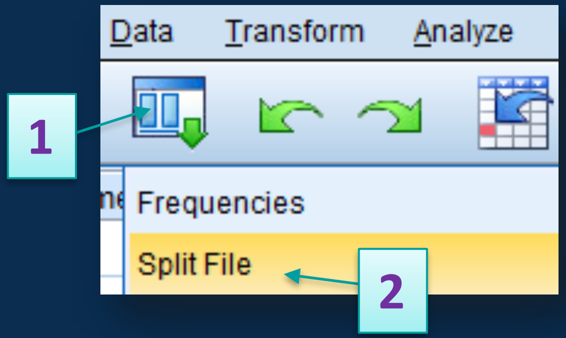
They are not perfect bells, but are fairly symmetrical. The t-test will work just fine for small departures from normality (next topic we will see problematic cases).
Therefore we may use the ‘two independent samples t-test’ for the hypotheses:

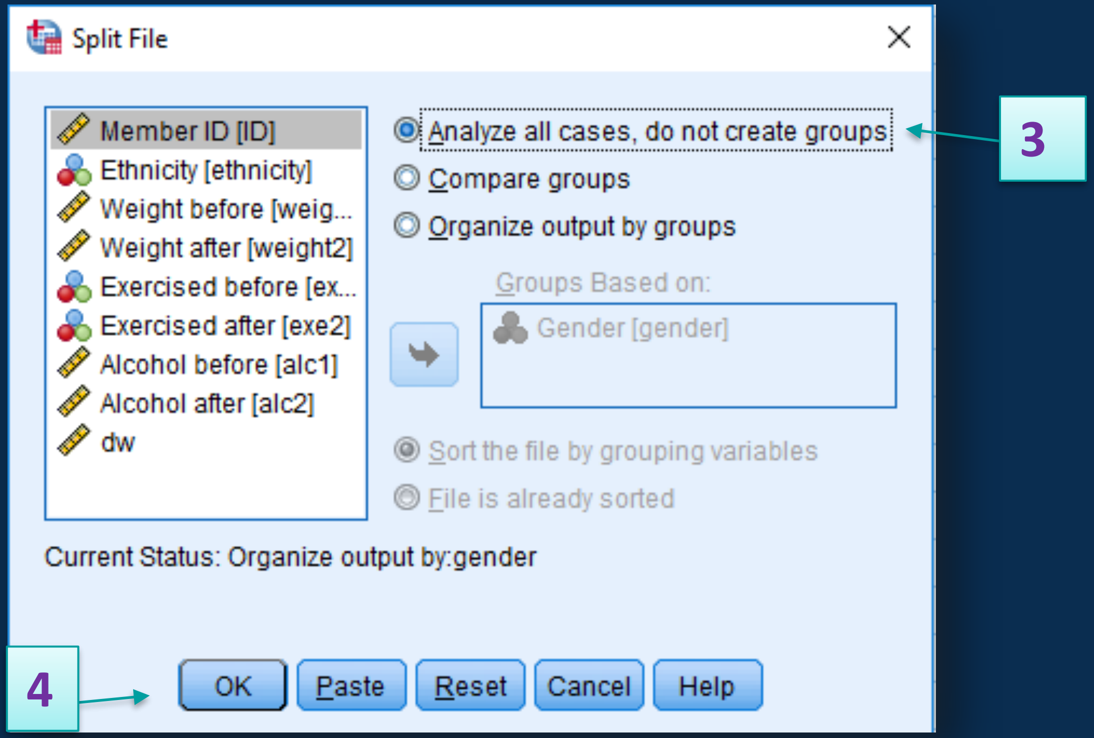
$$H_0: \mu_{males} = \mu_{females}$$

$$H_a: \mu_{males} \neq \mu_{females}$$

SPSS Slide: ‘how to’

Before proceeding with the test, use the ‘recall button’ to go back to the ‘split file’ and re-unite the data.



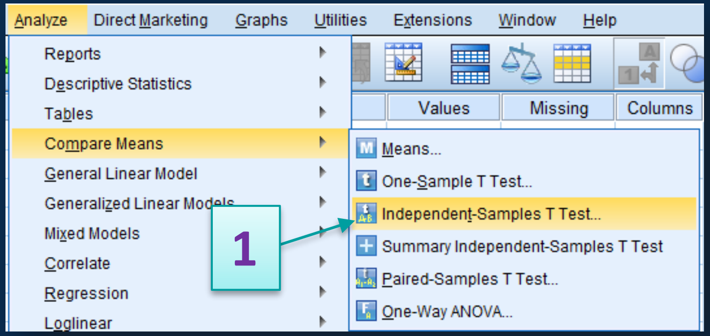


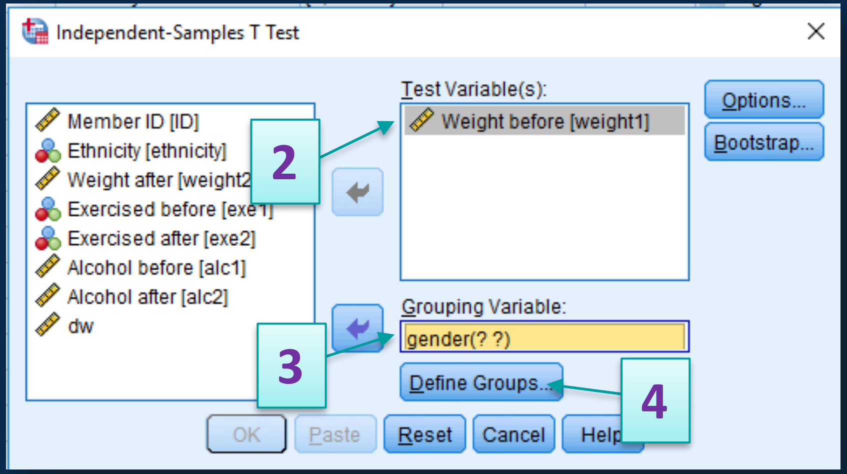
Go to ‘Data’ to use the ‘Split File’ function
‘Click on Analyse all cases’
Click on ‘OK’

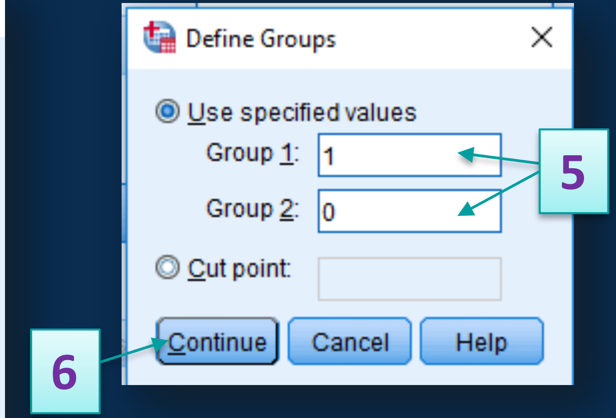
SPSS Slide: ‘how to’

Step 2: Use the appropriate test, here: ‘independent samples t-test’.

Analyse -> Compare means -> ‘Independent samples t-test’







Or click on the ‘Analyse Tab’→ ‘Compare means’ → ‘Independent samples T-Test’
Add the variable of interest (weight1) into the ‘Test Variable(s)’ box
Add the grouping variable (gender) into the ‘Grouping Variable’ box.
‘Define Groups’ Use the values that gender has been coded in the dataset
Click on ‘Continue’
Click on ‘OK’.

Output and Interpretation Slide

SPSS prints first a table with descriptive statistics

Group Statistics

	Gender	N	Mean	Std. Deviation	Std. Error Mean
Weight before	Male	157	66.5713	3.96616	.31653
	Female	143	64.6947	2.12732	.17790

In our sample, we have 157 men with mean weight 66.6kg (sd=3.97) and 143 women with mean weight 64.7kg (sd=2.13).

There is a (mathematical) difference on the average weight between men and women in our sample. But is this difference statistically significant? Is it by chance alone, or can we expect to see this difference in the population? We will need to see the results of the test.

Output and Interpretation Slide

SPSS prints a table with the t-test for the equality of means, but gives two rows of results: equal variances assumed and equal variances not assumed.

To decide which one to use we need to see the results of the **Levene's test for the equality of variances**.

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Weight before	Equal variances assumed **1**	46.712	.000	5.036	298	.000	1.87666	.37263	1.14335	2.60998
	Equal variances not assumed **2**			5.168	243.430	.000	1.87666	.36310	1.16144	2.59188

Levene's test for the equality of variances hypotheses:

H_0: $\sigma_{males} = \sigma_{females}$

H_a: $\sigma_{males} \neq \sigma_{females}$

Remember: 'in our sample, we have 157 men with mean weight 66.6kg (sd=3.97) and 143 women with mean weight 64.7kg (sd=2.13)'.

The **p-value** for **Levene's** test was <0.001, therefore we **reject** the null hypothesis, and '**equal variances** are not assumed' (go with line 2).

Output and Interpretation Slide

We are now ready to proceed with the Independent samples t-test and test for the equality of means between the groups.

We see that p<0.001, thus we reject the null hypothesis for equality of means and we conclude that there are strong evidence in our data that in the population men weigh more on average than women.

