The background is a dark teal color with various financial symbols and numbers scattered across it. Symbols include the dollar sign (\$), euro sign (€), pound sign (£), and yen sign (¥). Numbers include 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Some numbers are in a light teal color, while others are in a darker teal. There are also some white arrows pointing up and down. The text "Monte Carlo Method for Simulating Baccarat" is centered in a large, white, sans-serif font.

Monte Carlo Method for Simulating Baccarat

A Final Project for IS 590PR Programming for Analytics & Data Processing

Xiaoliang Jiang

Te Lin

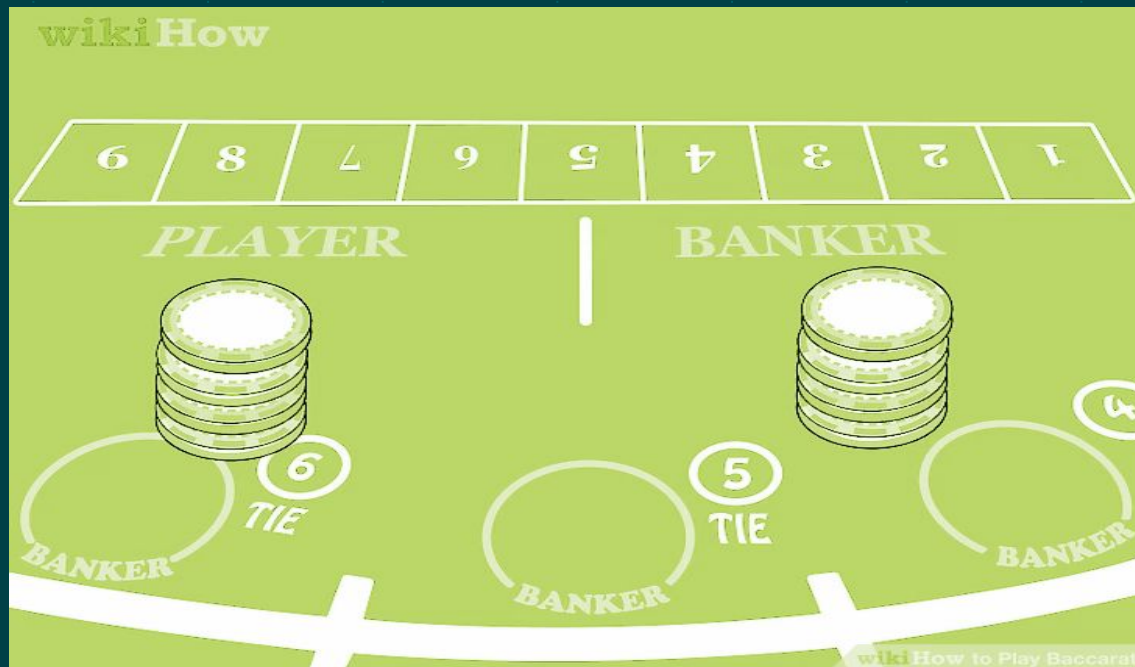


1. Baccarat (card game)

A popular card game in Asian casinos



Gamblers can bet on two hands



PLAYER

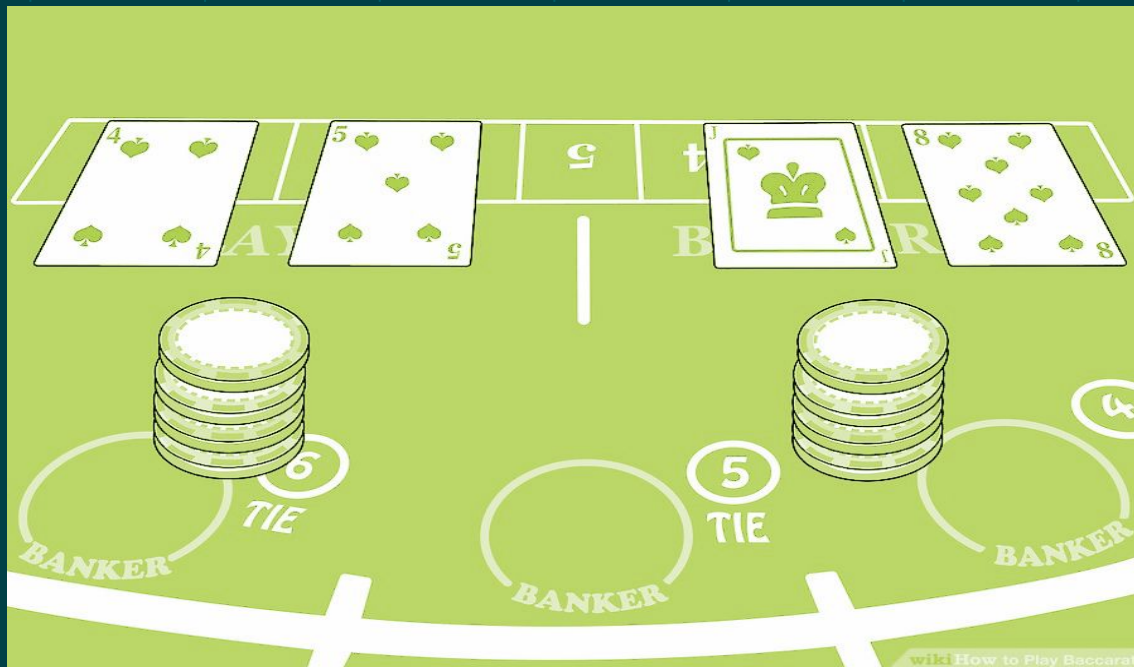
vs.

BANKER

Reference:

1. wikiHow: How to Play Baccarat, <https://www.wikihow.com/Play-Baccarat>

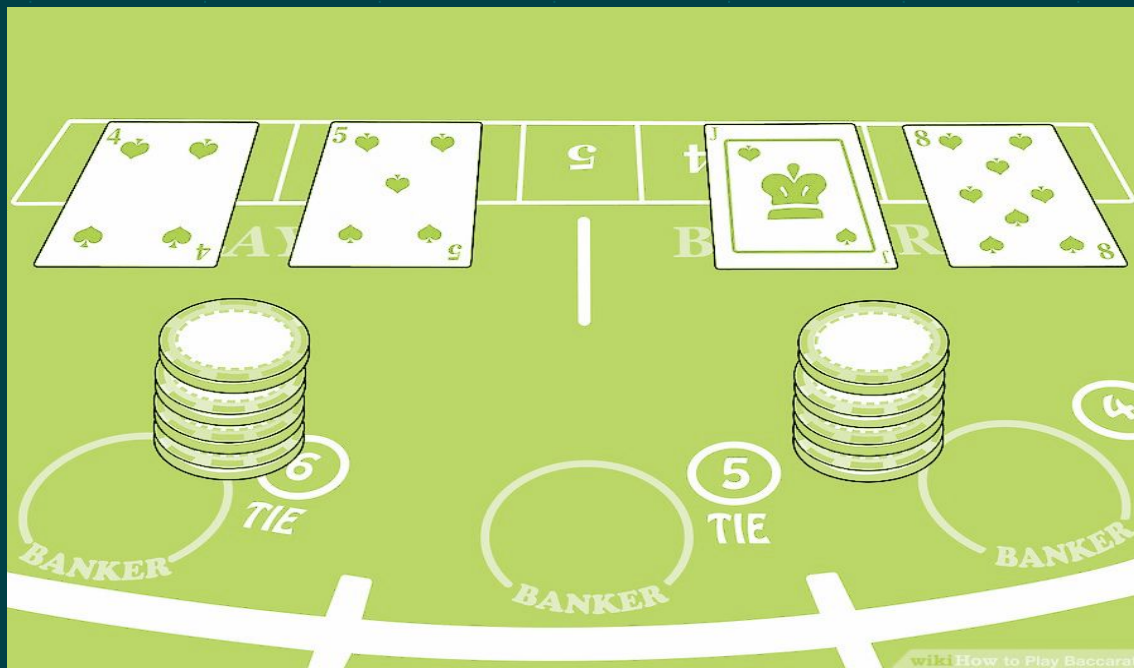
Each hand has two (or three) cards



Reference:

1. wikiHow: How to Play Baccarat, <https://www.wikihow.com/Play-Baccarat>

Point of A Hand



Points = unit decimal place of the sum of two (or three) cards

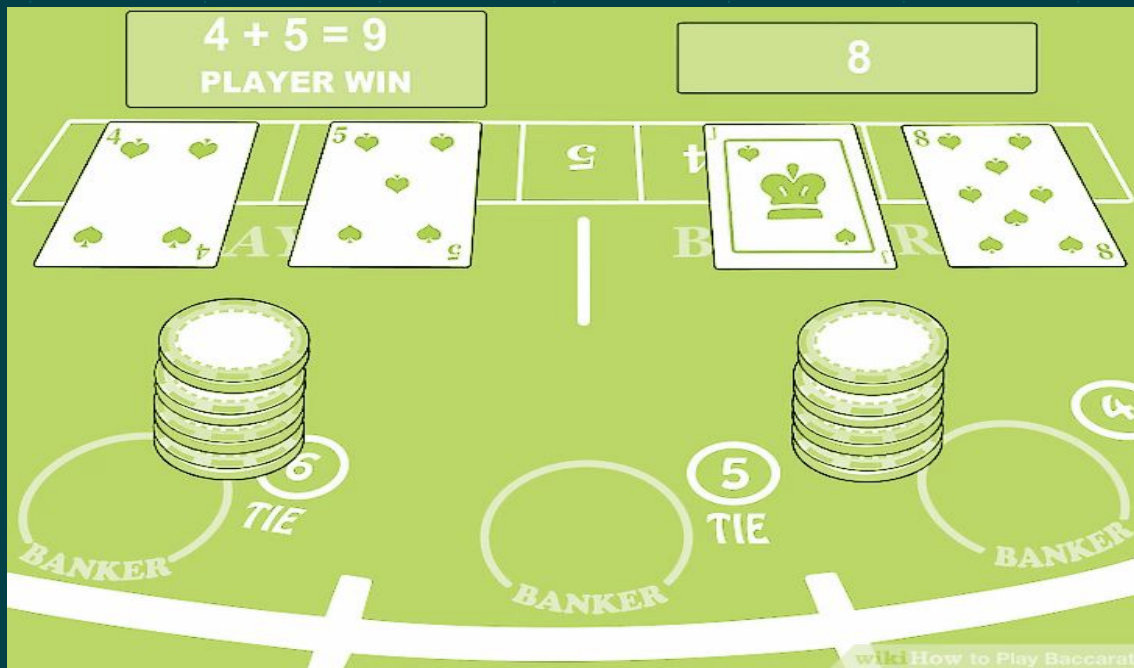
e.g. cards: 4,5,8, sum = 17, point = 7;

cards: J (10), 1, sum = 11, point = 1.

Reference:

1. wikiHow: How to Play Baccarat, <https://www.wikihow.com/Play-Baccarat>

Natural Wins

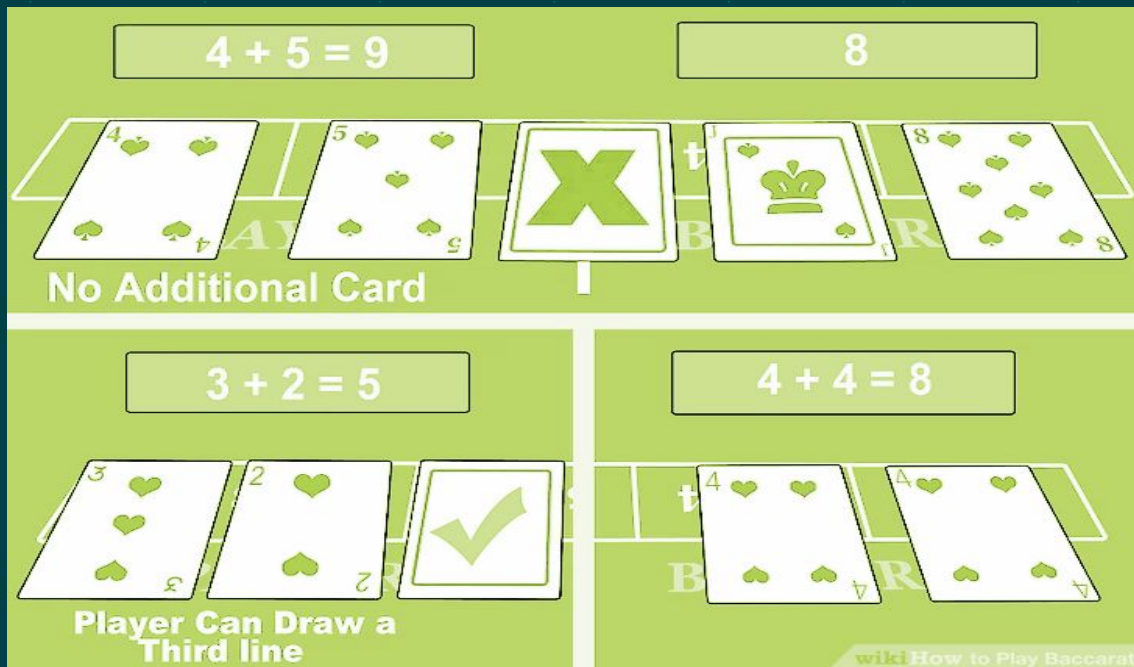


Either hand has point ≥ 8 , whoever is closer to 9 wins.

Reference:

1. wikiHow: How to Play Baccarat, <https://www.wikihow.com/Play-Baccarat>

Third card or not?



Player's point > 7, no additional cards;

Player's point = 6 or 7, stands;

Player's point < 6, Player draw the 3rd card.

