Electronic Companion for FO-RDDP

EC.1. Specific Problem Formulation

Consider the piecewise linear fuel cost functions of generators, the robust economic dispatch problem can be written as (EC.1):

$$
\begin{aligned}
&\min_{\boldsymbol{p}(\cdot)} \max \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{N}_g} C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}))t_\delta \\
\text{s.t. }\ & E_{s,t}(\boldsymbol{\xi}_{[t]}) = E_{s,t-1}(\boldsymbol{\xi}_{[t-1]}) + p_{s,t}^{\mathrm{sc}}(\boldsymbol{\xi}_{[t]})\alpha^{\mathrm{sc}} t_\delta - p_{s,t}^{\mathrm{sd}}(\boldsymbol{\xi}_{[t]}) t_\delta/\alpha^{\mathrm{sd}},\ s\in\mathcal{N}_s,\ t\in\mathcal{T} \\
& p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}) - p_{i,t-1}^{\mathrm{g}}(\boldsymbol{\xi}_{[t-1]}) \le \Delta_i^{\mathrm{u}},\ \forall i\in\mathcal{N}_g, t\in\mathcal{T} \\
& p_{i,t-1}^{\mathrm{g}}(\boldsymbol{\xi}_{[t-1]}) - p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}) \le \Delta_i^{\mathrm{d}},\ \forall i\in\mathcal{N}_g, t\in\mathcal{T} \\
& -\bar{G}_\ell \le \sum_{i\in\mathcal{N}_g} \Gamma_{\ell,i}^{\mathrm{g}} p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}) + \sum_{r\in\mathcal{N}_r} \Gamma_{\ell,r}^{\mathrm{r}} \xi_{r,t} + \sum_{s\in\mathcal{N}_s} \Gamma_{\ell,s}^{\mathrm{ss}}(p_{s,t}^{\mathrm{sd}}(\boldsymbol{\xi}_{[t]}) - p_{s,t}^{\mathrm{sc}}(\boldsymbol{\xi}_{[t]})) \\
& \qquad - \sum_{d\in\mathcal{N}_d} \Gamma_{\ell,d}^{\mathrm{dm}} p_{d,t}^{\mathrm{dm}} \le \bar{G}_\ell, \forall \ell\in\mathcal{N}_\ell, t\in\mathcal{T} \\
& \sum_{i\in\mathcal{N}_g} p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}) + \sum_{s\in\mathcal{N}_s} (p_{s,t}^{\mathrm{sd}}(\boldsymbol{\xi}_{[t]}) - p_{s,t}^{\mathrm{sc}}(\boldsymbol{\xi}_{[t]})) + \sum_{r\in\mathcal{N}_r} \xi_{r,t} = \sum_{d\in\mathcal{N}_d} p_{d,t}^{\mathrm{dm}}, \forall t\in\mathcal{T} \\
& x_{i,t}^{\mathrm{g}} \underline{p}_i^{\mathrm{g}} \le p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}) \le x_{i,t}^{\mathrm{g}} \bar{p}_i^{\mathrm{g}},\ \forall i\in\mathcal{N}_g, t\in\mathcal{T} \\
& \underline{E}_s \le E_{s,t}(\boldsymbol{\xi}_{[t]}) \le \bar{E}_s,\ s\in\mathcal{N}_s,\ t\in\mathcal{T} \\
& 0 \le p_{s,t}^{\mathrm{sc}}(\boldsymbol{\xi}_{[t]}) \le \bar{p}_s^{\mathrm{sc}},\ s\in\mathcal{N}_s,\ t\in\mathcal{T} \\
& 0 \le p_{s,t}^{\mathrm{sd}}(\boldsymbol{\xi}_{[t]}) \le \bar{p}_s^{\mathrm{sd}},\ s\in\mathcal{N}_s,\ t\in\mathcal{T} \\
& C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]})) \ge \theta_{i,a}^1 p_{i,t}^{\mathrm{g}}(\boldsymbol{\xi}_{[t]}) + \theta_{i,a}^2 x_{i,t}^{\mathrm{g}},\ a = 1,\cdots,A.
\end{aligned} \tag{EC.1}
$$

where $\theta_{i,a}^1, \theta_{i,a}^2$ are the piecewise linear parameters for generator i, A is the number of the pieces for the fuel cost functions.