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Weight before	Equal variances assumed	46.712	.000	5.036	298	.000	1.87666	.37263	1.14335	2.60998
	Equal variances not assumed			5.168	243.430	.000	1.87666	.36310	1.16144	2.59188

H_0: $\mu_{males} = \mu_{females}$

H_a: $\mu_{males} \neq \mu_{females}$

Based on our sample, the expected mean difference in the 'weight before' between women and men was 1.9kg (95% CI for the difference: [1.16, 2.59]). This difference was statistically significant (t=5.168, df=243.430, p<0.001).

Equality of Means: The Three t-tests

Hypotheses	Suitable test	Decision
H_0: is equal H_a: not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses

H_0: $\mu=\mu_0$

H_a: $\mu\neq\mu_0$

One sample t-test

$$t = \frac{\bar{x}-\mu_0}{s.e.}, \text{ df=n-1}$$

$$s.e. = \sqrt{s^2/n}$$

In the population, is the population mean (μ) equal to a certain value (μ_0)?

Hypotheses

H_0: $\mu_A=\mu_B$
H_a: $\mu_A\neq\mu_B$

Independent samples t-test

$$t = \frac{\bar{x}_A-\bar{x}_B}{\sqrt{s_A^2/n_A+s_B^2/n_B}}, \text{ df= } n_A+n_B\text{-2}$$

In the population, is the mean of group A (μ_A) equal to the mean of group B (μ_B)?

Hypotheses

H_0: $\mu_1=\mu_2$

H_a: $\mu_1\neq\mu_2$

Paired samples t-test

$$t = \frac{\bar{x}_{diff}}{\sqrt{s_{diff}^2/n}}, \text{ df=n-1}$$

In the population, is the mean of a group in one condition (μ_1) equal to the mean of the same (or paired) group in another condition (μ_2)?

Two Paired Samples t-test

<u>When to use</u>

To test if, according to the current data, the mean in the population differs across matched groups (e.g. weight before-weight after, weight in cases vs weight in matched controls).

<u>Hypotheses:</u>

H_0: the mean of the (paired) difference is zero $\mu_1 = \mu_2$
H_a: the mean of the (paired) difference is different than zero $\mu_1 \neq \mu_2$

<u>Assumptions:</u>

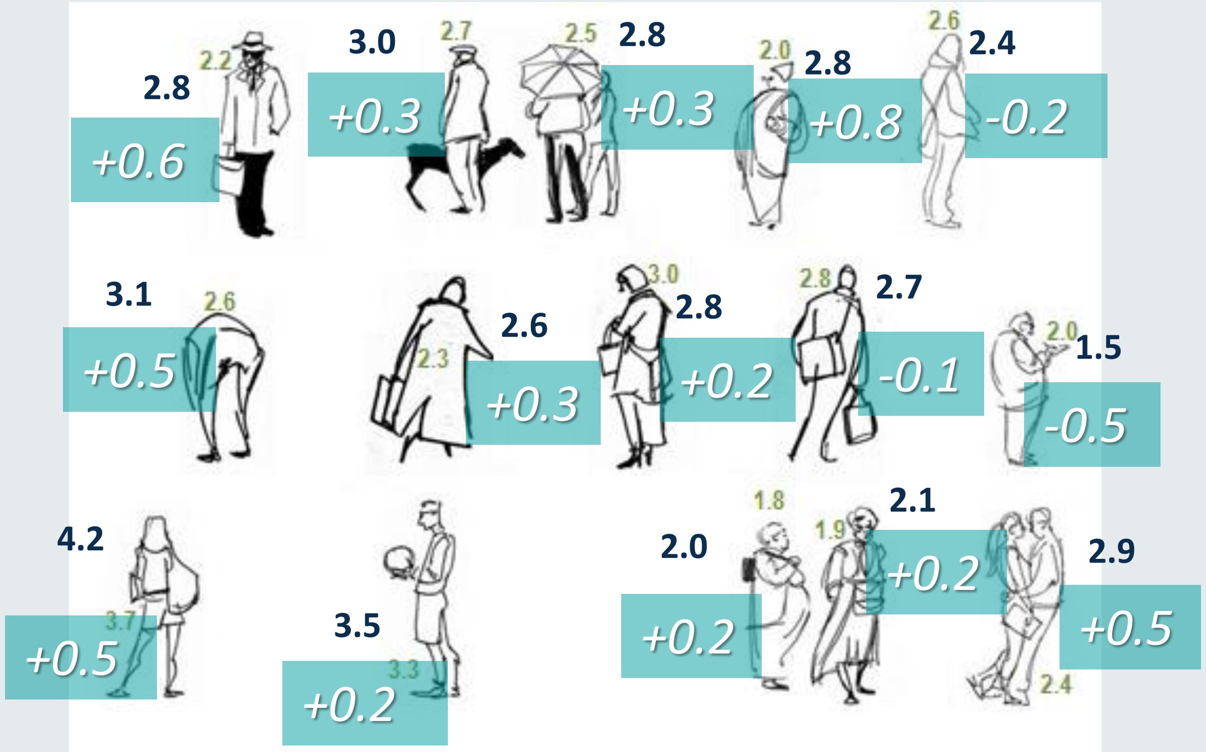
- The (paired) observations are randomly and independently drawn
- The (paired) difference are is symmetrical continuous variable
- There are no outliers in the difference

Two Paired Samples t-test

After the campaign, in the population, do people exercise more than before?

before and after , that is, pairs of observed observations

$H_0: \mu_{before} = \mu_{after}$



$$t = \frac{0.23}{\sqrt{\frac{0.35}{15}}}, \qquad df = 15 - 1$$

Sample mean diff $\bar{x}_{diff}=0.23$

Sample stand. dev. $s_{diff}=0.35$

Sample size $n=15$

P– value

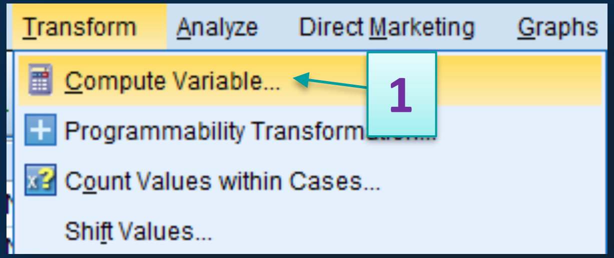
$p>0.05$ Fail to reject the null hypothesis and conclude μ_{before} is not significantly different to μ_{after}

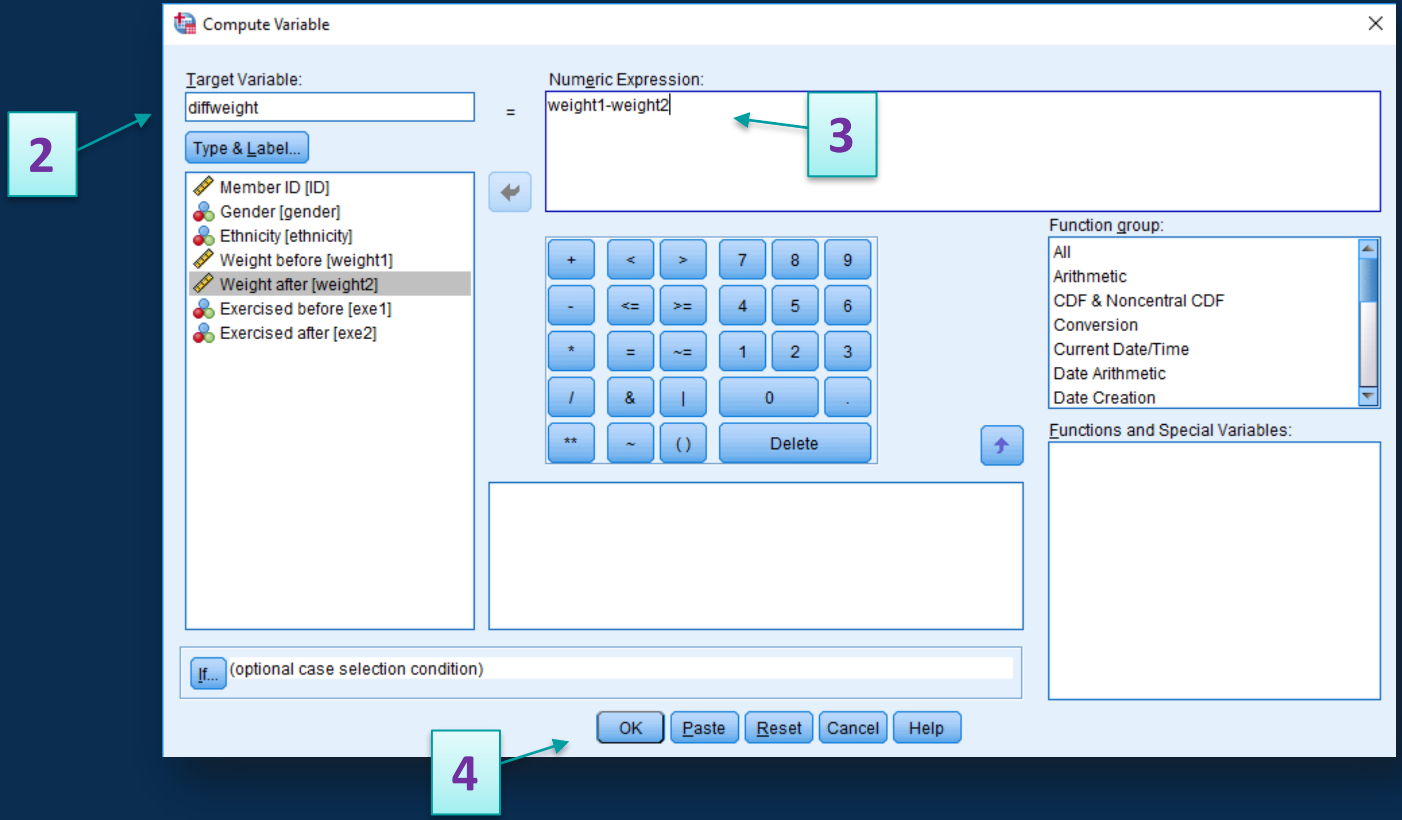
$p \leq 0.05$ Reject the null hypothesis as true and conclude μ_{before} is significantly different to μ_{after}

SPSS Slide: ‘how to’

The next question is whether the ‘weight before’ was different than the ‘weight after’.