Reference:

1. wikiHow: How to Play Baccarat, <https://www.wikihow.com/Play-Baccarat>

When to Draw the 3rd Card?

Player third Card	Banker	
8	0-2 Draws	3-7, Stays
6 or 7	0-6 Draws	7, Stays
4 or 5	0-5 Draws	6-7, Stays
2 or 3	0-4 Draws	5-7, Stays
9, 10	0-3 Draws	4-7, Stays

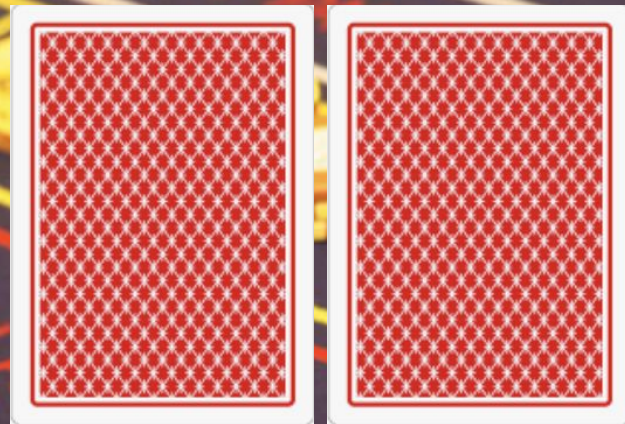
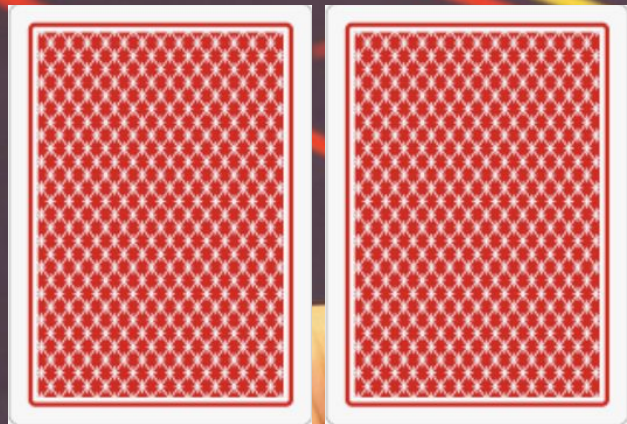
wikiHow to Play Baccarat

Reference:

1. wikiHow: How to Play Baccarat, <https://www.wikihow.com/Play-Baccarat>

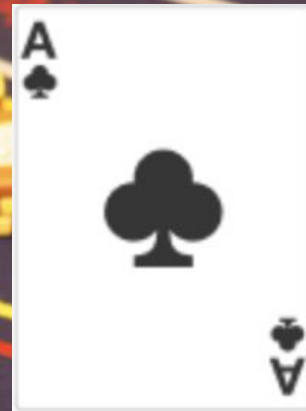
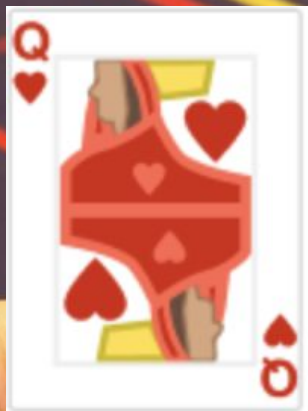
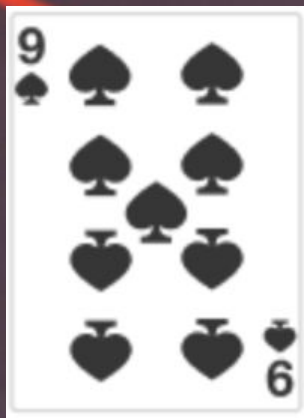
Banker

Player



Banker 😊

Player 😐



2. Monte Carlo Simulations



2.0 Our Goals

1. Testing the **theoretical expectation** of each situation.
2. Comparing different **bet ratios**.
3. Comparing different **bet strategies**.
4. Exploring the **possibility of earning money** from this game.



Theoretical Expectation

2.1 Tested Theoretical possibilities

Settings:

- 1,000 rounds in a game
- 1,000 times game
- 0.1% bet ratio
- $E = P(\text{Win})$
- $o = \text{Gaming odds}$:
 - Banker: 1 (-5% to casino)
 - Player: 1
 - Tie: 8

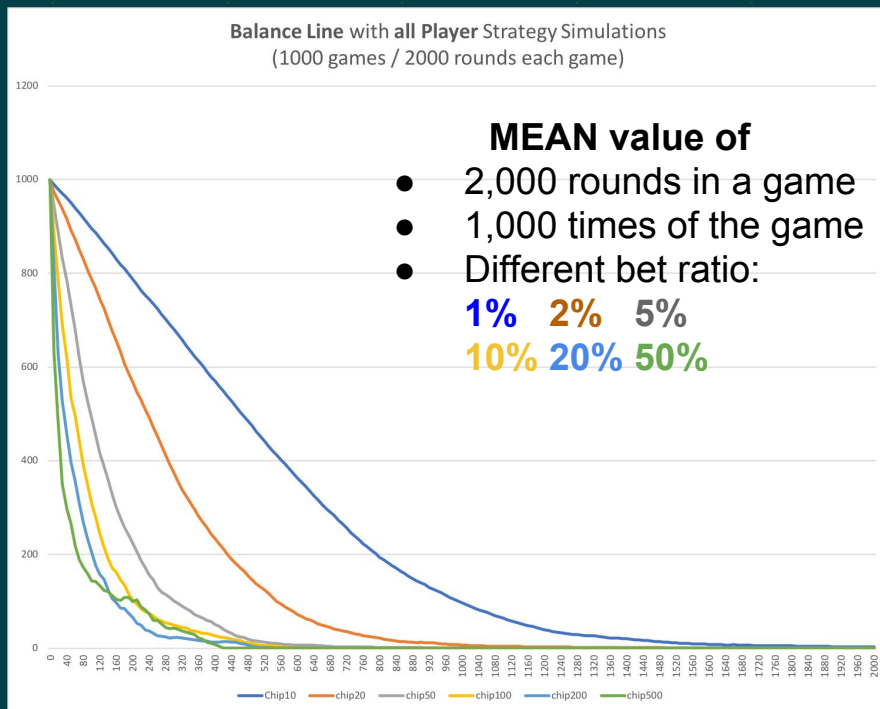
Expected gains: $E^*(o+1)$	Gains based on Theoretical possibility ¹	Results from Monte Carlo 
Banker	0.8943	0.8956
Player	0.8925	0.8912
Tie	0.8564	0.8564

Reference:

1. <https://zh.wikipedia.org/wiki/%E7%99%BE%E5%AE%B6%E6%A8%82>

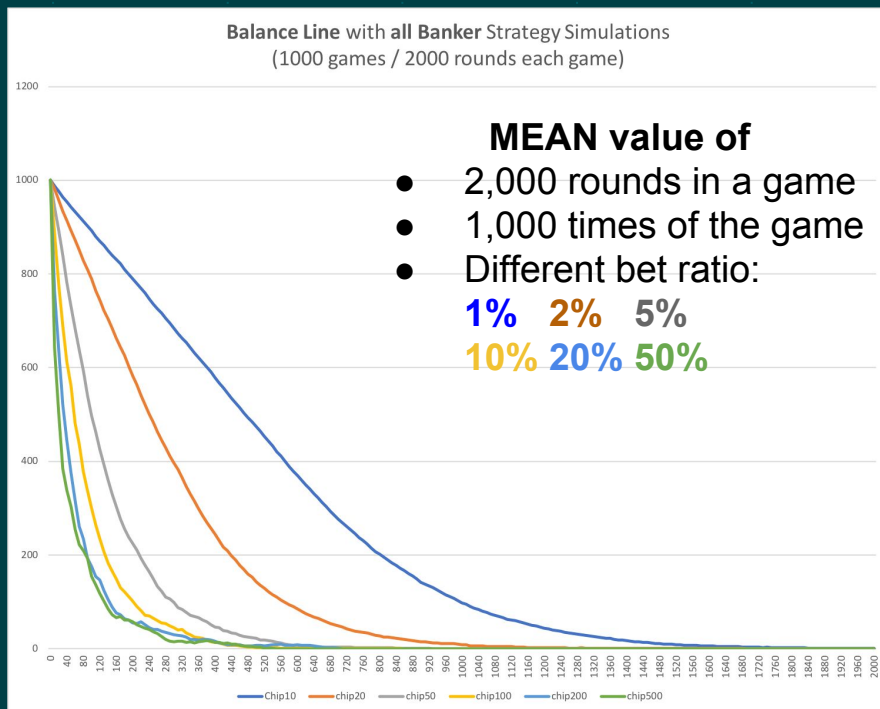
Different Bet Ratios

2.2.1 Simulations - Always Bet on Player



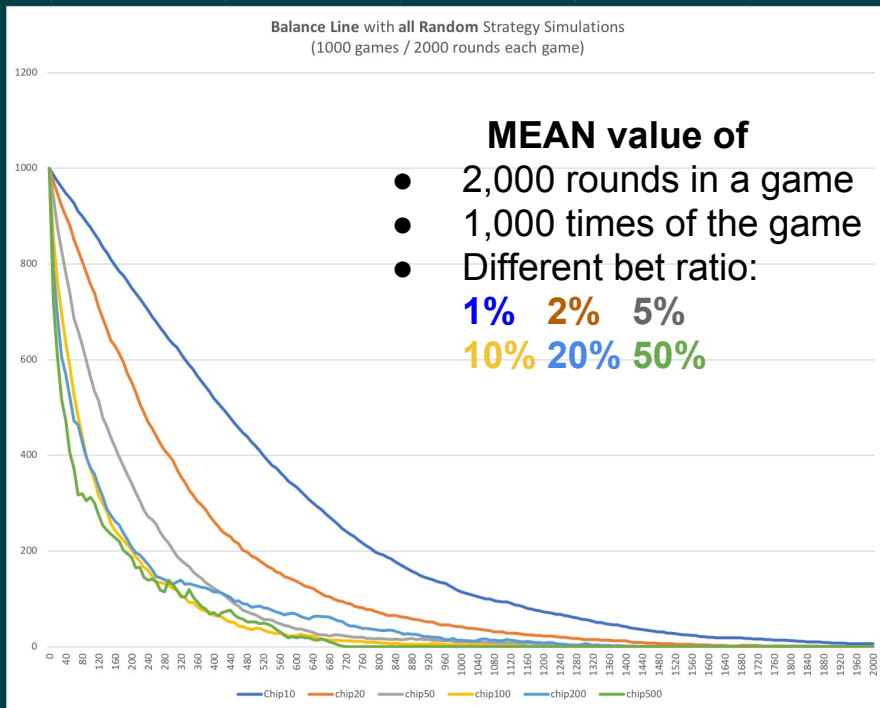
Different Bet Ratios

2.2.2 Simulations - Always Bet on Banker



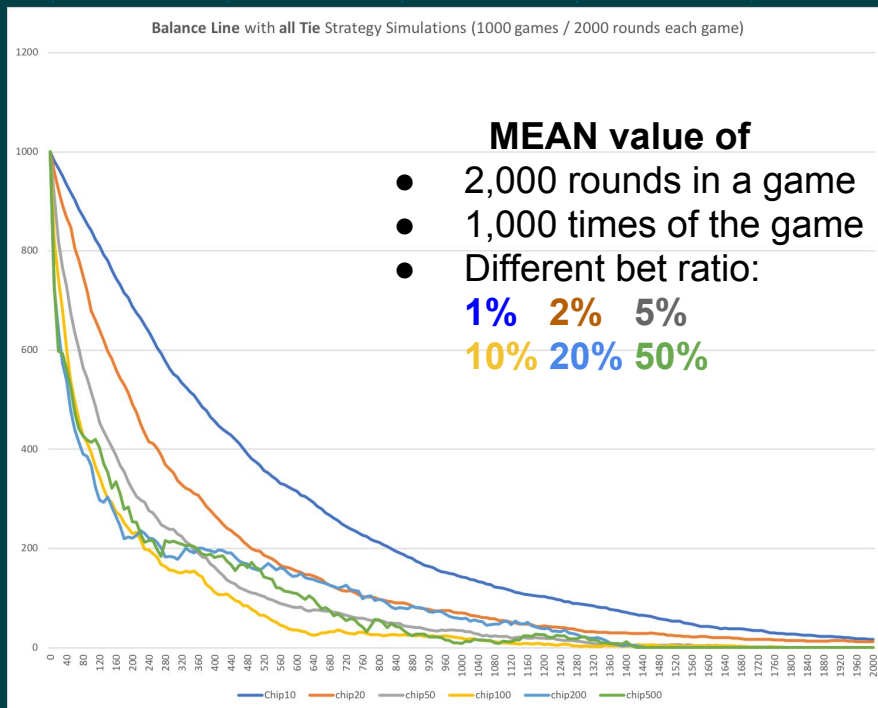
Different Bet Ratios

2.2.3 Simulation - Place Bet Randomly



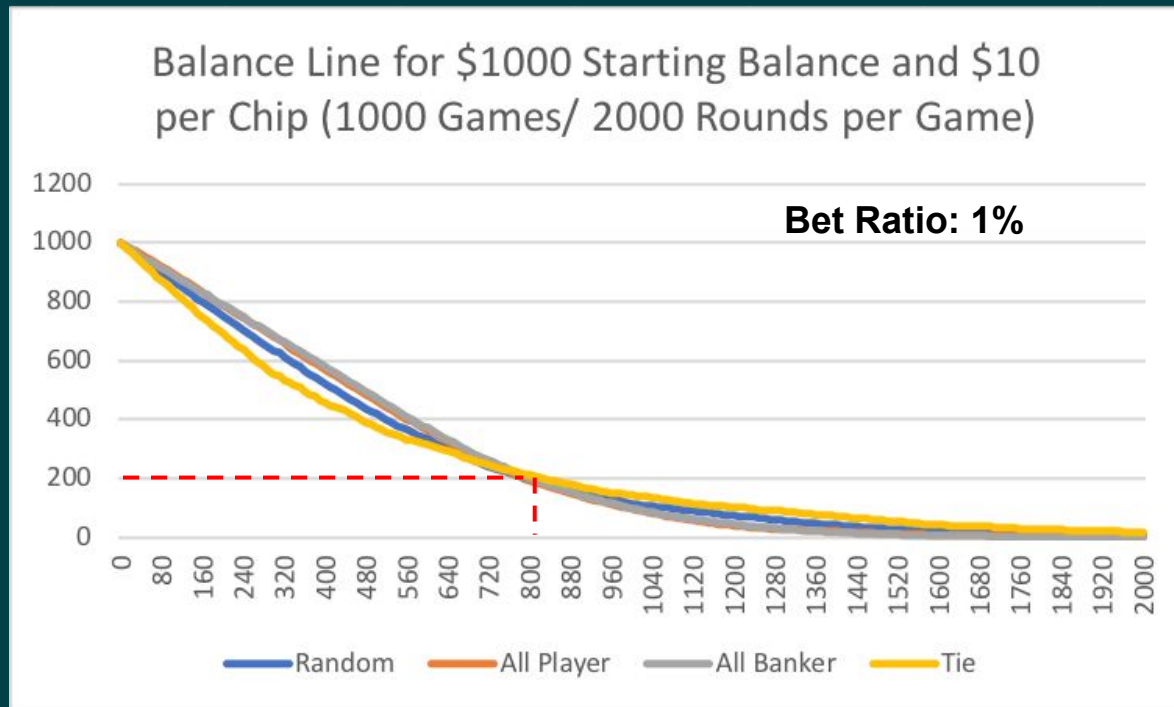
Different Bet Ratios

2.2.4 Simulations - Always Bet on Tie



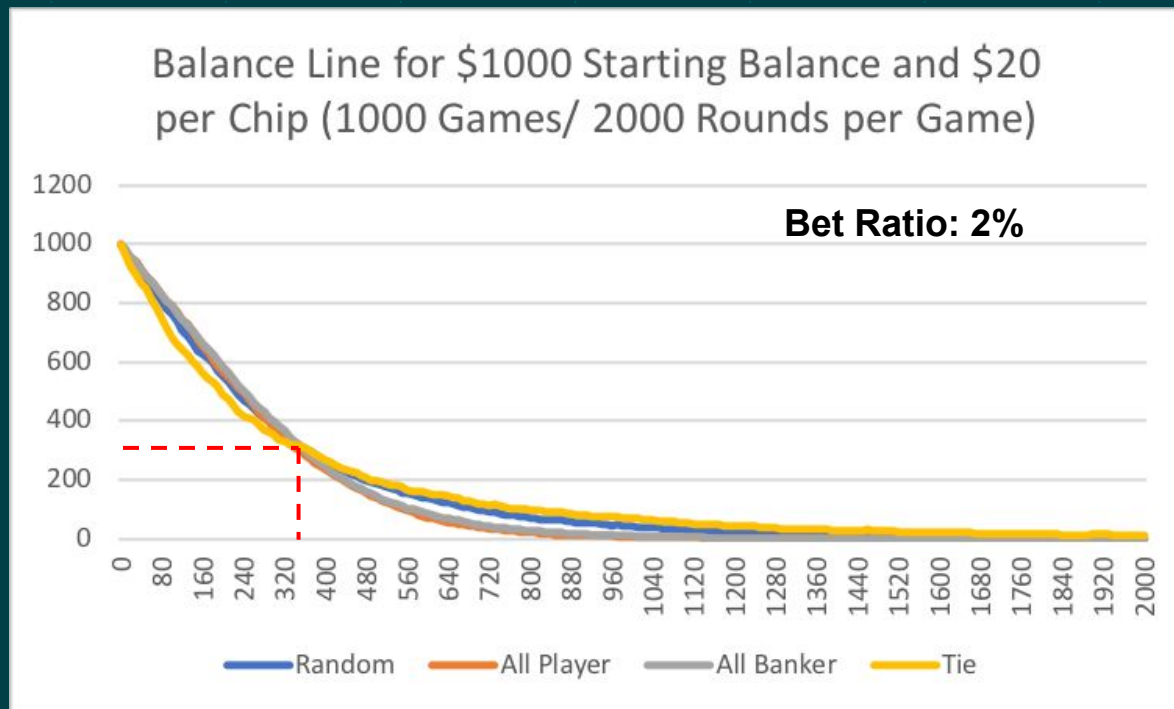
Different Bet Strategies

2.3.1 Comparison of Strategies (1%)



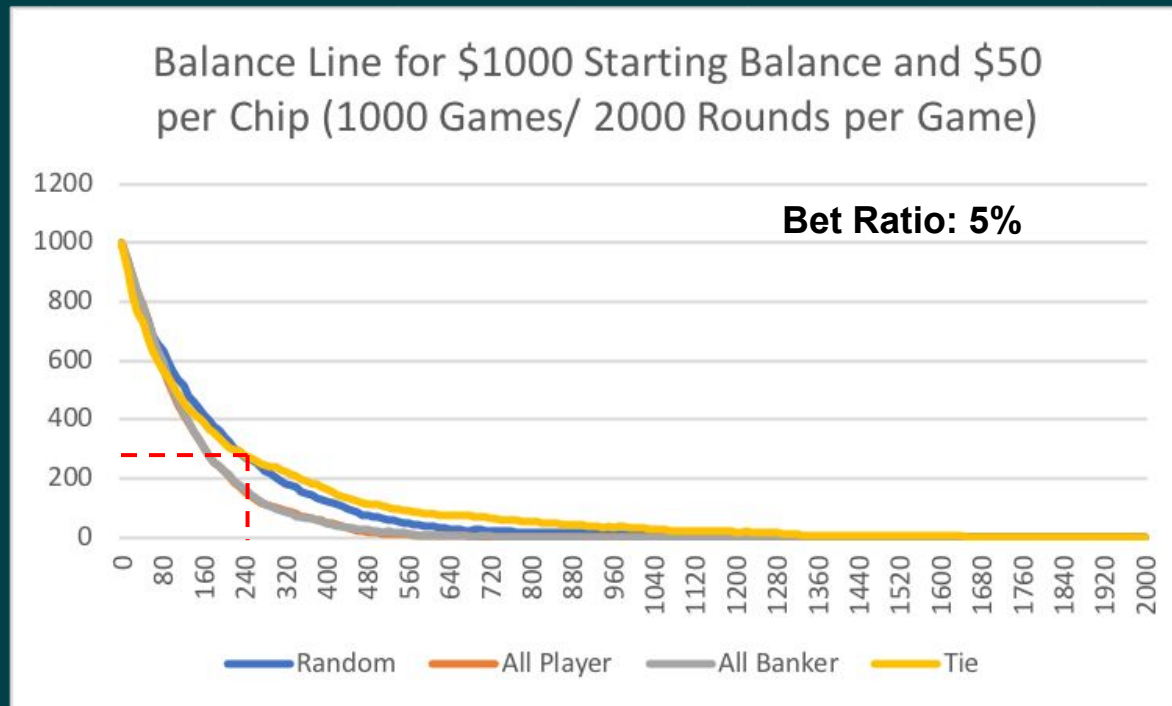
Different Bet Strategies

2.3.2 Comparison of Strategies (2%)



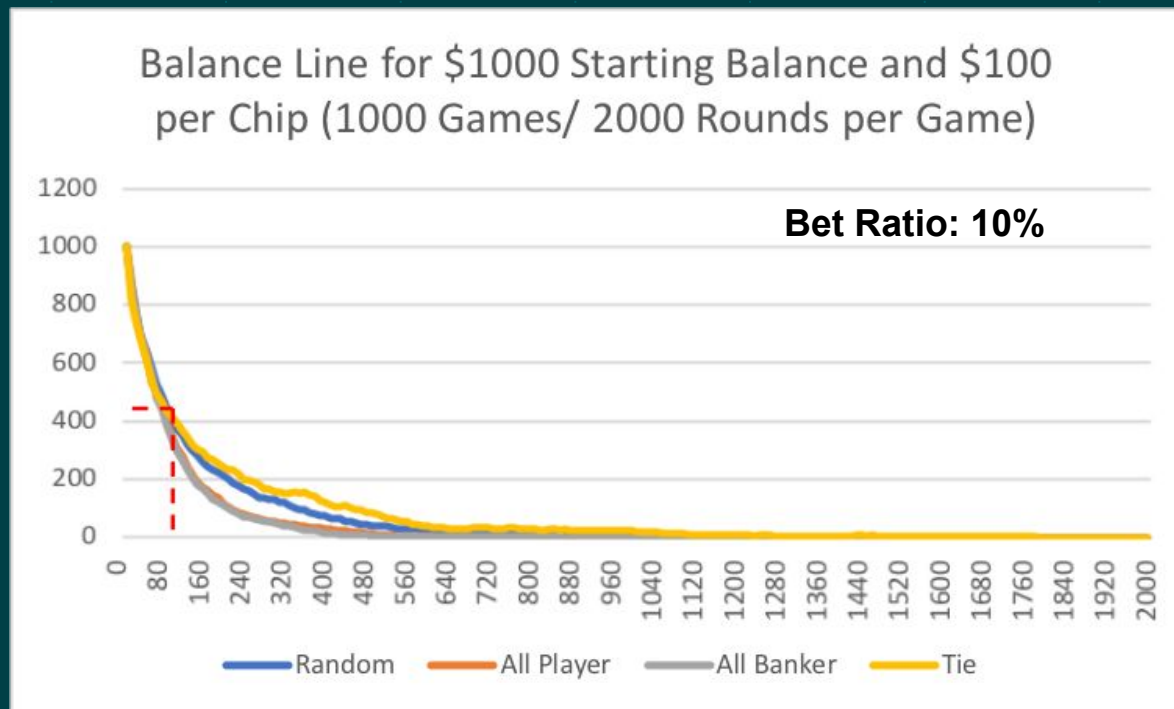
Different Bet Strategies

2.3.3 Comparison of Strategies (5%)



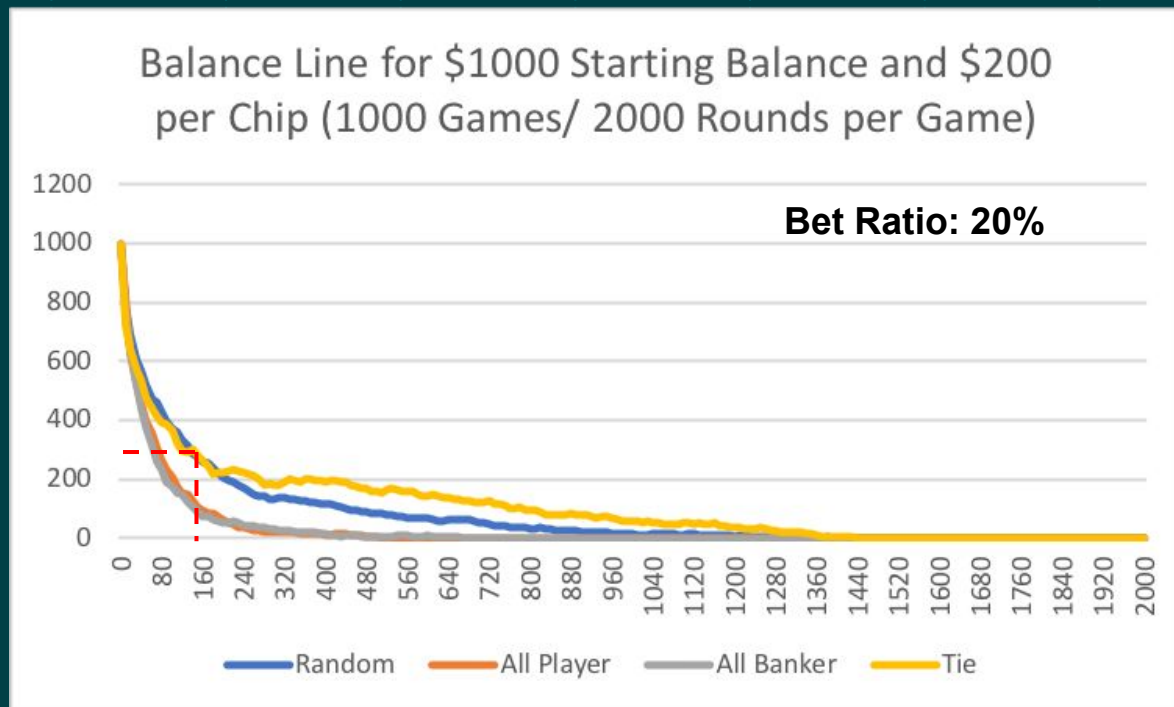
Different Bet Strategies

2.3.4 Comparison of Strategies (10%)



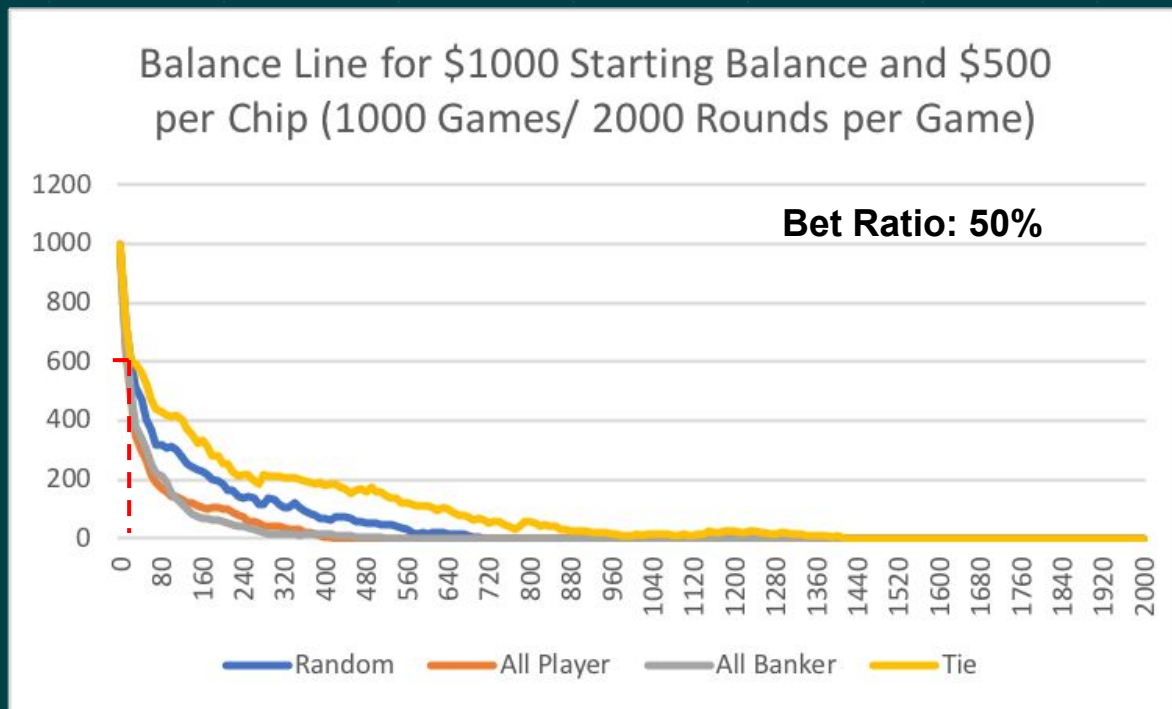
Different Bet Strategies

2.3.5 Comparison of Strategies (20%)



Different Bet Strategies

