Plotting

January 28, 2020

1 MECH 510: Final Projects

Dec. 2, 2019

Athena Liu

[1]:

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

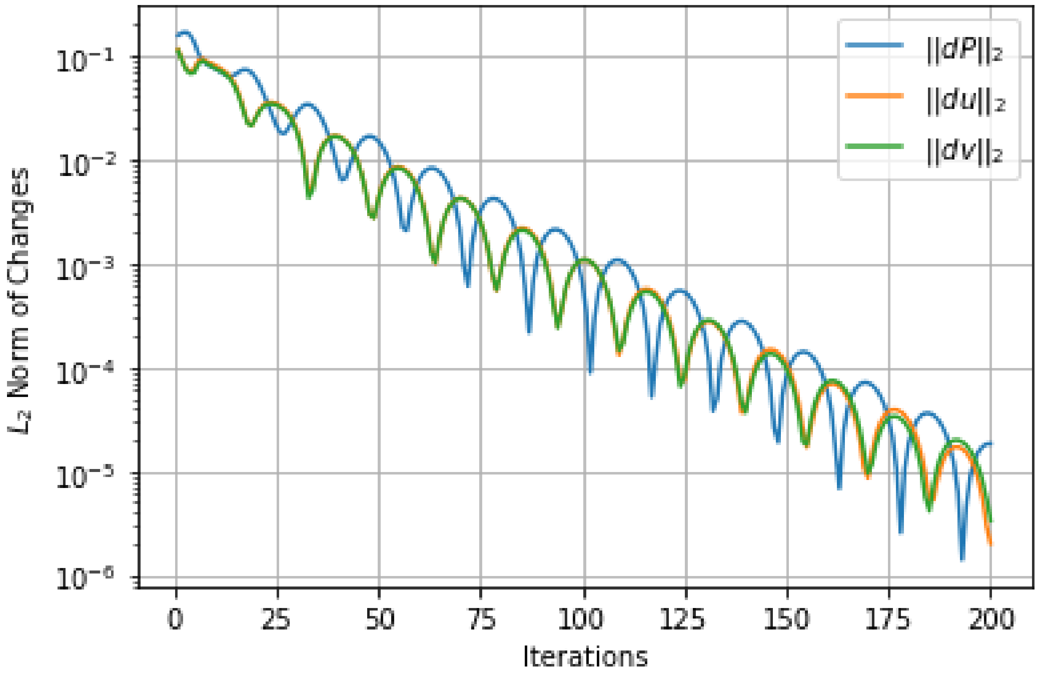
1.1 3.1 Validation: Stability

[14]:

```python
 def convergence(f_name):
    f = pd.read_csv("./outputs/" + f_name + ".csv")
    plt.plot(f["Iter"], f["dP"])
    plt.plot(f["Iter"], f["du"])
    plt.plot(f["Iter"], f["dv"])

    plt.yscale('log')
    plt.grid(True)
    plt.xlabel("Iterations")
    plt.ylabel("$L_2$ Norm of Changes")
    plt.legend(["$||dP||_2$", "$||du||_2$", "$||dv||_2$"])
    plt.show()
```

[15]:

```python
convergence("log-3_1_validation")
```



1.2 2 Things to watch out for (Optimization)

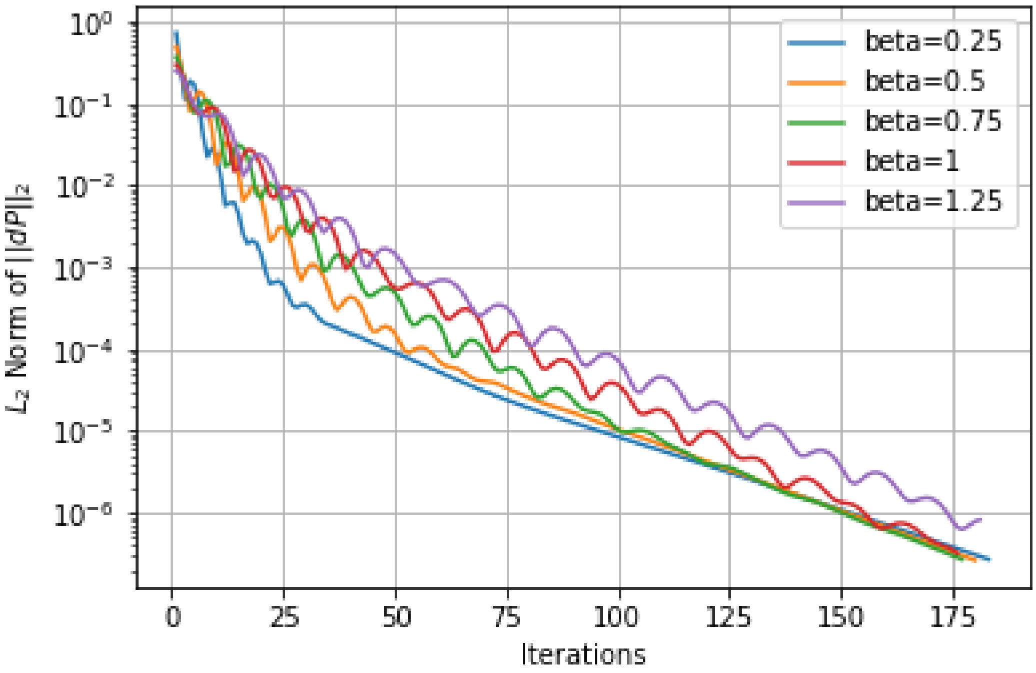
Use Re = 100, U_top = 1, 20 by 20 mesh, tolerance 10^{-6}, dt = 0.1, h = 1.

[167]:
```
def compare_convergence(parameter, values, component):
    for value in values:
        s_value = str(value).replace(".", "_")
        f = pd.read_csv("./outputs/log-" + parameter + "-" + s_value + ".csv")
        lb = parameter + "=" +str(value)
        plt.plot(f["Iter"], f["d"+component], label=lb)
    plt.yscale('log')
    plt.grid(True)
    plt.xlabel("Iterations")
    plt.ylabel("$L_2$ Norm of $||d"+component+"||_2$")
    plt.legend()
    plt.show()
```

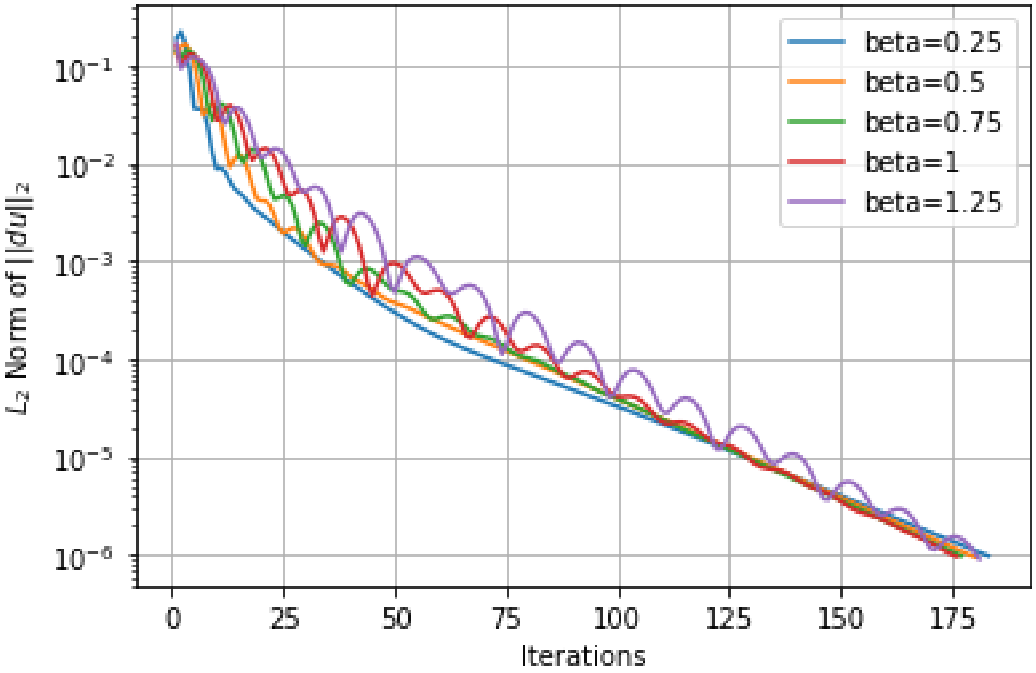
1.2.1 2.2 Choice of beta

beta = 0.75 seems to be a good choice.