Assuming the uncertainty is stagewise independent, the t-stage upper-bound problem is formulated as (EC.2):

$$\bar{F}_t(\boldsymbol{y}_{t-1}) = \max_{\boldsymbol{\xi}_t\in\Xi_t} \min_{\boldsymbol{y}_t} \sum_{i\in\mathcal{N}_g} C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}})t_\delta + \tau_t \tag{EC.2a}$$

$$\text{s.t. }\ E_{s,t} = E_{s,t-1} + p_{s,t}^{\mathrm{sc}}\alpha^{\mathrm{sc}} t_\delta - p_{s,t}^{\mathrm{sd}} t_\delta/\alpha^{\mathrm{sd}},\ s\in\mathcal{N}_s, \tag{EC.2b}$$

$$p_{i,t}^{\mathrm{g}} - p_{i,t-1}^{\mathrm{g}} \le \Delta_i^{\mathrm{u}},\ i\in\mathcal{N}_g, \tag{EC.2c}$$

$$p_{i,t-1}^{\mathrm{g}} - p_{i,t}^{\mathrm{g}} \le \Delta_i^{\mathrm{d}},\ i\in\mathcal{N}_g, \tag{EC.2d}$$

$$-\bar{G}_\ell \le \sum_{i\in\mathcal{N}_g} \Gamma_{\ell,i}^{\mathrm{g}} p_{i,t}^{\mathrm{g}} + \sum_{r\in\mathcal{N}_r} \Gamma_{\ell,r}^{\mathrm{r}} \xi_{r,t} + \sum_{s\in\mathcal{N}_s} \Gamma_{\ell,s}^{\mathrm{ss}}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) - \sum_{d\in\mathcal{N}_d} \Gamma_{\ell,d}^{\mathrm{dm}} p_{d,t}^{\mathrm{dm}} \le \bar{G}_\ell, \ell\in\mathcal{N}_\ell, \tag{EC.2e}$$

$$\sum_{i\in\mathcal{N}_g} p_{i,t}^{\mathrm{g}} + \sum_{s\in\mathcal{N}_s} (p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) + \sum_{r\in\mathcal{N}_r} \xi_{r,t} = \sum_{d\in\mathcal{N}_d} p_{d,t}^{\mathrm{dm}}, \tag{EC.2f}$$

$$x_{i,t}^{\mathrm{g}}\underline{p}_{i}^{\mathrm{g}} \leq p_{i,t}^{\mathrm{g}} \leq x_{i,t}^{\mathrm{g}}\bar{p}_{i}^{\mathrm{g}},\ i \in \mathcal{N}_g, \tag{EC.2g}$$

$$\underline{E}_s \leq E_{s,t} \leq \bar{E}_s,\ s \in \mathcal{N}_s, \tag{EC.2h}$$

$$0 \leq p_{s,t}^{\mathrm{sc}} \leq \bar{p}_{s}^{\mathrm{sc}},\ s \in \mathcal{N}_s, \tag{EC.2i}$$

$$0 \leq p_{s,t}^{\mathrm{sd}} \leq \bar{p}_{s}^{\mathrm{sd}},\ s \in \mathcal{N}_s, \tag{EC.2j}$$

$$C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}}) \geq \theta_{i,a}^{1}p_{i,t}^{\mathrm{g}} + \theta_{i,a}^{2}x_{i,t}^{\mathrm{g}},\ a = 1, \cdots, A. \tag{EC.2k}$$

$$p_{i,t}^{\mathrm{g}} = \sum\nolimits_{j} \vartheta_j p_{i,t}^{\mathrm{g},j},\ i \in \mathcal{N}_g, \tag{EC.2l}$$