2.3.6 Comparison of Strategies (50%)



Possibility of Earning Money

Kelly Criterion

$$f^* = \frac{bp - q}{b} = \frac{p(b + 1) - 1}{b}$$

Kelly criterion is a formula used to determine the optimal size of a series of bets in order to maximise the logarithm of wealth

- f^* : how much to bet
- b : you could win \$ b (on top of getting back your \$1 wagered) for a \$1 bet
- p : probability of winning
- q is the probability of losing, which is $1 - p$.
- Banker: -0.1057
- Planer: -0.1075
- Tie: -0.1436



Reference:

1. https://en.wikipedia.org/wiki/Kelly_criterion

Possibility of Earning Money



2.4.1 Random Bet

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	40.48%	20.70%	10.18%	4.65%	2.25%	1.10%	0.59%	0.32%	0.16%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	57.89%	40.46%	28.88%	20.50%	14.30%	10.18%	7.08%	4.98%	3.44%	2.44%	0.28%	0.04%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%
5%	63.17%	55.50%	48.92%	41.87%	35.10%	30.30%	26.62%	23.35%	20.12%	17.60%	8.97%	4.41%	1.99%	1.02%	0.47%	0.18%	0.01%	0.00%
10%	63.35%	53.11%	48.43%	45.14%	42.26%	39.35%	36.01%	32.41%	28.95%	26.10%	17.75%	11.44%	8.02%	5.33%	3.53%	2.54%	0.43%	0.07%
20%	58.08%	49.17%	42.74%	37.51%	36.32%	33.77%	33.66%	32.04%	31.65%	30.09%	24.55%	16.70%	13.72%	10.70%	8.95%	7.06%	2.77%	1.13%
50%	44.56%	44.56%	44.56%	44.56%	33.88%	26.06%	26.06%	26.06%	26.02%	20.65%	17.10%	15.79%	15.32%	15.16%	14.63%	12.76%	6.82%	4.22%

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	0.44528	0.2484	0.13234	0.0651	0.03375	0.0176	0.01003	0.00576	0.00304	0.001	0	0	0	0	0	0	0	0
2%	0.63679	0.48552	0.37544	0.287	0.2145	0.16288	0.12036	0.08964	0.06536	0.0488	0.007	0.0012	0.0007	0	0	0	0	0
5%	0.69487	0.666	0.63596	0.58618	0.5265	0.4848	0.45254	0.4203	0.38228	0.352	0.22425	0.1323	0.06965	0.0408	0.02115	0.009	0.00075	0
10%	0.69685	0.63732	0.62959	0.63196	0.6339	0.6296	0.61217	0.58338	0.55005	0.522	0.44375	0.3432	0.2807	0.2132	0.15885	0.127	0.03225	0.007
20%	0.63888	0.59004	0.55562	0.52514	0.5448	0.54032	0.57222	0.57672	0.60135	0.6018	0.61375	0.501	0.4802	0.428	0.40275	0.353	0.20775	0.113