[168]:
```
values=[0.25, 0.5, 0.75, 1, 1.25]
compare_convergence("beta", values, 'P')
```



```
[169]: compare_convergence("beta", values, 'u')
```

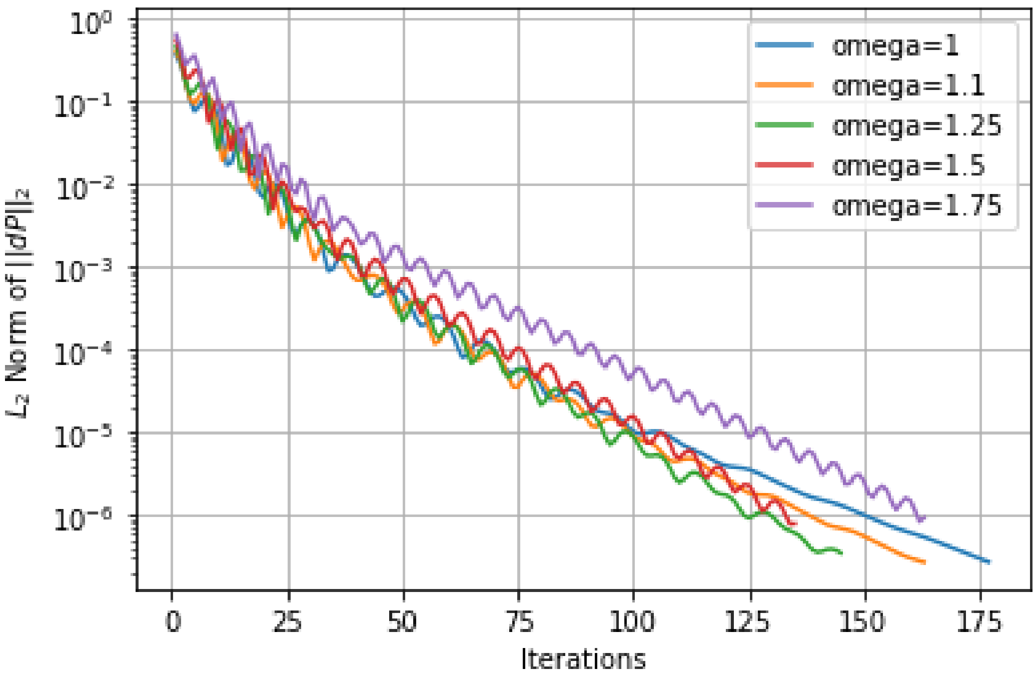


1.2.2 2.3 Overrelaxation

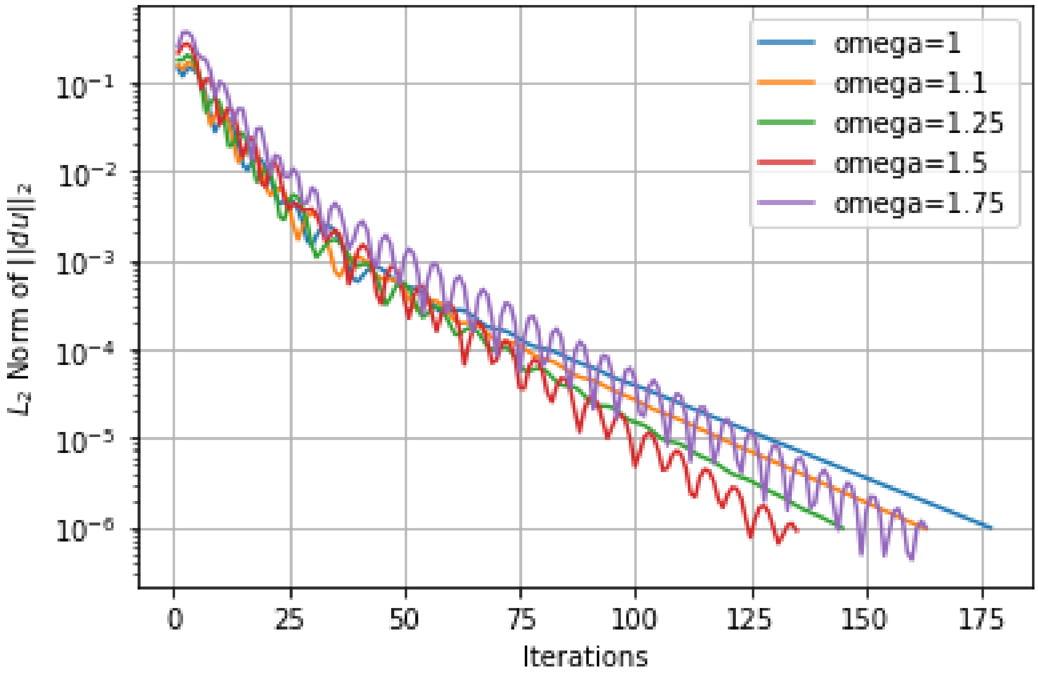
U = U + omega * dU

omega=1.25 seems the optimal

[170]:
```
values=[1, 1.1, 1.25, 1.5, 1.75]
compare_convergence("omega", values, "P")
```



[171]:
```
values=[1, 1.1, 1.25, 1.5, 1.75]
compare_convergence("omega", values, "P")
```

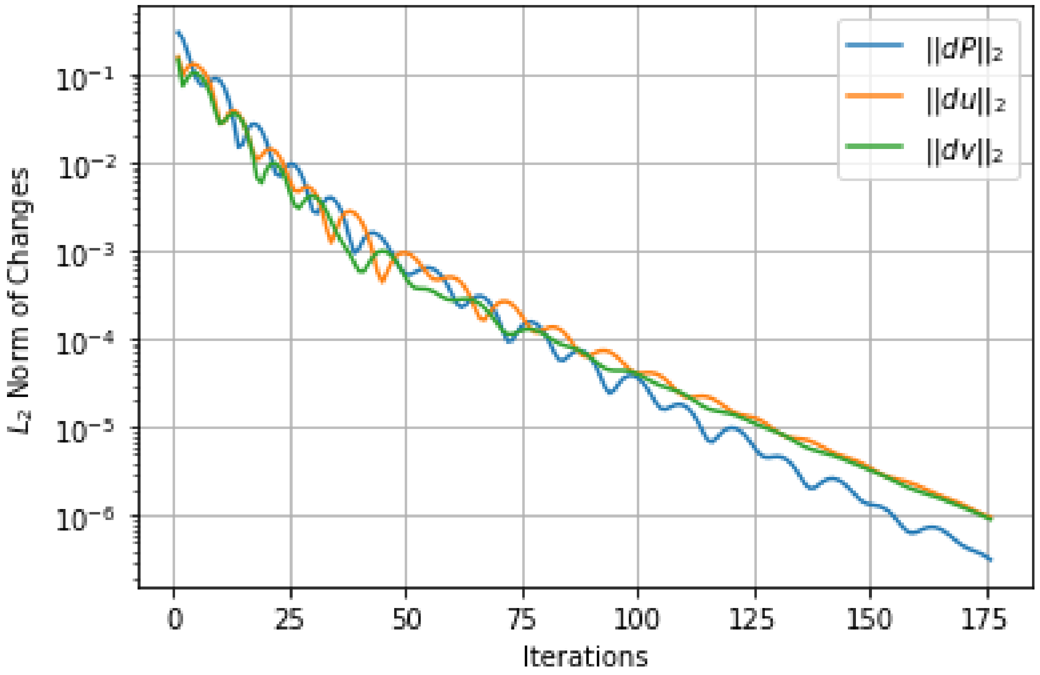


1.3 3.2 Basic solutions

1.3.1 3.2.1 Solution

Convergence history of $U_{top} = 1$ with $\Delta t = 0.1$ and tolerance$= 10^{-6}$.

```
[17]: convergence("log-3_2")
```



[3]:

```
def read_field(f_name):
    f = pd.read_csv("./outputs/" + f_name + ".csv", header=None)
    f = np.array(f)
    return f.T
```

[ ]:

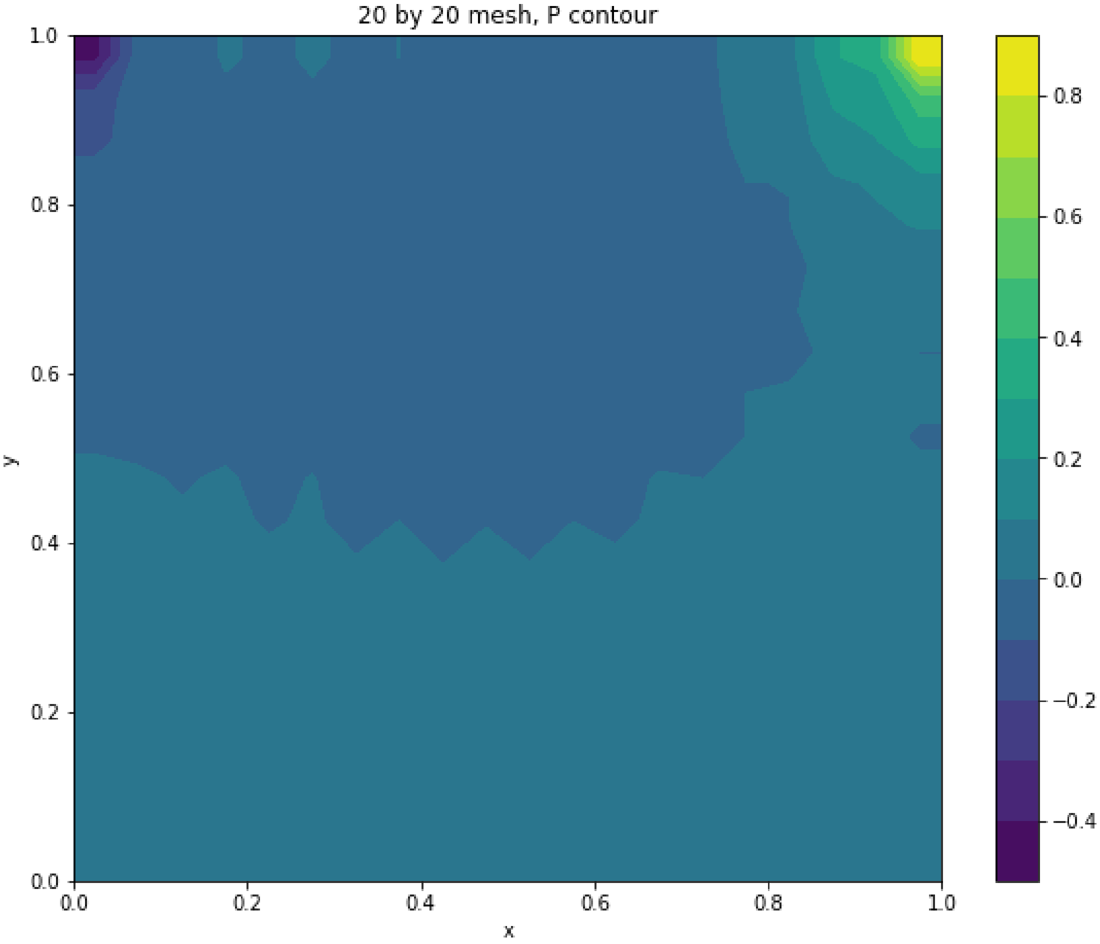
```
Lx = 1
Ly = 1
Factor = 20
Ni = Lx*Factor
Nj = Ly*Factor
dx = Lx/Ni
dy = Ly/Nj
```