$$E_{s,t} = \sum\nolimits_{j} \vartheta_j E_{s,t}^{j},\ s \in \mathcal{N}_s, \tag{EC.2m}$$

$$\tau_t - \sum\nolimits_{j} \vartheta_j \bar{\mathcal{F}}_{t+1}(p_{i,t}^{\mathrm{g},j}, E_{s,t}^{j}) = 0, \tag{EC.2n}$$

$$\vartheta_j \geq 0, \sum\nolimits_{j} \vartheta_j = 1,\ j = 1, \cdots, J_t, \tag{EC.2o}$$

$$\underline{\xi}_{r,t} \leq \xi_{r,t} \leq \bar{\xi}_{r,t},\ r \in \mathcal{N}_r. \tag{EC.2p}$$

where (EC.2l)-(EC.2o) stand for the inner approximation method used in the upper-bound problem, J_t is the number of candidate points at stage t.

Using the outer approximation method, the t-stage lower-bound problem is written as (EC.3):

$$\underline{F}_t(\boldsymbol{y}_{t-1}, \boldsymbol{\xi}_t) = \min \sum_{i \in \mathcal{N}_g} C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}})t_\delta + \tau_t \tag{EC.3a}$$

$$\text{s.t. } E_{s,t} = E_{s,t-1} + p_{s,t}^{\mathrm{sc}}\alpha^{\mathrm{sc}}t_\delta - p_{s,t}^{\mathrm{sd}}t_\delta/\alpha^{\mathrm{sd}},\ s \in \mathcal{N}_s, \tag{EC.3b}$$

$$p_{i,t}^{\mathrm{g}} = p_{i,t-1}^{\mathrm{g}} + p_{i,t}^{\mathrm{ramp}},\ i \in \mathcal{N}_g, \tag{EC.3c}$$

$$-\Delta_i^{\mathrm{d}} \leq p_{i,t}^{\mathrm{ramp}} \leq \Delta_i^{\mathrm{u}},\ i \in \mathcal{N}_g, \tag{EC.3d}$$

$$-\bar{G}_\ell \leq \sum_{i \in \mathcal{N}_g} \Gamma_{\ell,i}^{\mathrm{g}}p_{i,t}^{\mathrm{g}} + \sum_{r \in \mathcal{N}_r} \Gamma_{\ell,r}^{\mathrm{r}}\xi_{r,t} + \sum_{s \in \mathcal{N}_s} \Gamma_{\ell,s}^{\mathrm{ss}}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) - \sum_{d \in \mathcal{N}_d} \Gamma_{\ell,d}^{\mathrm{dm}}p_{d,t}^{\mathrm{dm}} \leq \bar{G}_\ell, \ell \in \mathcal{N}_\ell, \tag{EC.3e}$$

$$\sum_{i \in \mathcal{N}_g} p_{i,t}^{\mathrm{g}} + \sum_{s \in \mathcal{N}_s}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) + \sum_{r \in \mathcal{N}_r} \xi_{r,t} = \sum_{d \in \mathcal{N}_d} p_{d,t}^{\mathrm{dm}}, \tag{EC.3f}$$

$$x_{i,t}^{\mathrm{g}}\underline{p}_{i}^{\mathrm{g}} \leq p_{i,t}^{\mathrm{g}} \leq x_{i,t}^{\mathrm{g}}\bar{p}_{i}^{\mathrm{g}},\ i \in \mathcal{N}_g, \tag{EC.3g}$$

$$\underline{E}_s \leq E_{s,t} \leq \bar{E}_s,\ s \in \mathcal{N}_s, \tag{EC.3h}$$

$$0 \leq p_{s,t}^{\mathrm{sc}} \leq \bar{p}_{s}^{\mathrm{sc}},\ s \in \mathcal{N}_s, \tag{EC.3i}$$

$$0 \leq p_{s,t}^{\mathrm{sd}} \leq \bar{p}_{s}^{\mathrm{sd}},\ s \in \mathcal{N}_s, \tag{EC.3j}$$