Step 1: Check the suitability of the data, here: what type of variable is the differences between ‘weight1’ and ‘weight2’?



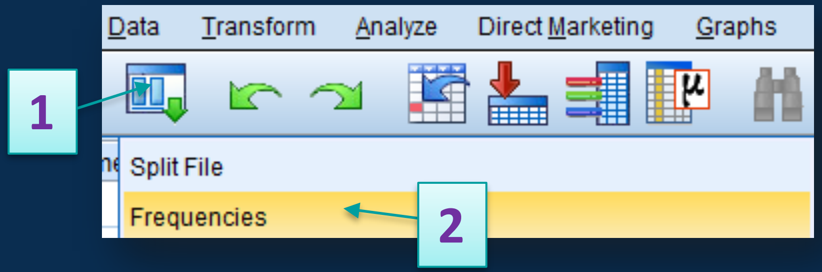


To calculate the difference click on the ‘Compute Variable’ Tab.
Give the new variable a name in ‘Target Variable’ box.
Add the two variables in the ‘Numeric Expression’ box separated by a **subtract** sign
Click on ‘OK’.

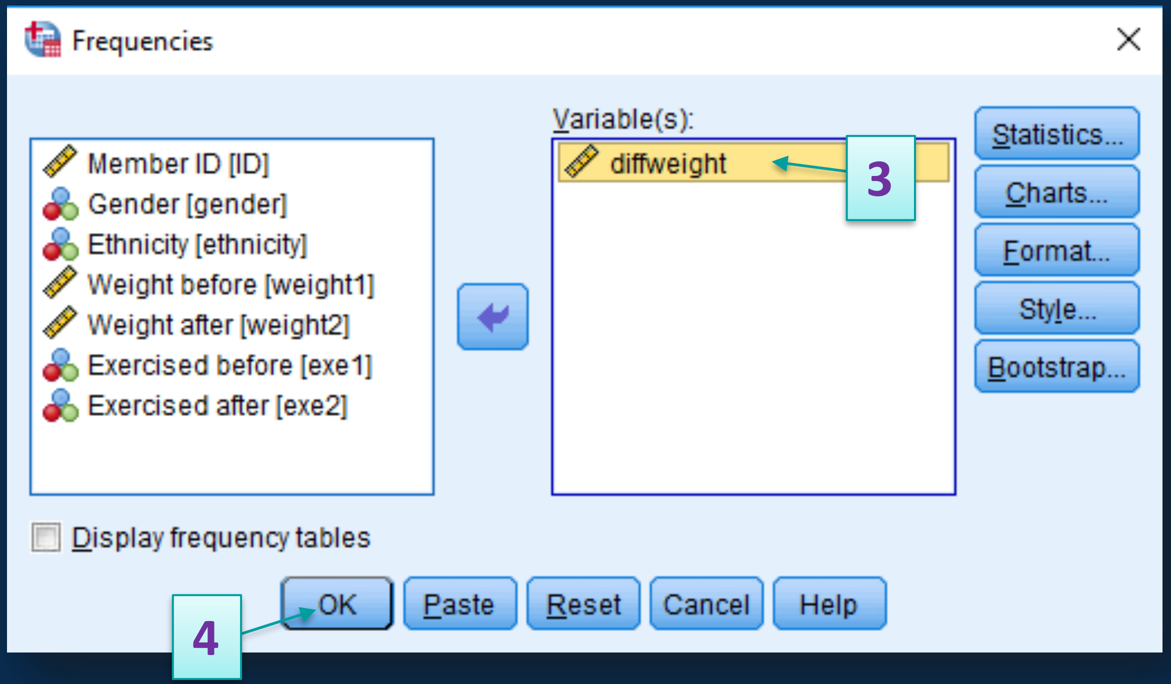
SPSS Slide: ‘how to’

Step 1: Check the suitability of the data, here: what type of variable is the differences between ‘weight1’ and ‘weight2’?

The new variable is now in your dataset and you can use the ‘recall’ button to see its descriptive indices.

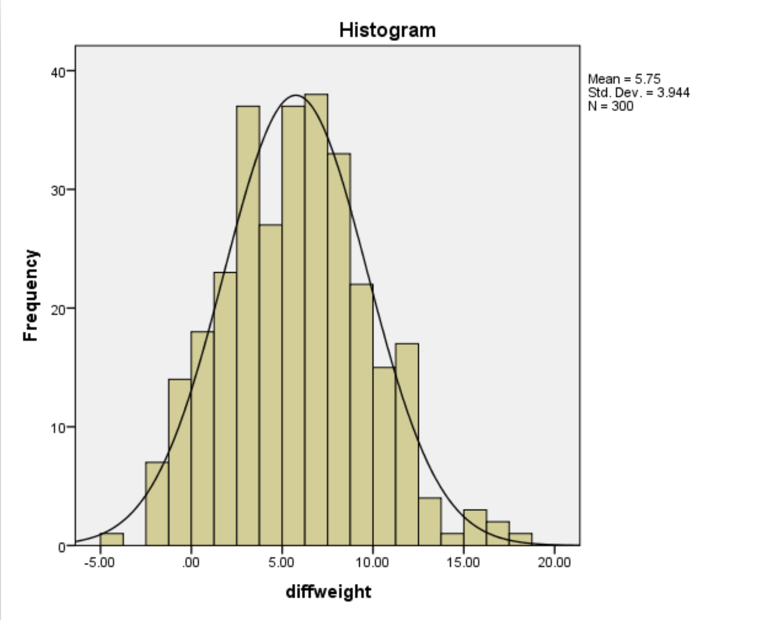


Or click on the ‘Analyse Tab’→ ‘Descriptive Statistics’ → ‘Frequencies’
Add the variable of interest (diffweight) into the ‘Variable(s)’ box
In ‘Charts’ choose to display histograms
Click on ‘OK.



Output & Interpretation Slide

Step 1: Check the suitability of the data, here: what type of variable is the differences between ‘weight1’ and ‘weight2’?



Almost a perfect bell, fairly symmetrical. The t-test will work just fine for small departures from normality (next topic we will see problematic cases).

Therefore we may use the ‘two paired samples t-test’ for the hypotheses:

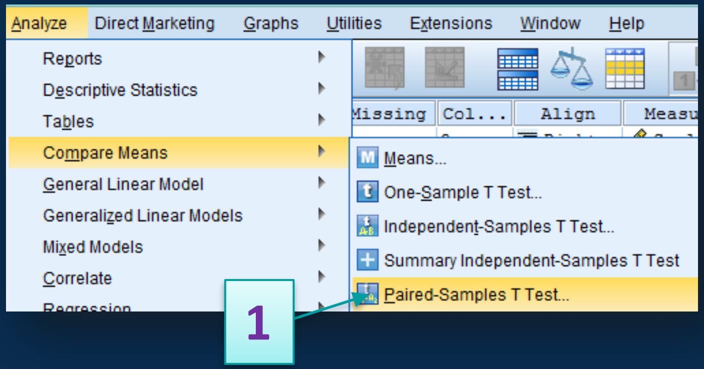
$H_0: \mu_{before} = \mu_{after}$

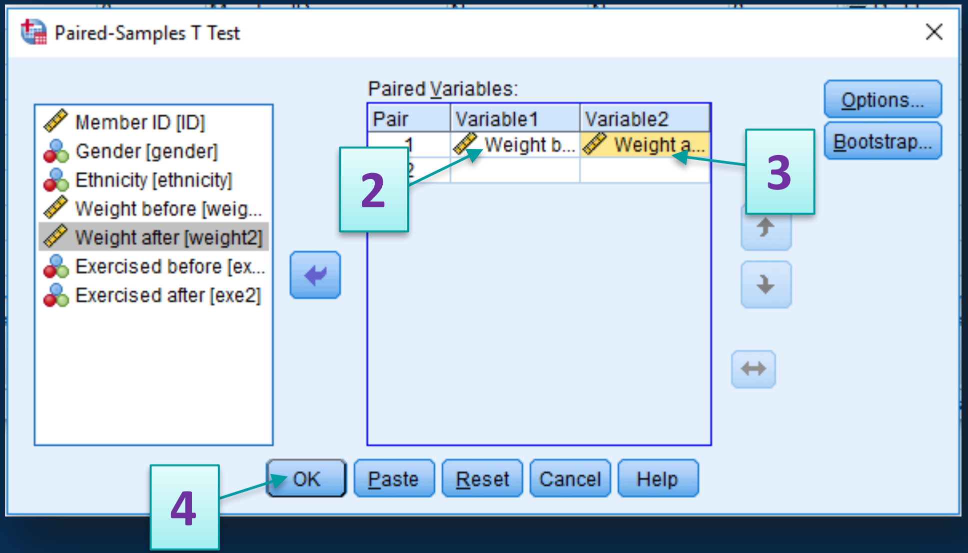
$H_a: \mu_{before} \neq \mu_{after}$

SPSS Slide: ‘how to’

Step 2: Use the appropriate test, here ‘paired samples t-test’.