[106]:

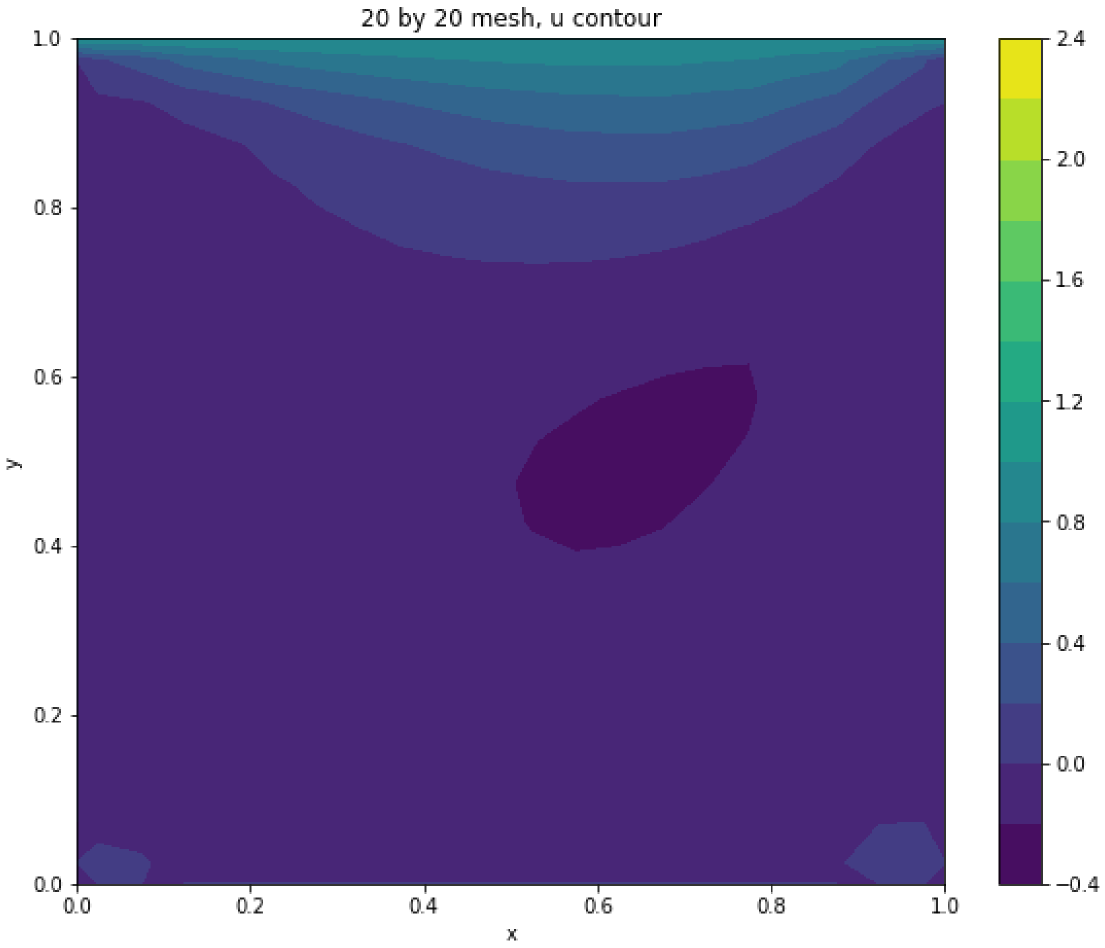
```
def contour_field(file_name, field_name):
    P = read_field(file_name)
#     P = P[1:-1, 1:-1]

#     x = np.arange(dx/2, Lx, dx)
#     y = np.arange(dy/2, Ly, dy)

    x = np.arange(-dx/2, Lx+dx, dx)
    y = np.arange(-dy/2, Ly+dy, dy)
    X, Y = np.meshgrid(x, y)

    plt.figure(figsize=(10, 8))
    # plt.pcolor(X, Y, P)
    plt.contourf(X, Y, P, 15)
    plt.colorbar()
    plt.axis([0, 1, 0, 1])
    plt.xlabel('x')
    plt.ylabel('y')
    ttl = str(Ni) + " by " + str(Nj) + " mesh, " + field_name + " contour"
    plt.title(ttl)
    plt.show()
```

Surface plot of pressure.

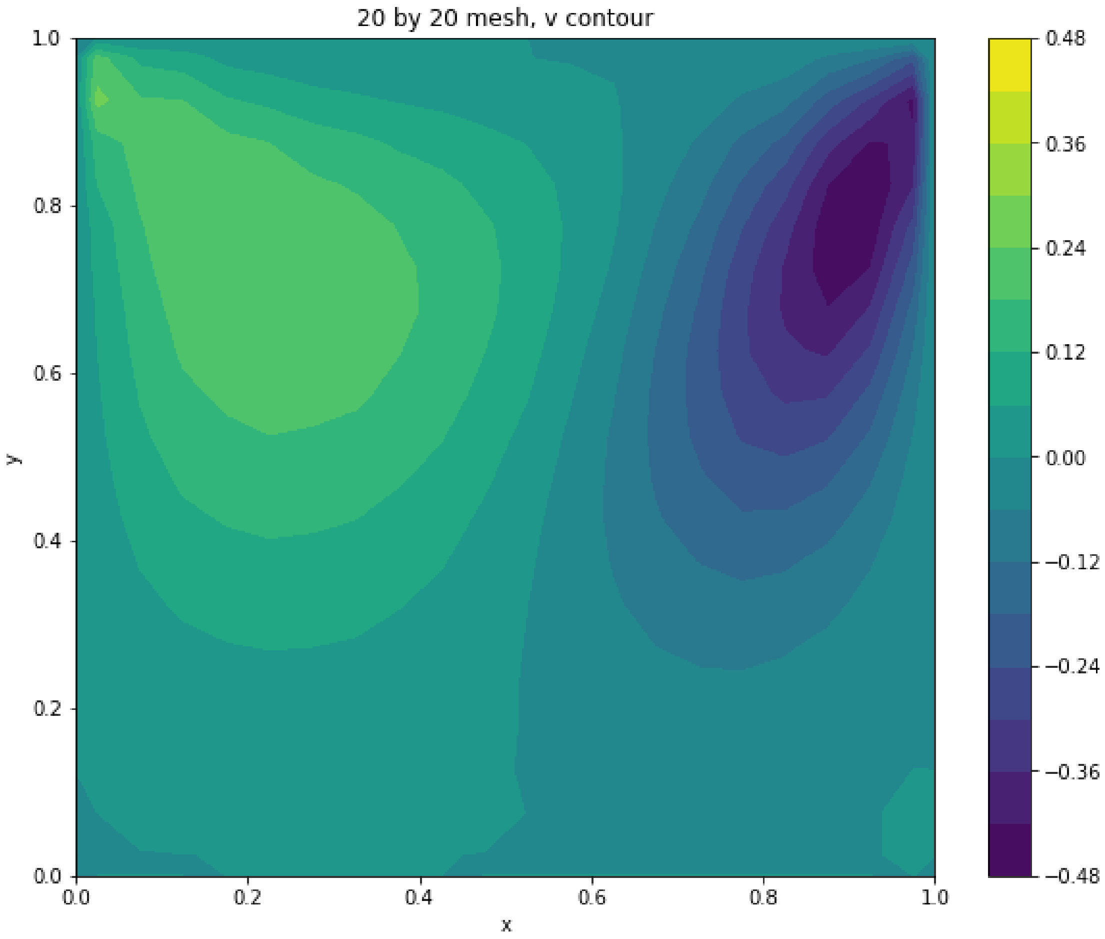
[107]:
```
contour_field("3_2-20by20-P", "P")
```



[108]:
```
contour_field("3_2-20by20-u", "u")
```



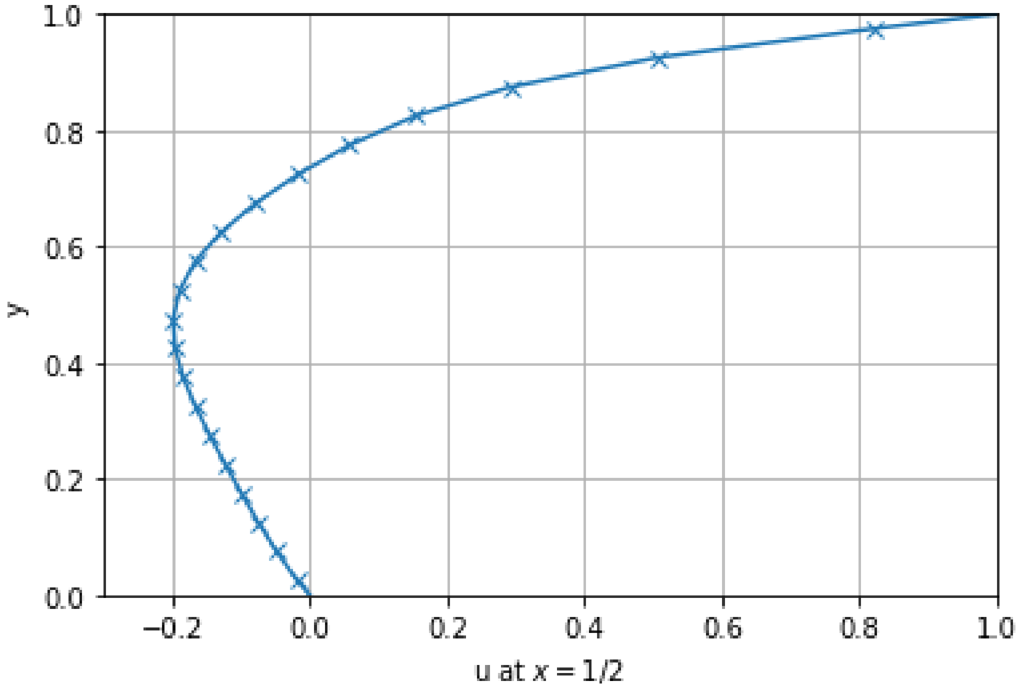
```
[109]: contour_field("3_2-20by20-v", "v")
```