50%	0.49016	0.53472	0.57928	0.62384	0.5082	0.41696	0.44302	0.46908	0.49438	0.413	0.4275	0.4737	0.5362	0.6064	0.65835	0.638	0.5115	0.422

Settings:

- 1,000 rounds in a game
- 10,000 times game
- bet ratio: 1%, 2%, 5%, 10%, 20%, 50%

Possibility of Earning Money

2.4.1 Bet on Player Only

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	11.34%	1.25%	0.15%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	33.30%	11.35%	3.71%	1.34%	0.43%	0.12%	0.05%	0.03%	0.03%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5%	63.86%	41.52%	27.25%	18.00%	11.54%	7.58%	5.16%	3.43%	2.26%	1.40%	0.19%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10%	78.45%	62.55%	49.08%	39.34%	30.98%	24.81%	20.08%	16.39%	13.05%	10.21%	3.43%	1.06%	0.26%	0.04%	0.00%	0.00%	0.00%	0.00%
20%	73.45%	73.45%	54.85%	54.85%	41.86%	41.86%	32.34%	32.34%	25.21%	25.21%	12.50%	7.72%	3.96%	2.70%	1.38%	0.89%	0.04%	0.01%
50%	59.73%	59.73%	59.73%	59.73%	59.73%	39.51%	39.51%	39.51%	39.51%	39.51%	27.91%	20.70%	15.68%	11.66%	9.04%	7.01%	2.15%	0.85%