Analyse -> Compare means -> ‘paired samples t-test’





Or click on the ‘Analyse Tab’→ ‘Compare means’ → ‘Paired samples T-Test’

Add the variable of interest (weight1 and weight 2) into the ‘Paired Variable(s)’ box

Click on ‘OK’.

Output and Interpretation Slide

SPSS prints a table with descriptive statistics and one with the 'paired samples t-test'

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Weight before	65.6768	300	3.35230	.19355
	Weight after	59.9268	300	1.97707	.11415

In our sample, the mean 'weight before' was 65.7kg (sd=3.35) and the mean 'weight after' was 59.9kg (sd=1.98).

There is a (mathematical) difference on the average weight before and after the programme, in our sample. But is this difference statistically significant? Is it by chance alone, or can we expect to see this difference in the population? We will need to see the results of the test.

Output and Interpretation Slide

H_0: $\mu_{before} = \mu_{after}$
H_a: $\mu_{before} \neq \mu_{after}$

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	95% Confidence Interval of the Difference Upper			
Pair 1	Weight before - Weight after	5.74996	3.94395	.22770	5.30186	6.19807	25.252	299	.000

We reject the null hypothesis of the equality of means, and we infer that people are expected to lose 5.75 on average, by the end of the programme.

Based on our sample, the expected mean difference in the weight was 5.75 (95% CI: [5.30, 6.20]). This difference was statistically significant ($t=25.252$, $df=299$, $p<0.001$).

P-value and the 95% CI

Equality of means   Null: the difference is zero

One-Sample Test

	Test Value = 66					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Weight before	-1.670	299	.096	-.32323	-.7041	.0577

zero included
in the 95% CI

Based on our sample, the expected mean weight was 0.32 lower than 66kg (95% CI for the difference: [-0.70, 0.06]). This difference was not statistically significant (t=-1.670, df=299, p=0.096).

t-test for Equality of Means						
					95% Confidence Interval of the Difference	
t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
5.036	298	.000	1.87666	.37263	1.14335	2.60998
5.168	243.430	.000	1.87666	.36310	1.16144	2.59188

zero not included
in the 95% CI

Based on our sample, the expected mean difference in the ‘weight before’ between women and men was 1.88kg (95% CI for the difference: [1.16, 2.59]). This difference was statistically significant (t=5.168, df=243.430, p<0.001).

Some Tips!

<u>Equality of means</u>

Instead of the p-value, we can also check the 95% CI to infer on whether there is a significant difference

The null hypothesis: $\mu = 0$

- can be rejected at the 5% level in favour of the two-sided alternative hypothesis if the null value (here 0) is not contained in the 95% confidence interval for μ.

- cannot be rejected if the confidence interval contains the null value.

Knowledge Test

Match the scenario with the correct test.

Tom wants to test if mother's reported ADHD scores for children are higher than those reported by fathers.

Tom wants to test if boys' ADHD scores are higher than those of girls.

Tom wants to test if children's ADHD scores are higher than 30.

One-sample t-test

Two independent samples t-test

Two paired samples t-test

Reflection

Write down three examples from your research that would require the use of each of the three t-tests.

Reference List

Agresti and Finlay (2009) Statistical Methods for the Social Sciences, 4th Edn, Pearson Hall, Upper Saddle River, NJ.

Comparison of Two Groups, Ch 7, pages 183-209

Analyzing Association between Categorical Variables, Ch 8, pages 221-239

Field (2005) Discovering Statistics using SPSS, 2nd Edn, Sage, London.

Comparing Two Means, Ch 7

Categorical Data, Ch 16

Thank you

Please contact your module leader or the course lecturer of your programme, or visit the module's forum for any questions you may have.

If you have comments on the materials (spotted typos or missing points) please contact Dr Vitoratou:

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Department: Biostatistics and Health Informatics

Module Title: Introduction to Statistics

Session Title: Equality of proportions: χ^2 tests

Topic title: Comparing groups I (parametric methods)

Topic materials:
Silia Vitoratou

Contributions:
Zahra Abdulla

Improvements:
Nick Beckley-Hoelscher
Kim Goldsmith
Professor Sabine Landau
Dr Silia Vitoratou

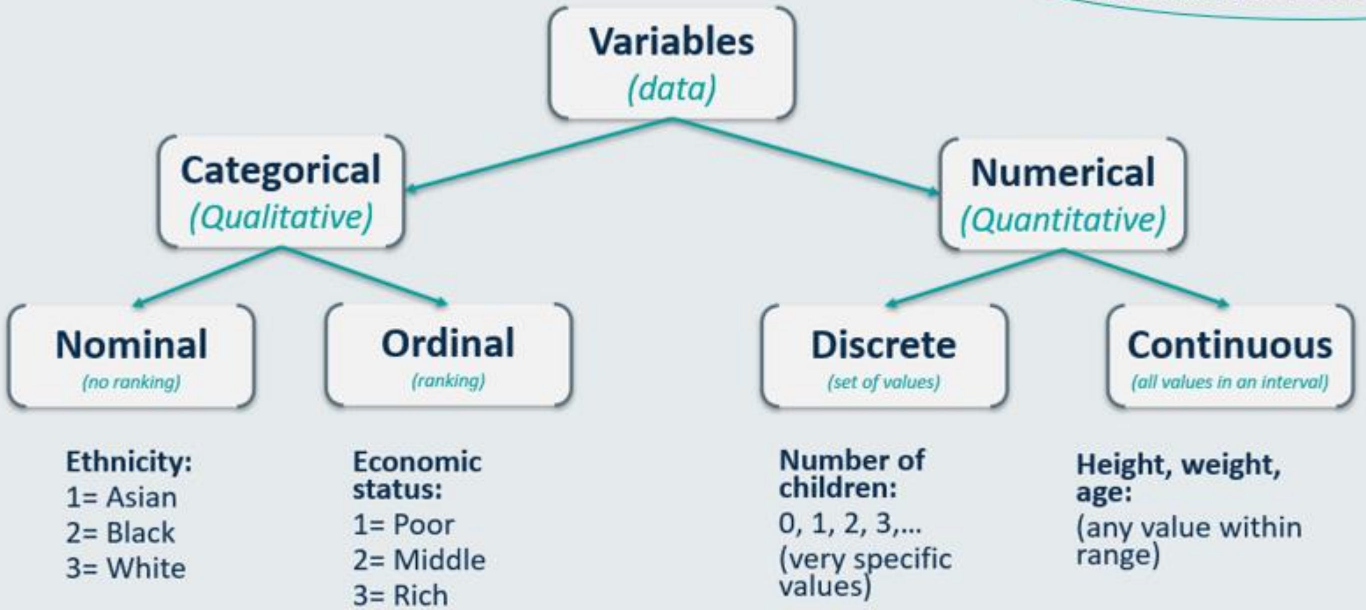
Learning Outcomes

- Learn when and how to use the χ^2 -tests for equality of proportions.
- Understand the assumptions of the various test of equality of proportions.
- Be able to conduct these tests in a statistical software.

Previously on 'Introduction to Statistical Methods'…

Based on the type of each variable, we use different ways to describe the data.

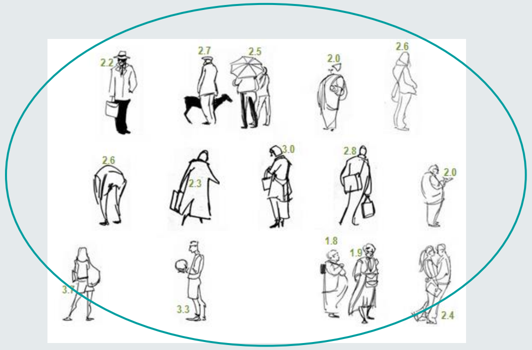
Blue for symmetrical (normal)
Purple italics for skewed or discrete data



	Categorical	Numerical
• Descriptive indices	Frequencies (Percentages %)	location: mean, *median*, mode Dispersion: SD, *min-max*, range
• Charts/plots	Bar Chart	Histogram, Box plot

Equality of Means: The Three t-tests

one sample t-test



$H_0: \mu = \mu_0$

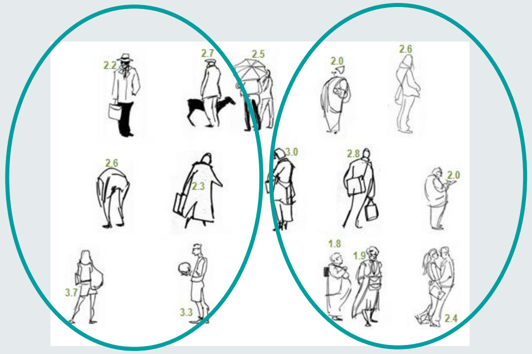
$H_a: \mu \neq \mu_0$

Examples

Difference from test value:

- age ≠ 25yo
- height ≠ 1.60cm
- weight ≠ 80kg

independent samples t-test



$H_0: \mu_A = \mu_B$

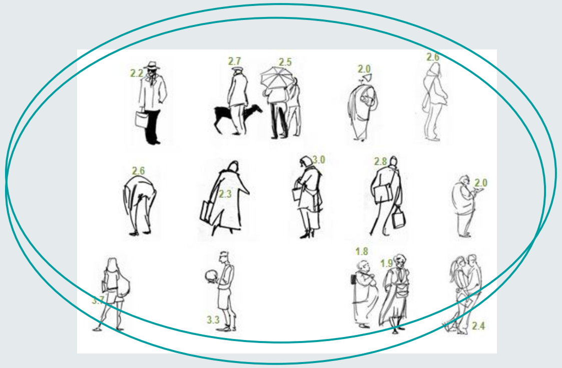
$H_a: \mu_A \neq \mu_B$

Examples

Difference in the means:

- young vs old
- males vs females
- City A vs City B

paired samples t-test



$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

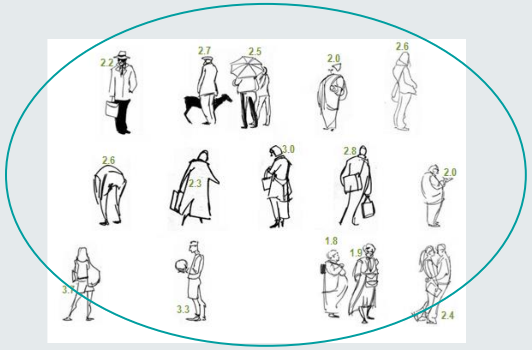
Examples

Difference in the means:

- before and after treatment
- twin studies
- matched cases vs controls

Equality of proportions: The three χ^2-tests

one sample χ^2-test



$H_0: \pi=\pi_0$

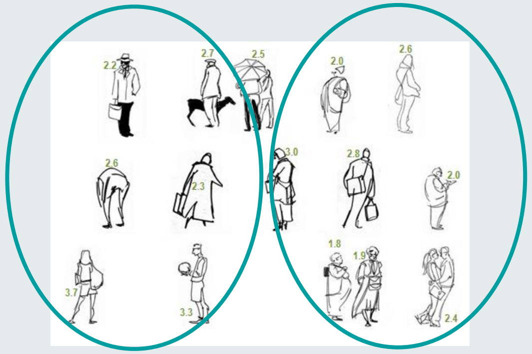
$H_a: \pi\neq\pi_0$

Examples

Difference from test value:

- % age 25yo ≠10%
- % taller ≠10%
- % gain weight ≠ 25%

independent samples χ^2-test



$H_0: \pi_A = \pi_B$

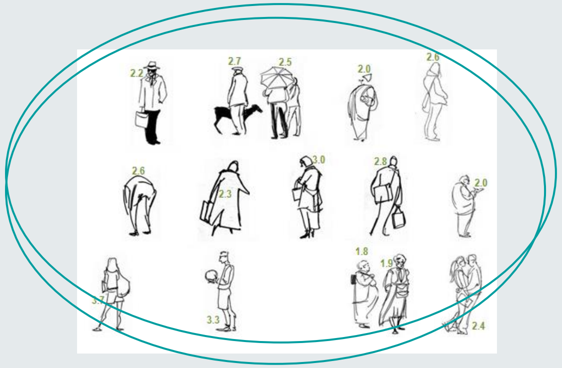
$H_a: \pi_A \neq \pi_B$

Examples

Difference in the proportions:

- % young vs %old
- % males vs %females
- %City A vs %City B

paired samples χ^2-test



$H_0: \pi_1 = \pi_2$

$H_a: \pi_1 \neq \pi_2$

Examples

Difference in the proportions:

- % before and %after treatment
- % twin studies
- % matched cases vs controls

Equality of Means: The Three t-tests

Hypotheses	**Suitable test**	**Decision**
H_0: is equal H_a: not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses	**One sample t-test**	
H_0: $\mu=\mu_0$ H_a: $\mu\neq\mu_0$	$t=\frac{\bar{x}-\mu_0}{s.e.}$, df=n-1 $s.e.=\sqrt{s^2/n}$	In the population, is the population mean (μ) equal to a certain value (μ_0)?

Hypotheses	**Independent samples t-test**	
H_0: $\mu_A=\mu_B$ H_a: $\mu_A\neq\mu_B$	$t=\frac{\bar{x}_A-\bar{x}_B}{\sqrt{s_A^2/n_A+s_B^2/n_B}}$, df= n_A+n_B-2	In the population, is the mean of group A (μ_A) equal to the mean of group B (μ_B)?

Hypotheses	**Paired samples t-test**	
H_0: $\mu_1=\mu_2$ H_a: $\mu_1\neq\mu_2$	$t=\frac{\bar{x}_{diff}}{\sqrt{s_{diff}^2/n}}$, df=n-1	In the population, is the mean of a group in one condition (μ_1) equal to the mean of the same (or paired) group in another condition (μ_2)?

Equality of proportions: The three χ^2-tests

Hypotheses	Suitable test	Decision
H_0: % is equal H_a: % not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses

H_0: $\pi=\pi_0$

H_a: $\pi\neq\pi_0$

One sample χ^2-test

$$\chi^2 = \sum \frac{(\mathrm{O}-\mathrm{E})^2}{\mathrm{E}}, \text{ df=c-1}$$

In the population, is the population proportion (π) equal to a certain value (π_0)?

Hypotheses

H_0: $\pi_A=\pi_B$

H_a: $\pi_A\neq\pi_B$

Pearson's χ^2 test

$$\chi^2 = \sum \frac{(\mathrm{O}-\mathrm{E})^2}{\mathrm{E}}, \text{ df=}(c_1\text{-1}) (c_2\text{-1})$$

In the population, is the proportion of group A (π_A) equal to the proportion of group B (π_B)?

Hypotheses

H_0: $\pi_1=\pi_2$

H_a: $\pi_1\neq\pi_2$

McNemar χ^2 test

$$\chi^2 = \frac{(b-c)^2}{b+c}, \text{ df=1}$$

In the population, is the proportion of a group in one condition (π_1) equal to the proportion of the same (or paired) group in another condition (π_2)?

Equality of proportions: The three χ^2-tests

Hypotheses	Suitable test	Decision
H_0: % is equal H_a: % not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses

H_0: $\pi=\pi_0$

H_a: $\pi\neq\pi_0$

One sample χ^2-test

$$\chi^2 = \sum \frac{(O-E)^2}{E}, \text{ df=c-1}$$

In the population, is the population proportion (π) equal to a certain value (π_0)?

One Sample Chi-Square Test

When to use

To test if according to the current data, the proportion in the population equals a certain, pre-specified, value.

Hypotheses:

H_0: the proportion in the population equals a certain pre-specified value
H_a: the proportion in the population is different than a certain pre-specified value

Assumptions:

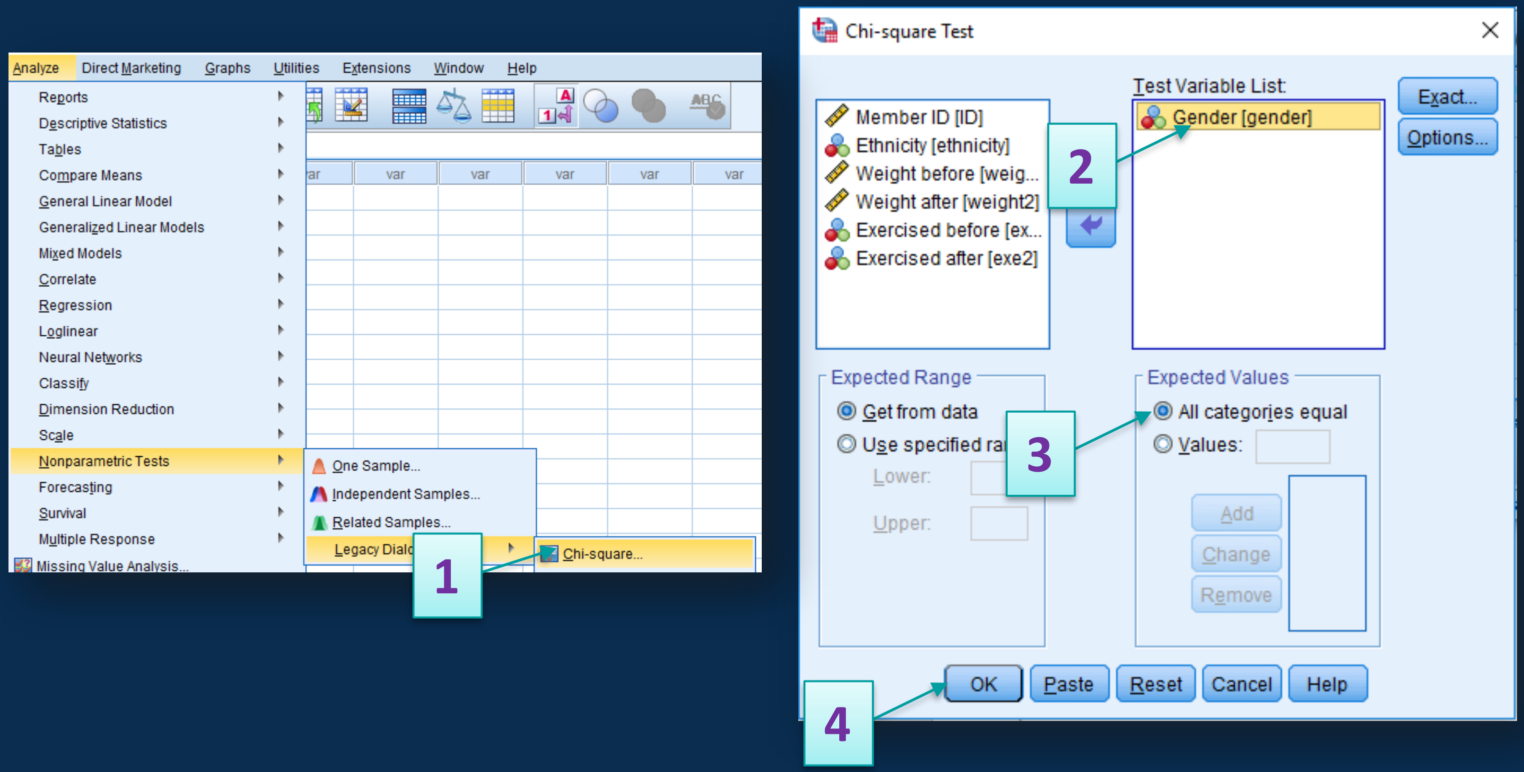
- The observations are randomly and independently drawn
- The number of cells with expected frequencies less than 5, are less than 20%
- The minimum expected frequency is at the very least 1.