Plot of u along $x = 1/2$.

```
[110]: u = read_field("3_2-20by20-u")
```

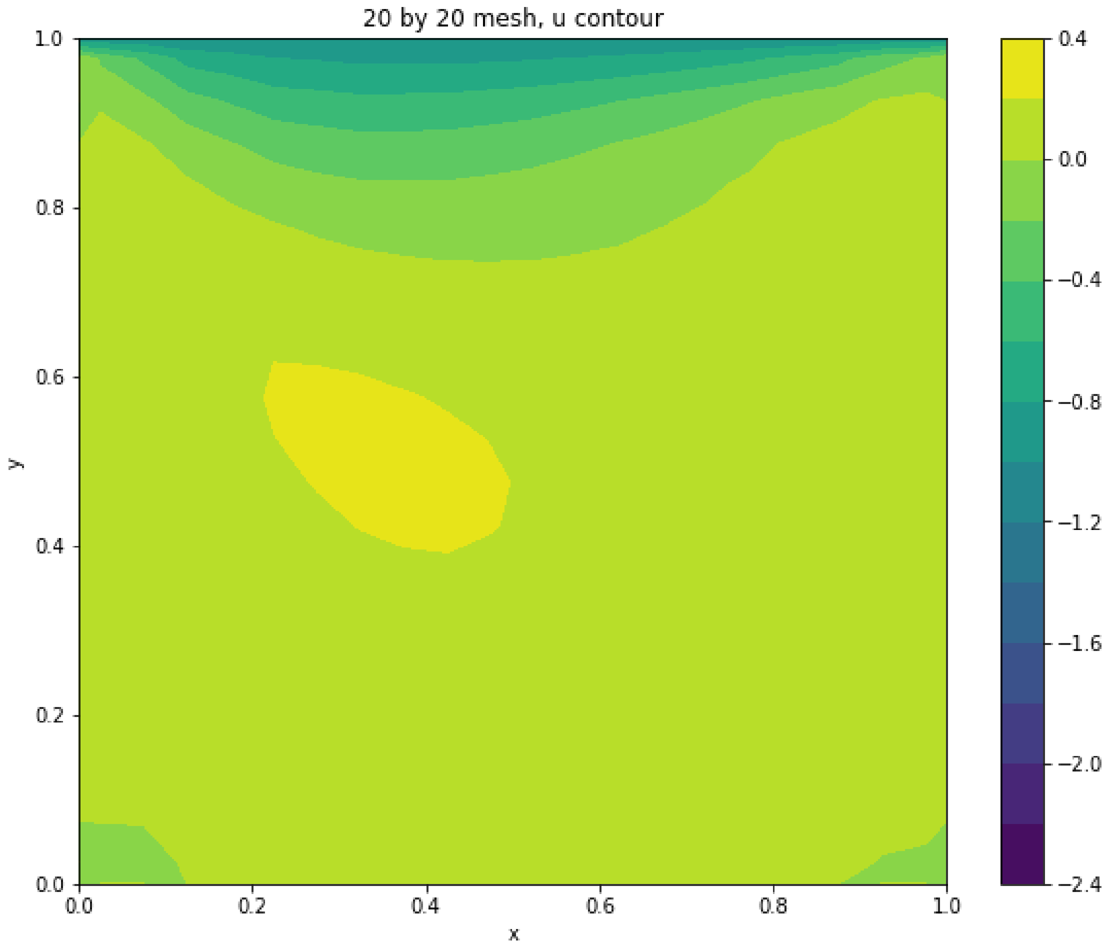
```
[111]: u_mid = (u[:,10] + u[:, 11])/2  # interpolate to get u at mid line
       y = np.arange(-dy/2, Ly+dy, dy)
       # y = np.arange(dy/2, Ly, dy)
       plt.plot(u_mid,y, 'x-')
       plt.xlabel('u at $x=1/2$')
       plt.ylabel('y')
       plt.axis([-0.3, 1.0, 0, 1])
       plt.grid(True)
       plt.show()
```



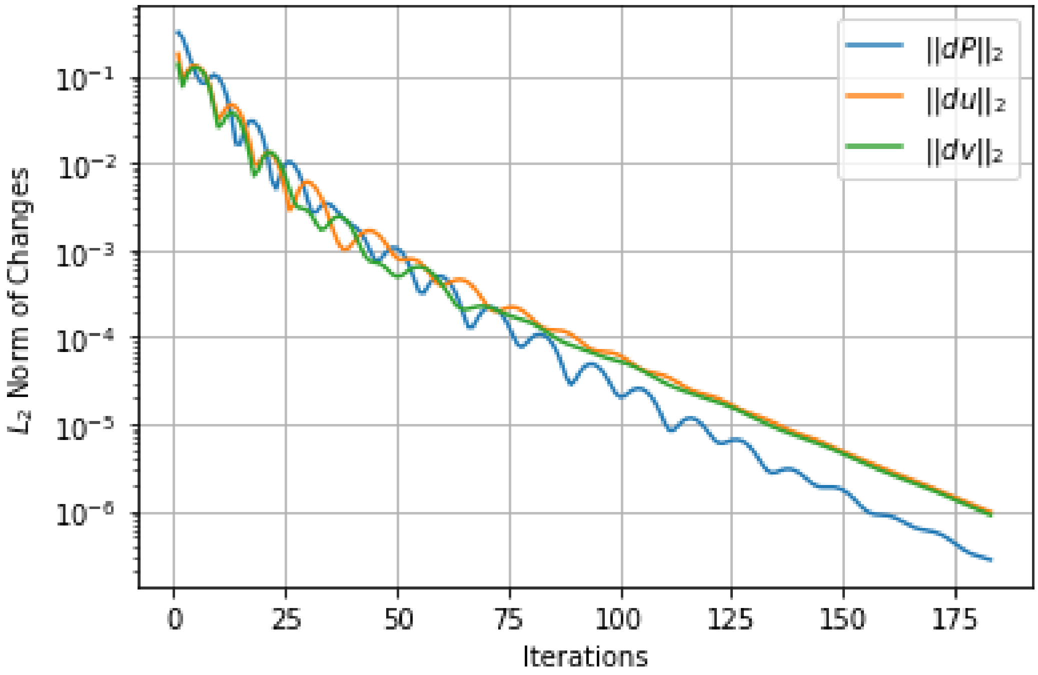
1.3.2 3.2.2 Sanity Check

$U_{top} = -1$

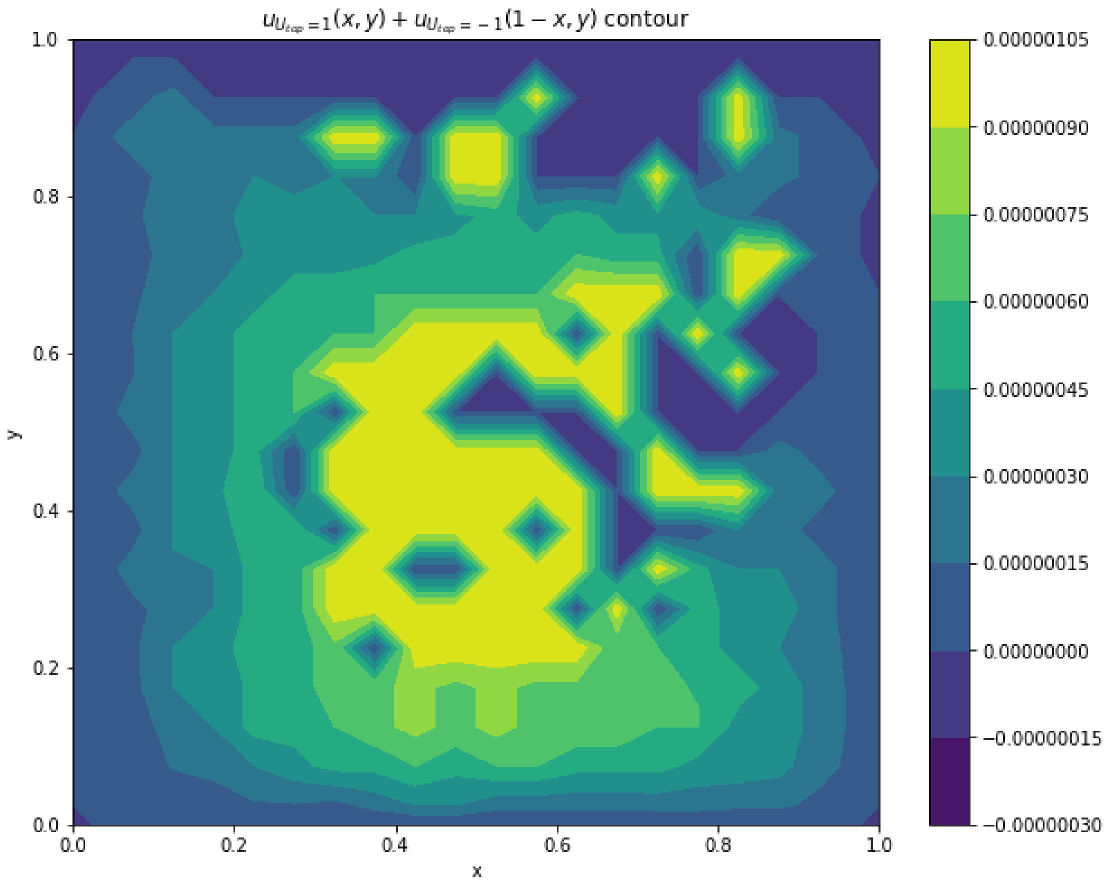
```
[112]: contour_field("3_2_2-20by20-u", "u")
```



