Question 1

i. $${}^{A}T_{C_1} = {}^{A}_{A_1}T \cdot {}^{A_1}_{C_1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c90 & -s90 & 0 \\ 0 & s90 & c90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. $${}^{A}T_{C_2} = {}^{A}T_{C_1} \cdot {}^{C_1}_{C_2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c90 & -s90 & 0 \\ 0 & s90 & c90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii $${}^{A_3}T_{C_2} = {}^{A_3}_{A}T \cdot {}^{A}T_{C_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

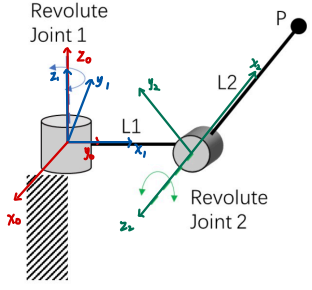
iv. $${}^{E_3}T_{C_2} = {}^{E_3}T_{A_3} \cdot {}^{A_3}T_{C_2} = {}^{E}T_{A} \cdot {}^{A_3}T_{C_2} = {}^{A}T_{E}^{-1} \cdot {}^{A_3}T_{C_2}$$

$$= ({}^{A}T_{B} \cdot {}^{B}T_{E})^{-1} \cdot {}^{A_3}T_{C_2} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v. $${}^{C_2}T_{E_4} = {}^{E_3}T_{C_2}^{-1} \cdot {}^{E_3}T_{E_4} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2

1. Schematic Diagram



2. DH Table

	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	90°	0
2	90°	2m	45°	0
3	0	1m	0	d

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \sqrt{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \sqrt{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-\frac{\sqrt{2}}{2}$$

$${}^{0}_{1}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Homogenous Transformation

$$R_x(\alpha_{i-1})D_x(a_{i-1})R_z(\theta_i)D_z(d_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i \cos\alpha_{i-1} & \cos\alpha_{i-1}\cos\theta_i & -\sin\alpha_{i-1} & -d_i \sin\alpha_{i-1} \\ \sin\alpha_{i-1}\sin\theta_i & \sin\alpha_{i-1}\cos\theta_i & \cos\alpha_{i-1} & d_i\cos\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^{1}_{2}T = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 2 \\ 0 & 0 & -1 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{3}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2}+2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}\tilde{P} = \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2}+2 \\ \frac{\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

$$\Rightarrow {}^{0}P = \left(0, \frac{\sqrt{2}}{2}+2, \frac{\sqrt{2}}{2}\right)$$