SPSS Slide: ‘how to’

We start with testing whether men or women are more likely to come to the programme. In other words, is the proportion of men equal to the proportion of women?

<u>Step 1</u>: Use the appropriate test, here ‘one-sample chi-square test’.

Analyse -> Non-parametric tests -> legacy dialogs -> ‘Chi-square’



Add the variable of interest Gender) in to the ‘Variables box’.
Choose to Test ‘All categories are equal’
(If you have specific proportions to test these would be entered in values in the order of the coding of the variable in the main dataset)
Click ‘OK’

Output & Interpretation Slide

SPSS prints a table with descriptive statistics and one with the one sample t-test

Gender

	Observed N	Expected N	Residual
Female	143	150.0	-7.0
Male	157	150.0	7.0
Total	300		

Test Statistics

	Gender
Chi-Square	.653[a]
df	1
Asymp. Sig.	.419

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 150.0.

Test statistic $\sum \frac{(O-E)^2}{E}$

Degrees of freedom: c-1

P-value

$$\sum \frac{(O-E)^2}{E} = \boxed{\frac{(143-150)^2}{150}} + \boxed{\frac{(157-150)^2}{150}} \quad \boxed{= 0.653}$$

H_0: $\pi=50\%$

H_a: $\pi\neq50\%$

Based on our sample, the expected proportion of men joining in the programme is not different than the expected proportion of women ($\chi^2=0.653$, df=1, p=0.419).

Output & Interpretation

Step 2: Check the suitability of the data, do the assumptions of the chi-square test hold?

Test Statistics

	Gender
Chi-Square	.653[a]
df	1
Asymp. Sig.	.419

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 150.0.

Gender

	Observed N	Expected N	Residual
Female	143	150.0	-7.0
Male	157	150.0	7.0
Total	300		

For the test to work properly we need to have enough data for all ‘cells’ (categories). Specifically:

- Only up to 20% of the cells are allowed to have expected frequencies less than 5.
- The minimum expected frequency needs to be larger than 1.

Equality of proportions: The three χ^2-tests

Hypotheses	**Suitable test**	**Decision**
H_0: % is equal H_a: % not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses	**One sample χ^2-test**
H_0: $\pi=\pi_0$ H_a: $\pi\neq\pi_0$	$\chi^2 = \sum \frac{(O-E)^2}{E}$, df=c-1

In the population, is the population proportion (π) equal to a certain value (π_0)?

Hypotheses	**Pearson's χ^2 test**
H_0: $\pi_A=\pi_B$ H_a: $\pi_A\neq\pi_B$	$\chi^2 = \sum \frac{(O-E)^2}{E}$, df=$(c_1\text{-}1)\ (c_2\text{-}1)$

In the population, is the proportion of group A (π_A) equal to the proportion of group B (π_B)?

Pearson's Chi-Square Test

When to use

To test if, according to the current data, the proportions in the population of one variable change based on another variable.

Hypotheses:

H_0: there is no association between the two variables
H_a: there is an association between the two variables

Assumptions:

- The observations are randomly and independently drawn
- The number of cells with expected frequencies less than 5, are less than 20%
- The minimum expected frequency is at the very least 1.
- The observations are not paired

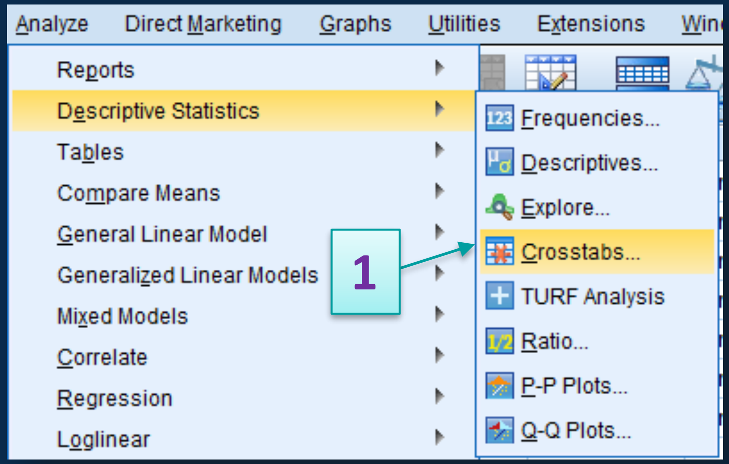
SPSS Slide: 'how to'

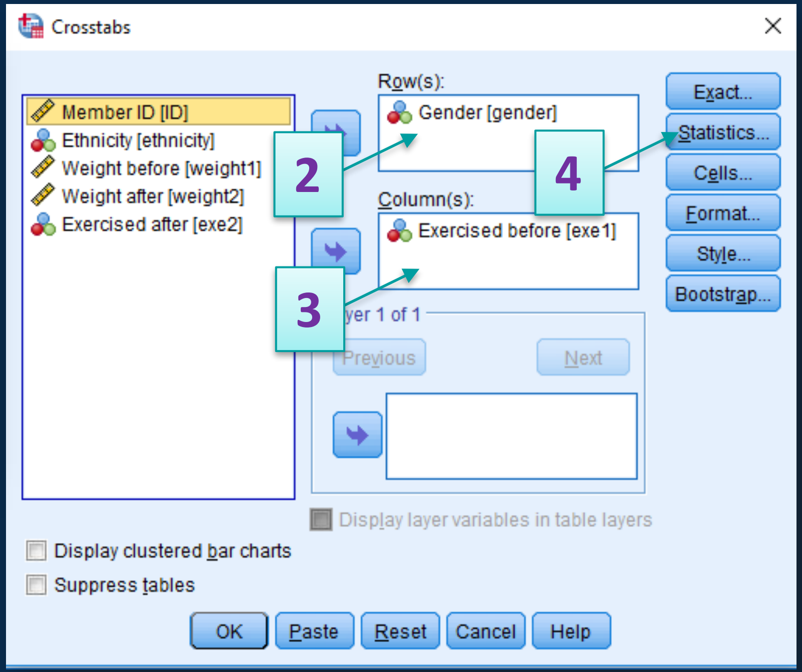
The next question is: do men exercise more than women prior to entering the programme?
Are the proportions of those exercised before the programme, different for men and women?

<u>Step 1</u>: Use the appropriate test, here: 'Pearson's chi-square test'.

Analyse -> Descriptive Statistics -> Crosstabs

H_0: $\pi_M = \pi_F$
H_a: $\pi_M \neq \pi_F$





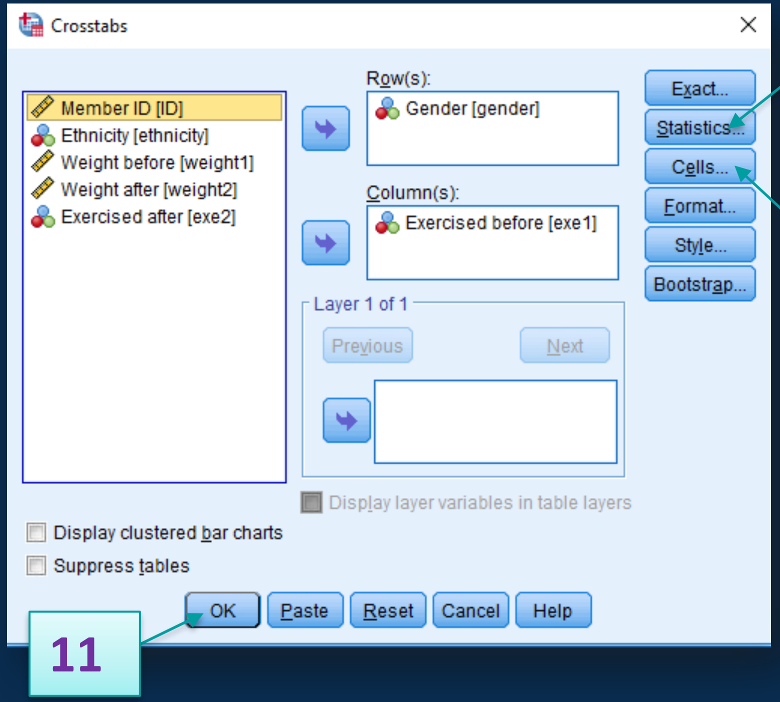
Add the variable of interest (Gender) in to the 'Rows box'.
Add the second variable interest (Exe before) in the 'columns box'.