[72]:
```
convergence("log-3_2_2")
```



```
[117]: u1 = read_field("3_2-20by20-u")
       u2 = read_field("3_2_2-20by20-u")
       u = np.flip(u2,1)+u1

       x = np.arange(-dx/2, Lx+dx, dx)
       y = np.arange(-dy/2, Ly+dy, dy)
       X, Y = np.meshgrid(x, y)

       plt.figure(figsize=(10, 8))
       # plt.pcolor(X, Y, P)
       plt.contourf(X, Y, u, 10)
       plt.colorbar()
       plt.xlabel('x')
       plt.ylabel('y')
       plt.axis([0, 1, 0, 1])
       ttl = "$u_{U_{top}=1}(x,y) + u_{U_{top}=-1}(1-x,y)$ contour"
       plt.title(ttl)
       plt.show()
```



The error is of size 10^{-6}, which is the same as the tolerance that I have set for these set of computations.

1.3.3 3.2.3 Grid Convergence

[259]:

```
jmin=np.argmin(u_mid)
xs = np.arange((jmin-3/2)*dy, (jmin+1)*dy, dy)
coeffs = np.polyfit(xs,u_mid[imax-1:imax+2] , 2)
p = np.poly1d(coeffs)
print(u_mid[jmin])
p(-coeffs[1]/2/coeffs[0])
```

```
-0.213792
```

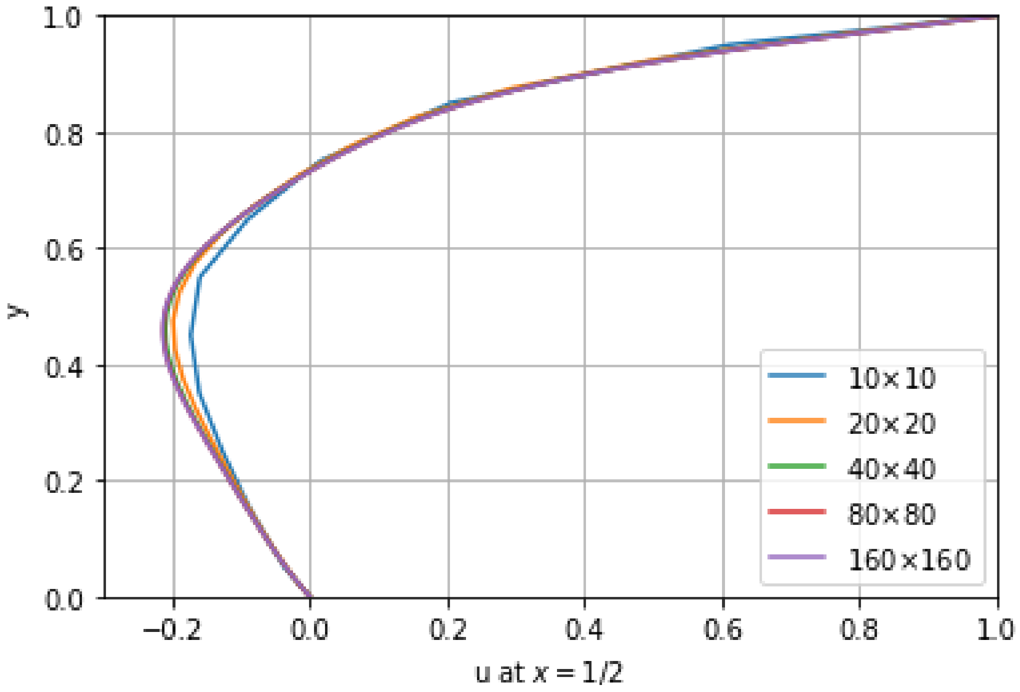
[259]:

```
-0.21379624002347541
```

[263]:

```
Ns = [10, 20, 40, 80, 160]
Ly = 1
```

```
umin = []
umin_interp = []
for n in Ns:
    file_name="u_midline-"+str(n)+"by" + str(n)
    u_mid = pd.read_csv("./outputs/" + file_name + ".csv", header=None)
    u_mid = np.array(u_mid[0])
    dy = 1/n
    jmin=np.argmin(u_mid)
    xs = np.arange((jmin-3/2)*dy, (jmin+1)*dy, dy)
    coeffs = np.polyfit(xs,u_mid[jmin-1:jmin+2] , 2)
    p = np.poly1d(coeffs)
    umin_interp.append(p(-coeffs[1]/2/coeffs[0]))
    umin.append(np.min(u_mid))

    y = np.arange(-dy/2, Ly+dy, dy)
    plt.plot(u_mid,y, label=str(n)+r"$\times$"+str(n))
plt.xlabel('u at $x=1/2$')
plt.ylabel('y')
plt.legend()
plt.axis([-0.3, 1.0, 0, 1])
# plt.axis([-0.3, -0.10, 0.4, 0.6])
plt.grid(True)
plt.show()
```



```
[264]: umin
```

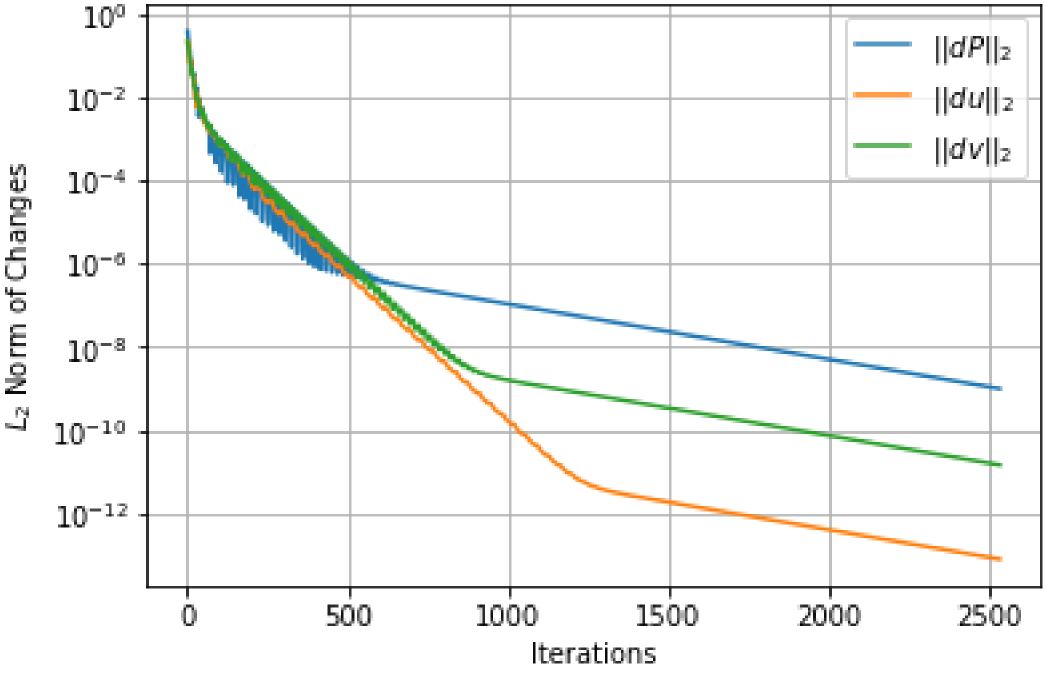
```
[264]: [-0.172967, -0.198794, -0.20995, -0.21302800000000002, -0.213792]
```

```
[265]: umin_interp
```

```
[265]: [-0.17296958722187883,
        -0.19922964086433204,
        -0.209994144051447,
        -0.21303685387768034,
        -0.21379624002347541]
```

2 4. h=2.5

```
[266]: convergence("log-4_mesh80")
```



```
[430]: Factor = 160 # mesh factor to import
       file = "4_mesh"+str(Factor)
       Lx = 1
       Ly = 2.5
       Ni = int(Lx*Factor)
       Nj = int(Ly*Factor)
       mesh=", "+str(Ni)+"*"+str(Nj)+" mesh"
```