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	0.12474	0.015	0.00195	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2%	0.3663	0.1362	0.04823	0.01876	0.00645	0.00192	0.00085	0.00054	0.00057	0.0004	0	0	0	0	0	0	0	0
5%	0.70246	0.49824	0.35425	0.252	0.1731	0.12128	0.08772	0.06174	0.04294	0.028	0.00475	0.0003	0	0	0	0	0	0
10%	0.86295	0.7506	0.63804	0.55076	0.4647	0.39696	0.34136	0.29502	0.24795	0.2042	0.08575	0.0318	0.0091	0.0016	0	0	0	0
20%	0.80795	0.8814	0.71305	0.7679	0.6279	0.66976	0.54978	0.58212	0.47899	0.5042	0.3125	0.2316	0.1386	0.108	0.0621	0.0445	0.003	0.001
50%	0.65703	0.71676	0.77649	0.83622	0.89595	0.63216	0.67167	0.71118	0.75069	0.7902	0.69775	0.621	0.5488	0.4664	0.4068	0.3505	0.16125	0.085

Settings:

- 1,000 rounds in a game
- 10,000 times game
- bet ratio: 1%, 2%, 5%, 10%, 20%, 50%

Possibility of Earning Money



2.4.1 Bet on Banker Only

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	10.16%	1.25%	0.17%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	29.94%	10.26%	3.04%	0.95%	0.29%	0.08%	0.02%	0.01%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5%	55.06%	36.37%	23.75%	15.18%	9.57%	6.25%	4.04%	2.74%	1.74%	1.01%	0.13%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10%	66.10%	53.41%	42.91%	34.62%	27.77%	22.21%	17.62%	14.11%	11.23%	9.02%	2.85%	0.94%	0.37%	0.12%	0.03%	0.01%	0.00%	0.00%
20%	75.17%	58.63%	57.53%	45.87%	44.28%	36.40%	34.07%	28.81%	26.04%	22.72%	12.63%	7.44%	4.19%	2.36%	1.36%	0.78%	0.04%	0.01%
50%	61.06%	61.06%	61.06%	60.97%	42.02%	42.02%	42.02%	41.92%	40.62%	31.15%	23.42%	17.64%	13.81%	10.88%	8.66%	6.74%	1.84%	0.54%

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	0.11176	0.015	0.00221	0.00028	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2%	0.32934	0.12312	0.03952	0.0133	0.00435	0.00128	0.00034	0.00018	0.00019	0	0	0	0	0	0	0	0	0
5%	0.60566	0.43644	0.30875	0.21252	0.14355	0.1	0.06868	0.04932	0.03306	0.0202	0.00325	0.0006	0	0	0	0	0	0
10%	0.7271	0.64092	0.55783	0.48468	0.41655	0.35536	0.29954	0.25398	0.21337	0.1804	0.07125	0.0282	0.01295	0.0048	0.00135	0.0005	0	0
20%	0.82687	0.70356	0.74789	0.64218	0.6642	0.5824	0.57919	0.51858	0.49476	0.4544	0.31575	0.2232	0.14665	0.0944	0.0612	0.039	0.003	0.001
50%	0.67166	0.73272	0.79378	0.85358	0.6303	0.67232	0.71434	0.75456	0.77178	0.623	0.5855	0.5292	0.48335	0.4352	0.3897	0.337	0.138	0.054

Settings:

- 1,000 rounds in a game
- 10,000 times game
- bet ratio: 1%, 2%, 5%, 10%, 20%, 50%

Possibility of Earning Money



2.4.1 Bet on Tie Only

BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	62.32%	43.08%	29.53%	20.12%	13.41%	9.02%	6.22%	4.40%	2.92%	2.02%	0.25%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2%	73.41%	58.72%	48.33%	39.77%	32.37%	26.38%	21.58%	17.92%	14.81%	12.11%	4.79%	1.72%	0.66%	0.26%	0.12%	0.03%	0.00%	0.00%
5%	70.44%	65.71%	59.99%	52.91%	46.93%	42.83%	38.84%	34.95%	31.62%	28.82%	18.41%	12.30%	8.11%	5.41%	3.62%	2.46%	0.35%	0.06%
10%	59.09%	57.13%	55.07%	52.41%	49.88%	46.63%	43.00%	39.62%	35.39%	33.67%	25.13%	19.11%	14.81%	12.02%	9.24%	7.48%	2.85%	1.12%
20%	40.04%	40.04%	40.04%	40.04%	40.04%	40.04%	40.04%	40.04%	37.49%	37.49%	26.76%	21.99%	18.75%	16.06%	13.29%	11.85%	6.03%	3.43%
50%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	17.72%	13.88%	8.16%	5.23%
BET	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4	4.5	5	7.5	10
1%	0.68552	0.51696	0.38389	0.28168	0.20115	0.14432	0.10574	0.0792	0.05548	0.0404	0.00625	0.0003	0	0	0	0	0	0
2%	0.80751	0.70464	0.62829	0.55678	0.48555	0.42208	0.36686	0.32256	0.28139	0.2422	0.11975	0.0516	0.0231	0.0104	0.0054	0.0015	0	0
5%	0.77484	0.78852	0.77987	0.74074	0.70395	0.68528	0.66028	0.6291	0.60078	0.5764	0.46025	0.369	0.28385	0.2164	0.1629	0.123	0.02625	0.006
10%	0.64999	0.68556	0.71591	0.73374	0.7482	0.74608	0.731	0.71316	0.67241	0.6734	0.62825	0.5733	0.51835	0.4808	0.4158	0.374	0.21375	0.112
20%	0.44044	0.48048	0.52052	0.56056	0.6006	0.64064	0.68068	0.72072	0.71231	0.7498	0.669	0.6597	0.65625	0.6424	0.59805	0.5925	0.45225	0.343
50%	0.19492	0.21264	0.23036	0.24808	0.2658	0.28352	0.30124	0.31896	0.33668	0.3544	0.443	0.5316	0.6202	0.7088	0.7974	0.694	0.612	0.523

Settings:

- 1,000 rounds in a game
- 10,000 times game
- bet ratio: 1%, 2%, 5%, 10%, 20%, 50%

3. Findings



3. Findings

- The results returned from our testing theoretical possibilities shows that they matched empirical possibilities;
- Surprisingly, in a long run, betting on tie can delay a gambler losing all his/her money to the casino;
- There is no strategy that would guarantee (or expect) a gambler would gain money from the casino playing Baccarat in a long run;
- Therefore, **DO NOT** expect to earn money from a casino!!!

4. To-does



4. To do next 🕒

1. Trying to build a master strategy and test it:
 - a. An algorithm can remember all cards played in the past, then make decision based on cards remain in decks.
 - b. possible parameter: number of cards remains in the decks before shuffle
2. Dragon strategy (strategy based on results)
3. Complete the readme in our repository.
4. Add comments and docstrings in our scripts.

141,221,110 rounds of game

That's a lot of trials

399,805,200 results of gambler

And a lot of testers

Hope you like our works

And enjoy this class :)

Thanks!

Any questions?

You can find our project on Github by scanning this QR code

Or contact us via email:

xjiang36@illinois.edu; telin2@illinois.edu



Thanks!

References

- wikiHow: How to Play Baccarat
<https://www.wikihow.com/Play-Baccarat>
- <https://zh.wikipedia.org/wiki/%E7%99%BE%E5%AE%B6%E6%A8%82>
- https://en.wikipedia.org/wiki/Kelly_criterion