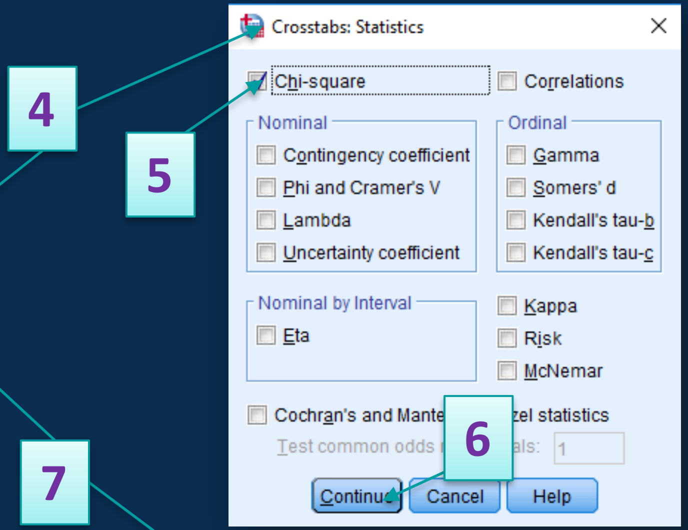
SPSS Slide: ‘how to’

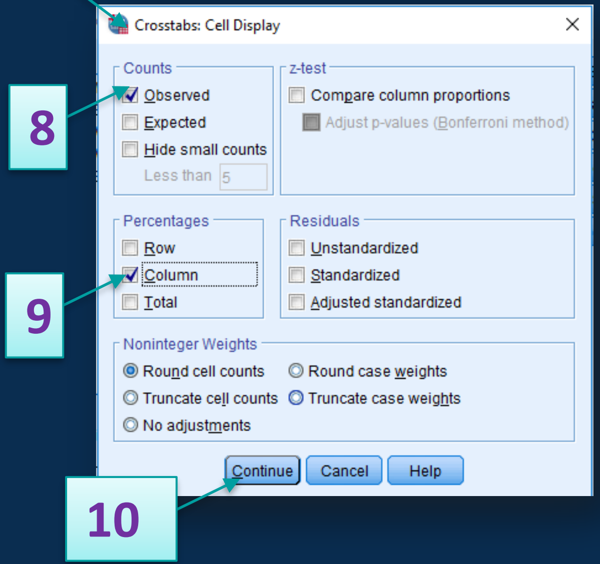
Step 1: Use the appropriate test, here: ‘Pearson’s chi-square test’.

Click on ‘Statistics’ and choose ‘Chi square’
Click ‘continue’

Click on ‘cells’ choose ‘Observed’ and ‘column’ percentages.
Click ‘Continue’
Click ‘OK’







Choosing row or column percentages depends on the interpretations you are hoping to make.

Output and Interpretation

SPSS prints a double entry table with descriptive statistics and one with the χ^2-test

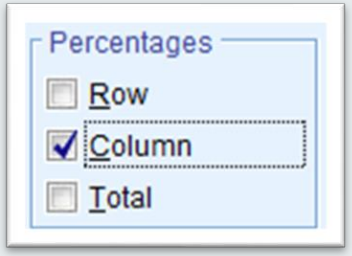
Gender * Exercised before Crosstabulation

			Exercised before No	Exercised before Yes	Total
Gender	Female	Count	119	24	143
		% within Exercised before	53.6%	30.8%	47.7%
	Male	Count	103	54	157
		% within Exercised before	46.4%	69.2%	52.3%
Total		Count	222	78	300
		% within Exercised before	100.0%	100.0%	100.0%

In our sample, we have

- 119 women who did not exercise before the programme
- 24 women who did exercise before the programme
- 103 men who did not exercise before the programme
- 54 men who did exercise before the programme.

Output and Interpretation



Gender * Exercised before Crosstabulation

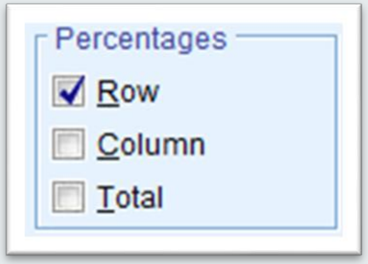
			Exercised before: No	Exercised before: Yes	Total
Gender	Female	Count	119	24	143
		% within Exercised before	53.6%	30.8%	47.7%
	Male	Count	103	54	157
		% within Exercised before	46.4%	69.2%	52.3%
Total		Count	222	78	300
		% within Exercised before	100.0%	100.0%	100.0%

Add by column -> interpret by row ! **NEVER compare percentages which add up to 100%!**

Among the ‘females’, the proportion of those who did ‘not exercise before’ was higher than the proportion of those who ‘exercise before’ (53.6% versus 30.8%, respectively).

(Alternatively, we could have used the row proportions for men)

Output and Interpretation



Gender * Exercised before Crosstabulation

			Exercised before		Total
			No	Yes	
Gender	Female	Count	119	24	143
		% within Gender	83.2%	16.8%	100.0%
	Male	Count	103	54	157
		% within Gender	65.6%	34.4%	100.0%
Total		Count	222	78	300
		% within Gender	74.0%	26.0%	100.0%

Add by row -> interpret by column! **NEVER compare percentages which add up to 100%!**

Among those who did not 'exercise before', the proportion of 'females' was higher than the proportion of 'males' (83.2% versus 65.6%, respectively).

(Alternatively, we could have used the column proportions for those who do exercise)

Output and Interpretation

What essentially happens here, is that when you compare proportions which add up to 100%, then you essentially work with one of the variables. Therefore the association is not described.

On the contrary, when you compare proportions which do not add up to 100, you use information by both variables, thus you highlight the association!

Gender * Exercised before Crosstabulation

			Exercised before		
			No	Yes	Total
Gender	Female	Count	119	24	143
		% within Gender	83.2%	16.8%	100.0%
	Male	Count	103	54	157
		% within Gender	65.6%	34.4%	100.0%
Total		Count	222	78	300
		% within Gender	74.0%	26.0%	100.0%

Output and Interpretation

Recent example: people’s wrong read of the benefit of the COVID vaccines against loss of life.

Let us assume a city has 1000 people and 80% are vaccinated against a virus.

Sadly over the course of a year, 30 people die due to the virus. Among them, 15 were vaccinated and 15 were not.

Headline: half of the people who died were vaccinated

FAKE NEWS

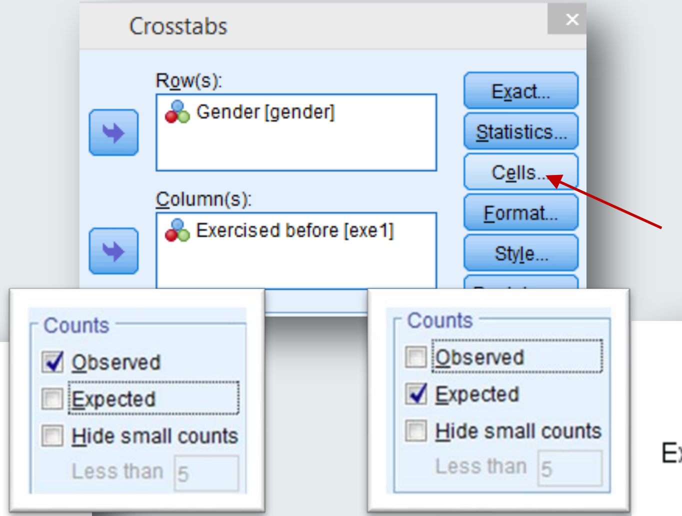
People naturally think: among those who died, 50% were vaccinated and 50% not. Thus equal proportions!

But we know, we do not compare proportions that add up to 100%

What we need to compare here, among those who died, it was the 15/800=2.25% of the vaccinated people, and the 7.5% of the unvaccinated people.

So we expect almost 2 out of 100 people who are vaccinated to be in danger, but almost 8 out of 100 people who are not vaccinated (4 times up!).

Output and Interpretation



Gender * Exercised before Crosstabulation

Count

		Exercised before		
		No	Yes	Total
Gender	Female	119	24	143
	Male	103	54	157
Total		222	78	300

Gender * Exercised before Crosstabulation

Expected Count

		Exercised before		
		No	Yes	Total
Gender	Female	105.8	37.2	143.0
	Male	116.2	40.8	157.0
Total		222.0	78.0	300.0

$$\sum \frac{(O-E)^2}{E} = \frac{(119-105.8)^2}{105.8} + \frac{(103-116.2)^2}{116.2} + \frac{(24-37.2)^2}{37.2} + \frac{(54-40.8)^2}{40.8}$$

$$= 12.07$$

$df=(c_1-1)\,(c_2-1)=1*1=1$

Output and Interpretation Slide

Gender * Exercised before Crosstabulation

			Exercised before No	Exercised before Yes	Total
Gender	Female	Count	119	24	143
		% within Gender	83.2%	16.8%	100.0%
	Male	Count	103	54	157
		% within Gender	65.6%	34.4%	100.0%
Total		Count	222	78	300
		% within Gender	74.0%	26.0%	100.0%

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	12.065[a]	1	.001		
Continuity Correction[b]	11.167	1	.001		
Likelihood Ratio	12.342	1	.000		
Fisher's Exact Test				.001	.000
Linear-by-Linear Association	12.024	1	.001		
N of Valid Cases	300				

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 37.18.

b. Computed only for a 2x2 table

Among those who ‘exercised before’ the programme, the proportion of ‘males’ was higher than the proportion of ‘females’ (34.4% versus 16.8%, respectively). This difference was statistically significant (Pearson χ^2=12.065, df=1, p<0.001).