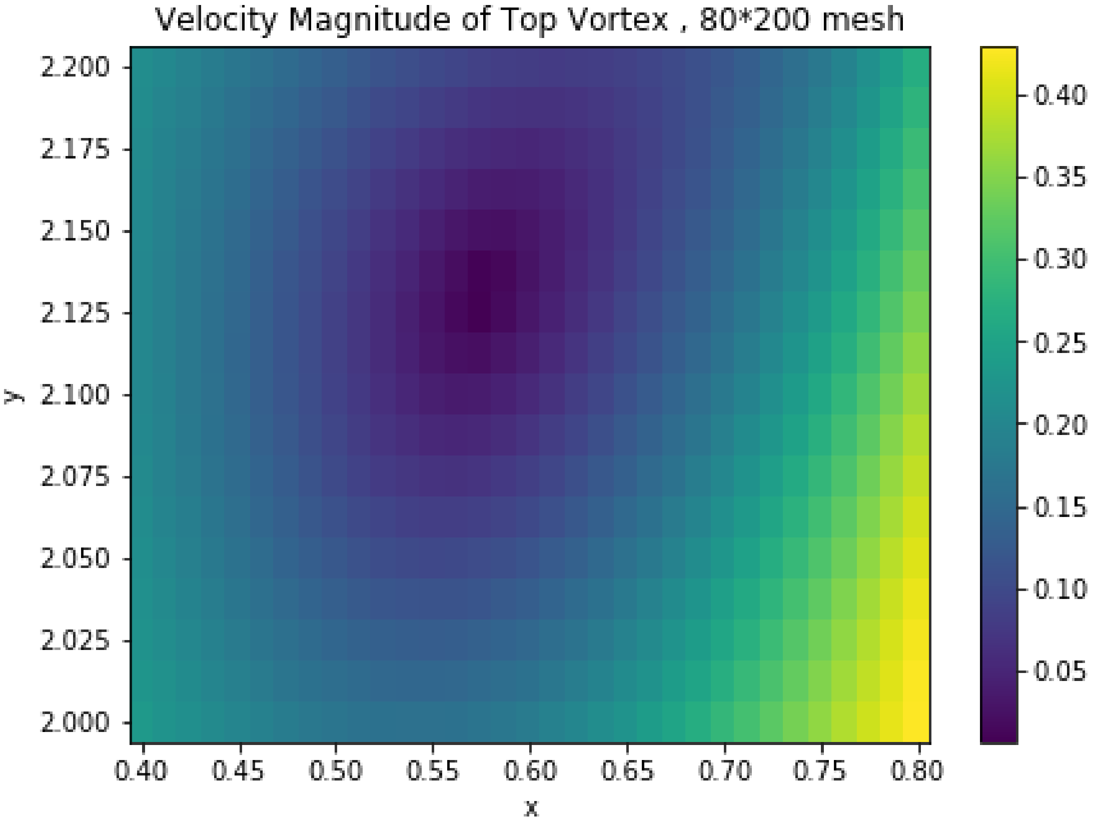
```
dx = Lx/Ni
dy = Ly/Nj
x = np.arange(-dx/2, Lx+dx, dx)
y = np.arange(-dy/2, Ly+dy, dy)
X, Y = np.meshgrid(x, y)
u = read_field(file+"-u")
v = read_field(file+"-v")
u_mag = np.sqrt(u**2+v**2) # Velocity magnitude
vorticity = np.zeros(np.shape(u)) # Vorticity field
for i in range(1, int(Ni)+1):
    for j in range(1, int(Nj)+1):
        vorticity[j,i] = ((v[j,i+1]-v[j,i-1])/2/dx - (u[j+1,i]-u[j-1,i])/2/dy)
print(mesh)
```

```
, 160*400 mesh
```

[429]:

```
# Find vortex given searching domain
def find_vortex(xmin, xmax, ymin, ymax, position):
    imin = int(np.floor(xmin/dx+0.5))
    imax = int(np.ceil(xmax/dx+0.5)+1)
    jmin = int(np.floor(ymin/dy+0.5))
    jmax = int(np.ceil(ymax/dy+0.5)+1)

    usec = u_mag[jmin:jmax, imin:imax] # section of u_mag to be examined
    jj, ii = ind = np.unravel_index(usec.argmin(), usec.shape) ## indices of cell with min u_mag among usec
    jv, iv = jmin + jj, imin + ii # indices of vortex location cell

    # u change direction along y/j, interpolate u to fine y_vort
    y_points = [Y[jv-1, iv], Y[jv, iv], Y[jv+1, iv]]
    [a, b, c] = np.polyfit(y_points,u[jv-1:jv+2, iv], 2)
    y_vort = (-b + np.sqrt(b*b-4*a*c))/2/a
    r1, r2 = (-b + np.sqrt(b*b-4*a*c))/2/a, (-b - np.sqrt(b*b-4*a*c))/2/a
    y_vort = r1 if Y[jv+1, iv] >= r1 >= Y[jv-1, iv] else r2

    # v change direction along x/i, interpolate v to find x_vort
    x_points = [X[jv, iv-1], X[jv, iv], X[jv, iv+1]]
    [a, b, c] = np.polyfit(x_points,v[jv, iv-1:iv+2], 2)
    r1, r2 = (-b + np.sqrt(b*b-4*a*c))/2/a, (-b - np.sqrt(b*b-4*a*c))/2/a
    x_vort = r1 if X[jv, iv+1] >= r1 >= X[jv, iv-1] else r2
    print("xloc:\t\t", x_vort) # location of vortex
    print("yloc:\t\t", y_vort) # location of vortex

    coeffs = np.polyfit(y_points,vorticity[jv-1:jv+2, iv], 2)
    py = np.poly1d(coeffs) # vorticity interpolated by y
    coeffs = np.polyfit(x_points,vorticity[jv, iv-1:iv+2], 2)
    px = np.poly1d(coeffs) # vorticity interpolated by x
    print("vorticity:\t", (py(y_vort)+px(x_vort))/2) # vorticity (vortex␣
↪strength)

    plt.figure(figsize=(7, 5))
    plt.pcolor(X[jmin:jmax, imin:imax], Y[jmin:jmax, imin:imax], u_mag[jmin:
↪jmax, imin:imax])
    plt.colorbar()
    plt.xlabel('x')
    plt.ylabel('y')
    plt.title("Velocity Magnitude of "+position+" Vortex "+mesh)
    plt.show()

find_vortex(xmin = 0.4, xmax = 0.8, ymin = 2, ymax = 2.2, position="Top")
```

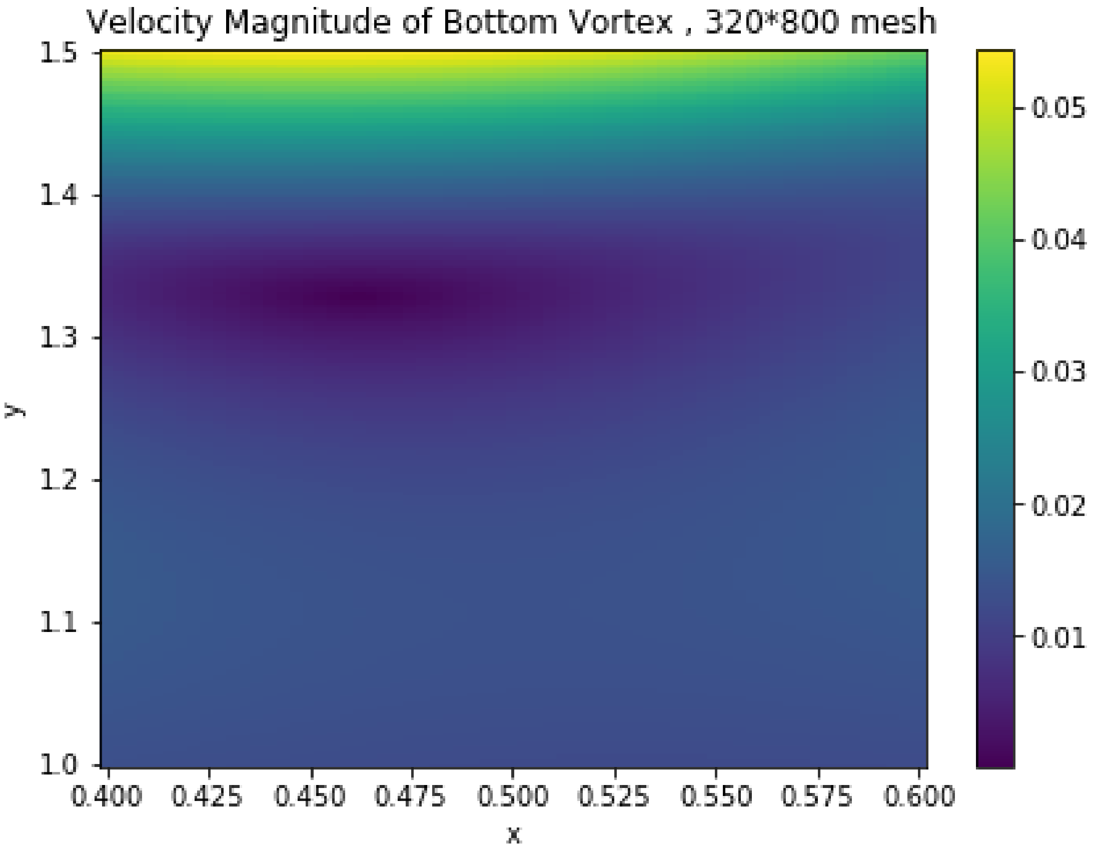
```
xloc:            0.568345051483043
yloc:            2.123922196861174
vorticity:       -2.401486276946871
```



```
[423]: find_vortex(xmin = 0.4, xmax = 0.6, ymin = 1, ymax = 1.5, position="Bottom")
```

