CS285: Assignment 1

Imitation Learning

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Problem 1

Suppose $X \mid p \sim \text{Binomial}(n, p)$ and $p \sim \text{Beta}(\alpha, \beta)$. Suppose also that the loss function $L(p, \hat{p}(X)) = (p - \hat{p}(X))^2$.

(a) Calculate the posterior risk $r(\hat{p} \mid X)$ for an arbitrary estimator $\hat{p}$.

(b) For a given x, what value of $\hat{p}(x)$ minimizes the posterior risk? Use this to construct a Bayes estimator. For this question do not rely on the general fact that the posterior mean minimizes squared error loss, but find the minimizer directly from minimizing the posterior risk from part a.

(c) Show that the posterior risk for the Bayes estimator you found in (b) is the posterior variance. Show that this going to be true generally for Bayes rules under squared error loss.

Subproblem 1

The posterior risk $r\left(\hat{p} \mid X\right)$ is

$$\begin{aligned} r\left(\hat{p} \mid X\right) &= \mathbb{E}\left(L\left(p, \hat{p}\right) \mid X\right) \\ &= \int_0^1 \left(p - \hat{p}\left(X\right)\right)^2 f\left(p \mid X\right) \mathrm{d}p \end{aligned}$$

where $f\left(p \mid X\right)$ is the density of the $\text{Beta}\left(\alpha + X, \beta + n - X\right)$.