Therefore, we conclude that men tend to exercise (before the programme) more often than women, in the population. The variables ‘gender’ and ‘exe1’ are related.

Output and Interpretation

Step 2: Check the suitability of the data, do the assumptions of the chi-square test hold?

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	12.065[a]	1	.001		
Continuity Correction[b]	11.167	1	.001		
Likelihood Ratio	12.342	1	.000		
Fisher's Exact Test				.001	.000
Linear-by-Linear Association	12.024	1	.001		
N of Valid Cases	300				

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 37.18.

b. Computed only for a 2x2 table

For the test to work properly we need to have enough data for all ‘cells’ (categories). Specifically:

- Only up to 20% of the cells are allowed to have expected frequencies less than 5.
- The minimum expected frequency needs to be larger than 1.

Equality of proportions: The three χ^2-tests

Hypotheses	Suitable test	Decision
H_0: % is equal H_a: % not equal	*test statistic*	p-value>0.05 do not reject the H_0 p-value≤0.05 reject the H_0

Hypotheses

H_0: $\pi=\pi_0$

H_a: $\pi\neq\pi_0$

One sample χ^2-test

$$\chi^2 = \sum \frac{(\mathrm{O}-\mathrm{E})^2}{\mathrm{E}}, \text{ df=c-1}$$

In the population, is the population proportion (π) equal to a certain value (π_0)?

Hypotheses

H_0: $\pi_A=\pi_B$

H_a: $\pi_A\neq\pi_B$

Pearson's χ^2 test

$$\chi^2 = \sum \frac{(\mathrm{O}-\mathrm{E})^2}{\mathrm{E}}, \text{ df=}(c_1\text{-1})\ (c_2\text{-1})$$

In the population, is the proportion of group A (π_A) equal to the proportion of group B (π_B)?

Hypotheses

H_0: $\pi_1=\pi_2$

H_a: $\pi_1\neq\pi_2$

Mc Nemar χ^2 test

$$\chi^2 = \frac{(b-c)^2}{b+c}, \text{ df=1}$$

In the population, is the proportion of a group in one condition (π_1) equal to the proportion of the same (or paired) group in another condition (π_2)?

McNemar χ^2-test

When to use:

To test if, according to the current data, the proportions in the population of a variable change based on another matched variable.

Hypotheses:

H_0: there is no association between the two (paired) variables
H_a: there is an association between the two (paired) variables

Assumptions:

- The observations are randomly and independently drawn
- There are at least 25 observations in the discordant cells
- The data are paired

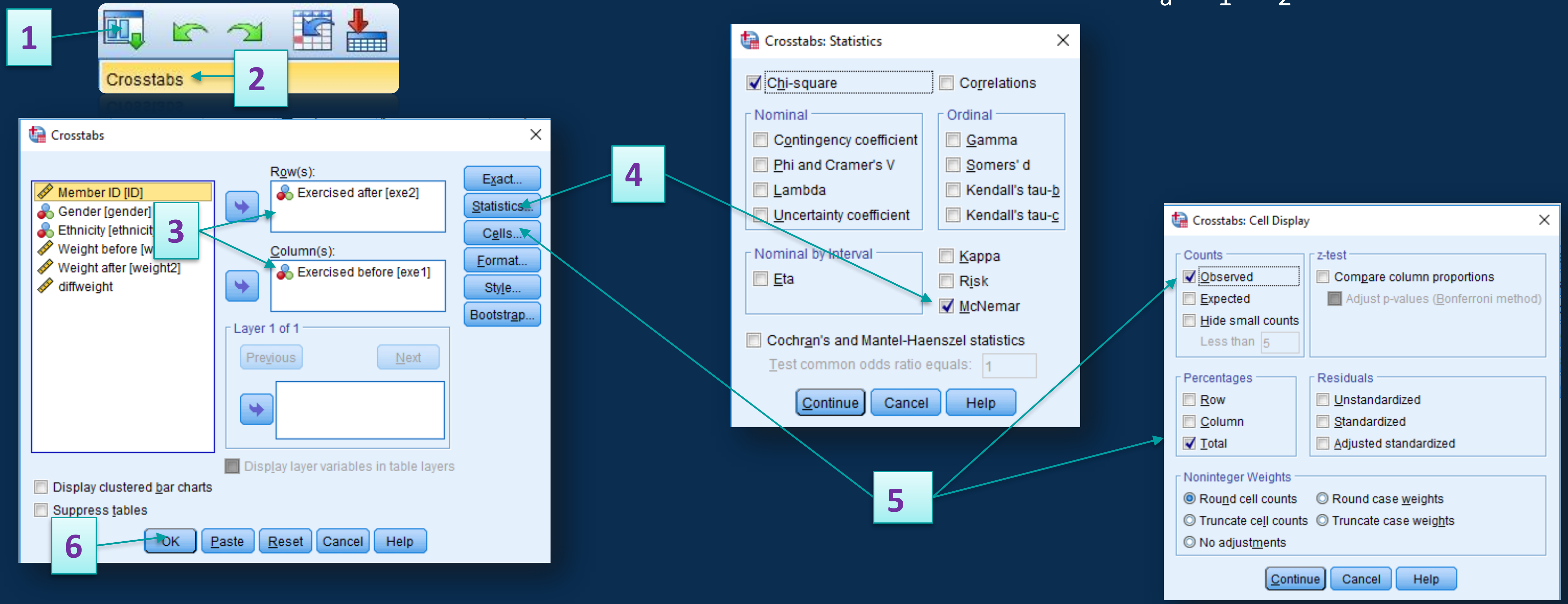
SPSS Slide: ‘how to’

Are the proportions of those exercised before the programme, different of those exercised after the programme?

<u>Step 1</u>: Use the appropriate test, here: ‘McNemar chi-square test’.

H_0: $\pi_1 = \pi_2$

H_a: $\pi_1 \neq \pi_2$



Output and Interpretation

Exercised after * Exercised before Crosstabulation

Count

		Exercised before No	Exercised before Yes	Total
Exercised after	No	a 119	b 48	167
	Yes	c 103	d 30	133
Total		222	78	300

discordant cells

concordant *cells*

only ***discordant*** *cells play a role in McNemar test!*

The test statistic:

$$\chi^2 = \frac{(b-c)^2}{b+c} = \frac{(48-103)^2}{151}$$

- The assumptions:
  - Paired data (here before-after)
  - b+c no less than 25 (here 151)

Output and Interpretation slide

Exercised before * Exercised after Crosstabulation

			Exercised after No	Exercised after Yes	Total
Exercised before	No	Count	119	103	222
		% of Total	39.7%	34.3%	74.0%
	Yes	Count	48	30	78
		% of Total	16.0%	10.0%	26.0%
Total		Count	167	133	300
		% of Total	55.7%	44.3%	100.0%

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)	Point Probability
Pearson Chi-Square	1.473[a]	1	.225	.236	.140	
Continuity Correction[b]	1.169	1	.280			
Likelihood Ratio	1.484	1	.223	.236	.140	
Fisher's Exact Test				.236	.140	
Linear-by-Linear Association	1.468[d]	1	.226	.236	.140	.051
McNemar Test				.000[c]	.000[c]	.000[c]
N of Valid Cases	300					

a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 34.58.

b. Computed only for a 2x2 table

c. Binomial distribution used.

d. The standardized statistic is -1.211.

Note, we would have concluded the wrong result if we had used Pearson's chi-square. Pearson's chi-square is not valid for paired data.

The percentage of those who 'exercised after' (44.3%) is higher than the percentage of those who 'exercised before' (26.0%). This difference was statistically significant according to the McNemar test ($p<0.001$).

Therefore, we conclude that the proportions of people who exercise before and after the programme are not the same.

Knowledge Test Solution

Match the scenario with the correct test.

Tom wants to test if boys' proportions in ADHD high/low classification groups are different than those of girls.

Tom wants to test if mothers' reported ADHD high/low classification for children are different than those reported by fathers.

Tom wants to test if children's high classification ADHD proportion is higher than 50%.

One-sample χ^2-test

Pearson's χ^2-test

McNemar test

Reflection

Write down three examples from your research that would require the use of each of the three chi square tests.

Reference List

- **Agresti and Finlay (2009) Statistical Methods for the Social Sciences, 4th Edn, Pearson Hall, Upper Saddle River, NJ.**
    - Comparison of Two Groups, Ch 7, pages 183-209
    - Analyzing Association between Categorical Variables, Ch 8, pages 221-239

- **Field (2005) Discovering Statistics using SPSS, 2nd Edn, Sage, London.**
    - Comparing Two Means, Ch 7
    - Categorical Data, Ch 16

Thank you

Please contact your module leader or the course lecturer of your programme, or visit the module's forum for any questions you may have.

If you have comments on the materials (spotted typos or missing points) please contact Dr Vitoratou:

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For any other comments or remarks on the module structure, please contact one of the three module leaders of the Biostatistics and Health Informatics department:

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