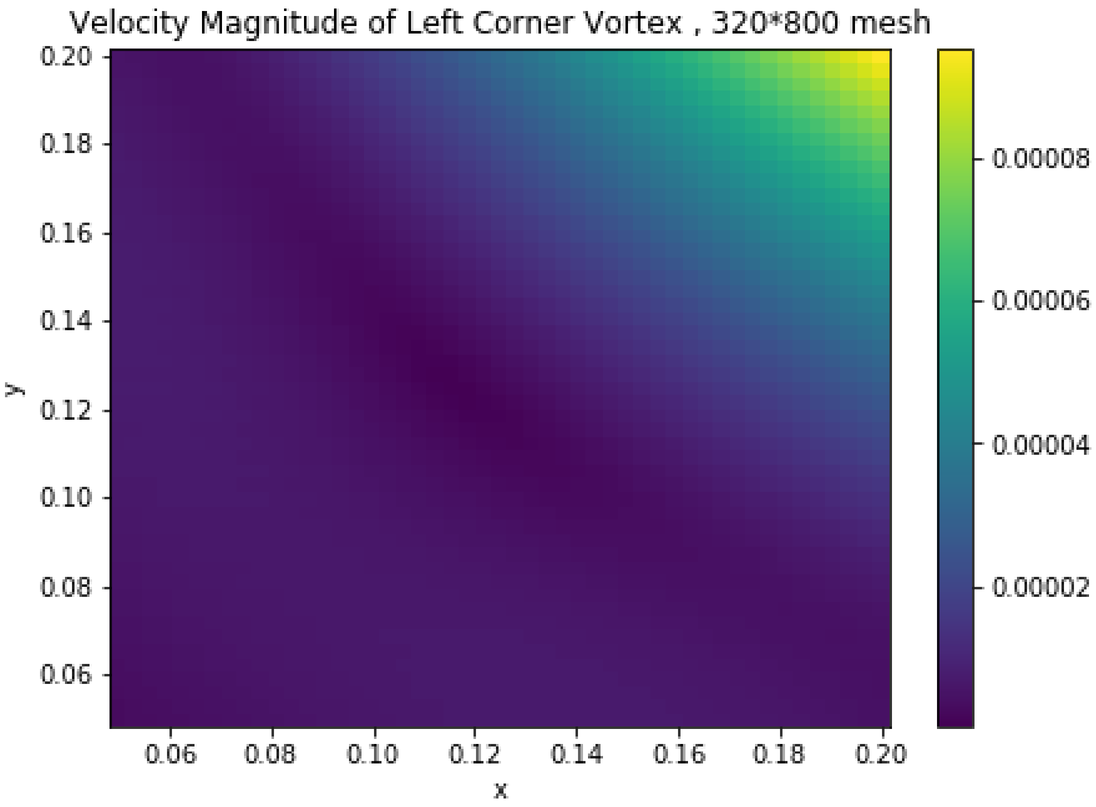
```
xloc:            0.4607602031212787
```

```
yloc:             1.3274775625190902
vorticity:        0.25486995755239994
```



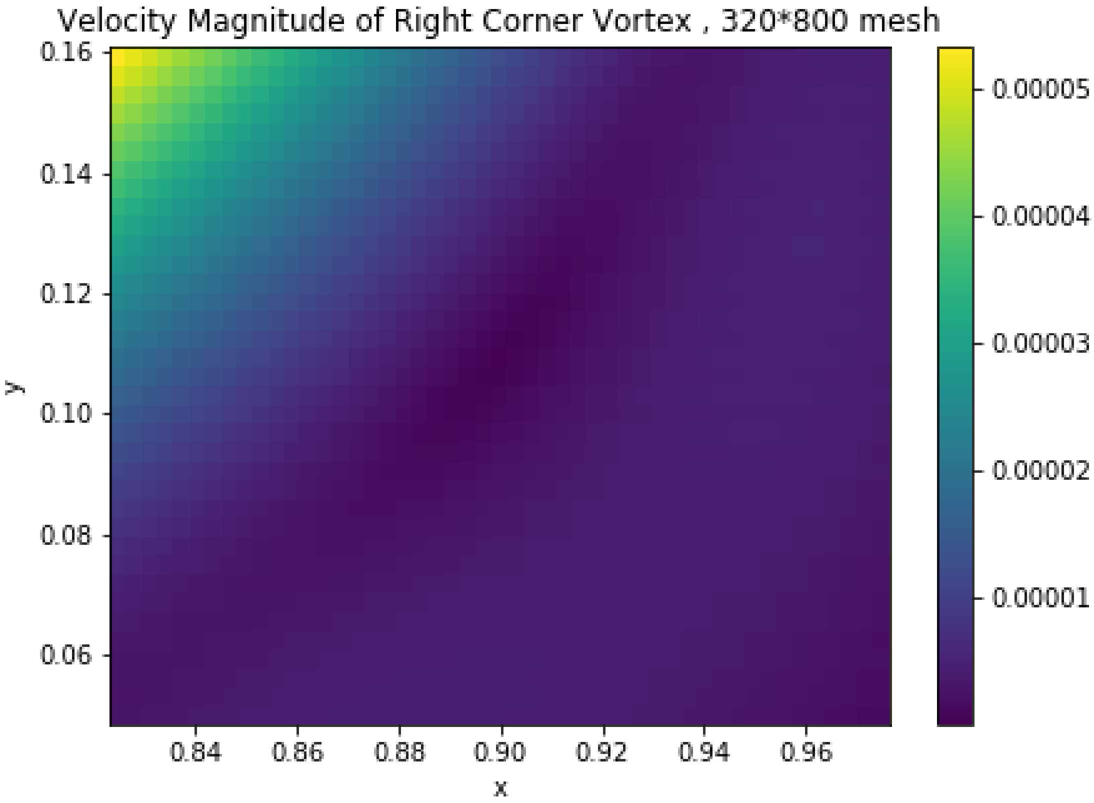
```
[424]: find_vortex(xmin = 0.05, xmax = 0.2, ymin = 0.05, ymax = 0.2, position="Left␣
        ↪Corner")
```

```
xloc:             0.11557120893259319
yloc:             0.12433667369498075
vorticity:        -0.000378108617751386
```

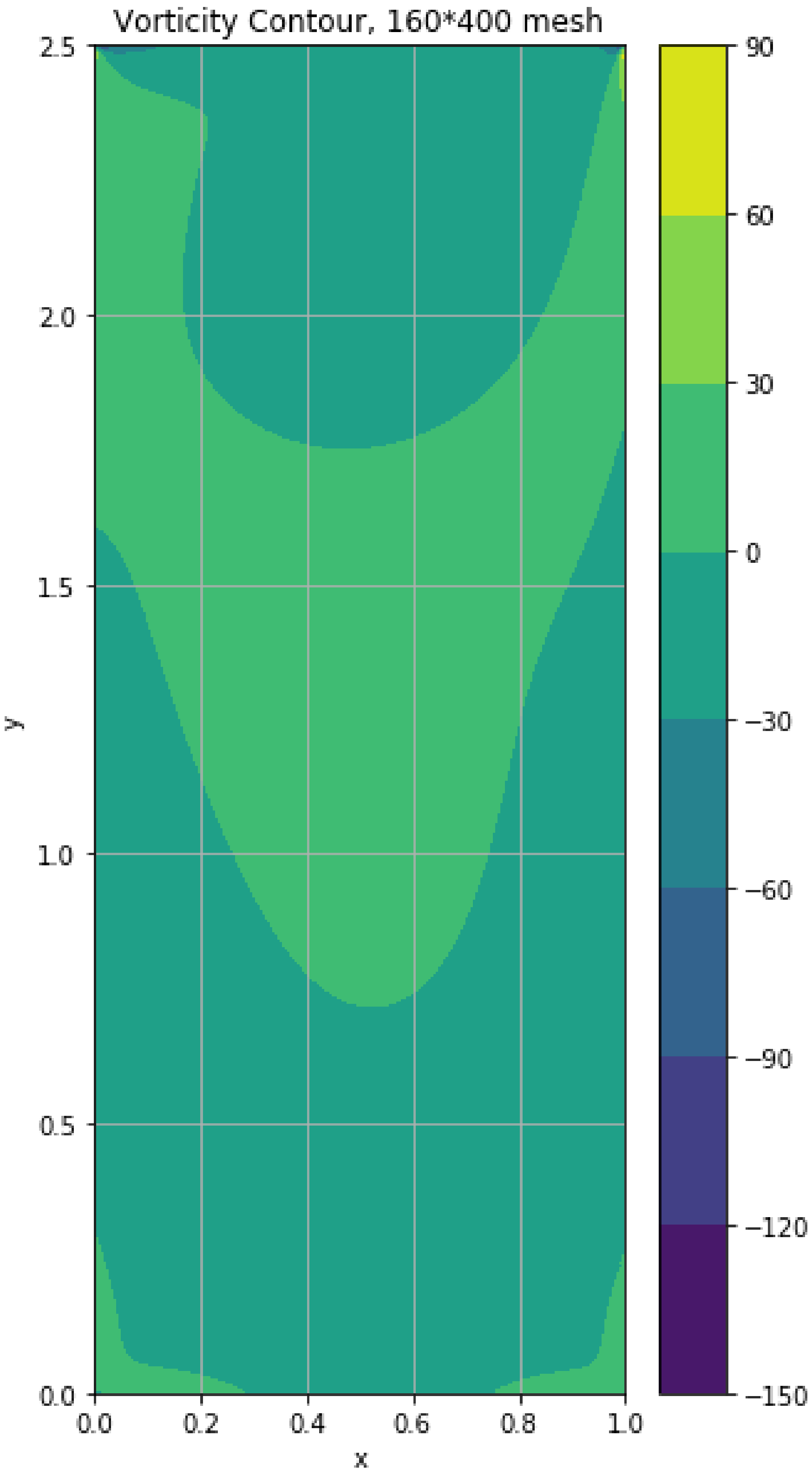


```
[425]: find_vortex(xmin = 0.825, xmax = 0.975, ymin = 0.05, ymax = 0.16,␣
        ↪position="Right Corner")
```

```
xloc:           0.8981052450177163
yloc:           0.10744806153756259
vorticity:      -0.00029925017387495505
```

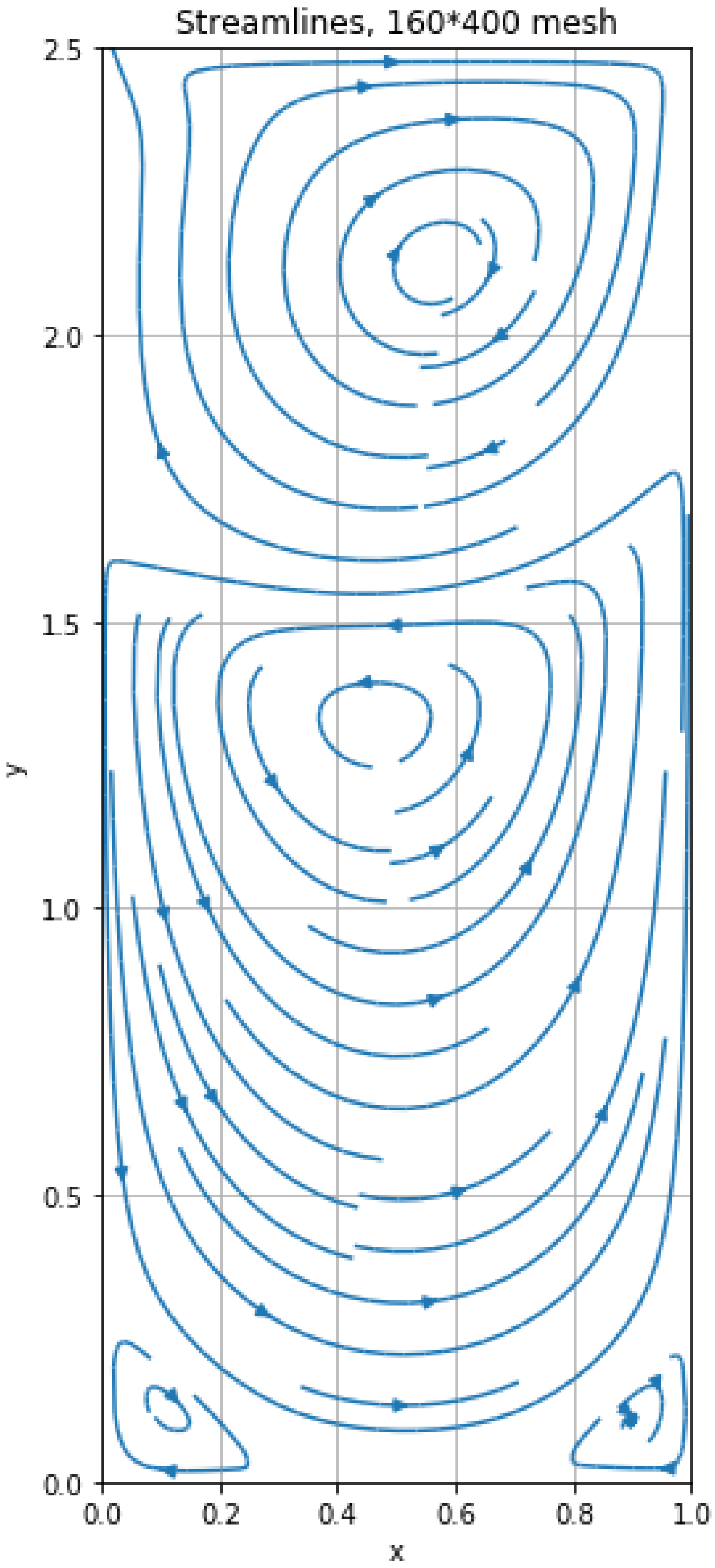


[436]:
```
# Plot vorticity
plt.figure(figsize=(12*Lx/Ly, 10))
plt.contourf(X, Y,vorticity)
plt.colorbar()
plt.xlabel('x')
plt.ylabel('y')
plt.axis([0, Lx, 0, Ly])
plt.title("Vorticity Contour"+mesh)
plt.grid(True)
plt.show()
```


Vorticity Contour, 160*400 mesh
2.5
2.0
1.5
1.0
0.5
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
90
60
30
0
−30
−60
−90
−120
−150

[370]:

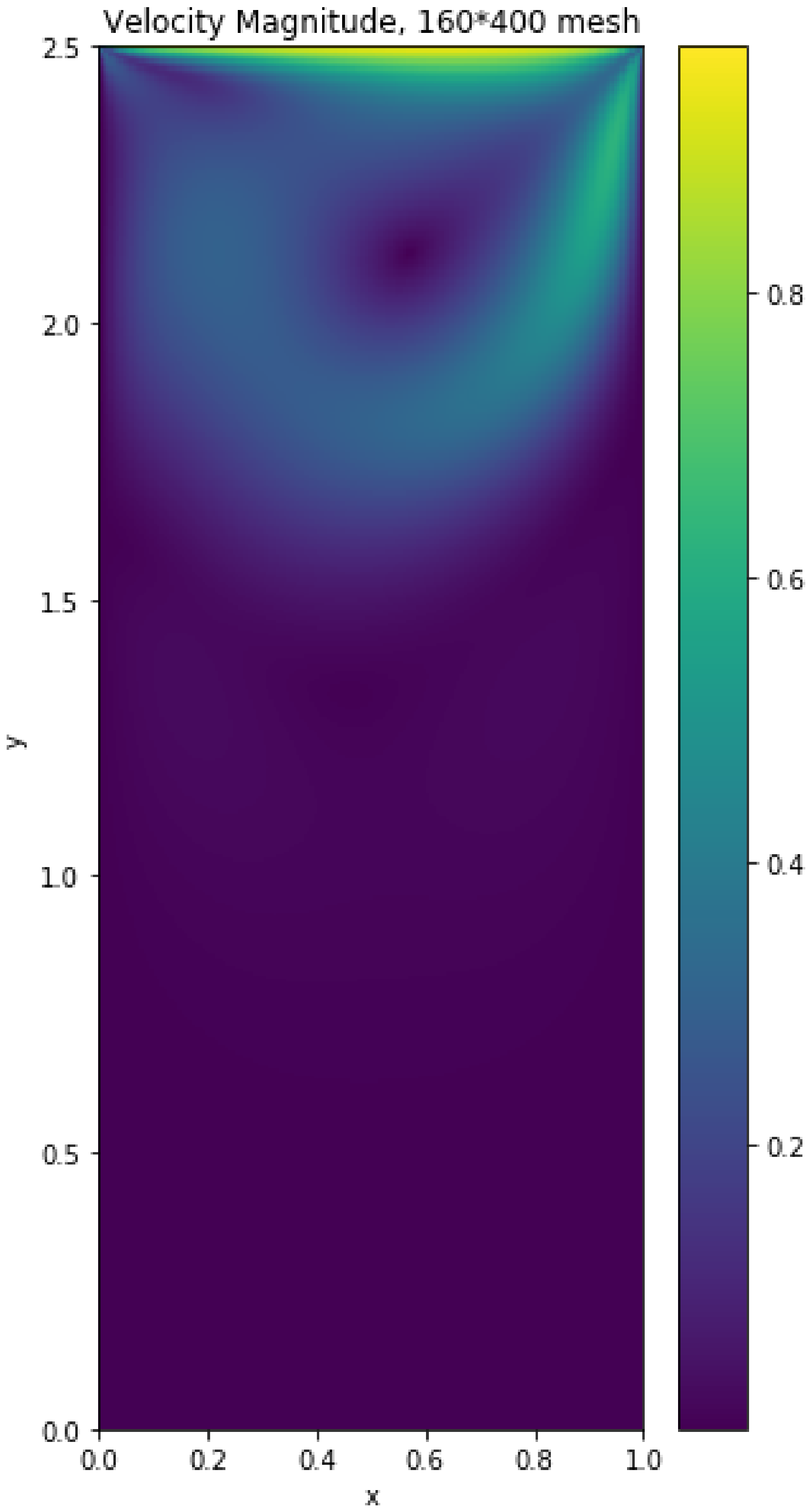
```
# Plot streamlines
plt.figure(figsize=(10*Lx/Ly, 10))
seed_points = np.array([np.linspace(0.4, 0.6,100), np.linspace(0.1, 2.3,100)])
seed_points = np.append(seed_points.T, [[0.1, 0.1],[0.02, 0.2], [0.9, 0.1], [0.
 ↪95, 0.12], [0.99, 0.13]], axis=0)
plt.streamplot(X, Y, u, v, start_points=seed_points)#, )
plt.xlabel('x')
plt.ylabel('y')
plt.axis([0, Lx, 0, Ly])
plt.title("Streamlines"+mesh)
plt.grid(True)
plt.show()
```


Streamlines, 160*400 mesh
2.5
2.0
1.5
1.0
0.5
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x

Top vortex in [0.4, 0.6] * [2, 2.2] Bottom vortex in [0.4, 0.6]*[1.2, 1.5] Left corner vortex (vortices?) Right corner vortex (vortices?)

[379]:

```
# Plot u_mag
plt.figure(figsize=(12*Lx/Ly, 10))
plt.pcolor(X, Y, u_mag)
plt.colorbar()
plt.xlabel('x')
plt.ylabel('y')
plt.axis([0, Lx, 0, Ly])
plt.title("Velocity Magnitude"+mesh)
plt.show()
```


Velocity Magnitude, 160*400 mesh
y
x
0.0
0.5
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
0.2
0.4
0.6
0.8