$$C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}}) \geq \theta_{i,a}^{1}p_{i,t}^{\mathrm{g}} + \theta_{i,a}^{2}x_{i,t}^{\mathrm{g}},\ a = 1, \cdots, A. \tag{EC.3k}$$

$$\tau_t \geq \sum_{i=1}^{N_g} \beta_{i,\kappa}^{1}p_{i,t}^{\mathrm{g}} + \sum_{s=1}^{N_s} \beta_{s,\kappa}^{2}E_{s,t} + \beta_\kappa^3, \kappa = 1, \cdots, \mathcal{K}. \tag{EC.3l}$$

where (EC.3l) stands for the supporting hyperplane based future-stage cost-to-go function, $\beta_{i,\kappa}^{1}, \beta_{s,\kappa}^{2}, \beta_\kappa^3$ represent the parameters for the hyperplanes, $\mathcal{K}$ is the number of the hyperplanes.

EC.2. PDBO Steps

Algorithm 1 Primal-Dual Bilevel Optimizer (PDBO)

Require: $\boldsymbol{\varphi}_t^{(1)}$, $\boldsymbol{z}_t^{(1)}$, θ, $\tau^{(1)}$, $\rho^{(1)}$

Ensure: $\boldsymbol{\varphi}_t^{(k)}$, $\boldsymbol{z}_t^{(k)}$

for $k = 1$ to K **do**

   Obtain $g^*(\boldsymbol{\xi}_t^{(k)})$ by solving (EC.7).

   $\boldsymbol{z}_t^+ = \boldsymbol{z}_t^{(k)} + \theta(\boldsymbol{z}_t^{(k)} - \boldsymbol{z}_t^{(k-1)})$.

   $\boldsymbol{\varphi}_t^{(k+1)} = \Pi_\Phi(\boldsymbol{\varphi}_t^{(k)} + \tau^{(k)}\nabla\mathcal{L}_{\boldsymbol{\varphi}_t}(\boldsymbol{\varphi}_t; \boldsymbol{z}_t^+))$.

   $\boldsymbol{z}_t^{(k+1)} = \Pi_{\mathcal{Z}}(\boldsymbol{z}_t^{(k)} - \rho^{(k)}\nabla\mathcal{L}_{\boldsymbol{z}_t}(\boldsymbol{\varphi}_t^{(k+1)}; \boldsymbol{z}_t))$.

   $\tau^{(k+1)} = \beta_1\tau^{(k)}$, $\rho^{(k+1)} = \beta_2\rho^{(k)}$.

end for

return $\boldsymbol{\varphi}_t^{(k)}$, $\boldsymbol{z}_t^{(k)}$

To solve the t-stage upper-bound problem using the primal-dual bilevel optimizer (PDBO) as the tabulated Algorithm 1, one needs to write out the primal-dual form for (EC.2) as:

$$
\begin{aligned}
&\mathcal{L}(\lambda_t, \boldsymbol{\pi}_t, \hat{\boldsymbol{y}}_t, \boldsymbol{\xi}_t) = -(\sum_{i\in\mathcal{N}_g} C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}})t_\delta + \tau_t) + \lambda_t[\sum_{i\in\mathcal{N}_g} C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}})t_\delta + \tau_t - g^*(\boldsymbol{\xi}_t) - \delta] \\
&+ \sum_{s\in\mathcal{N}_s}(E_{s,t} - E_{s,t-1} - p_{s,t}^{\mathrm{sc}}\alpha^{\mathrm{sc}}t_\delta + p_{s,t}^{\mathrm{sd}}t_\delta/\alpha^{\mathrm{sd}})\pi_s^{\mathrm{e}} \\
&+ \sum_{i\in\mathcal{N}_g}(p_{i,t}^{\mathrm{g}} - p_{i,t-1}^{\mathrm{g}} - \Delta_i^{\mathrm{u}})\pi_i^{\mathrm{u}} + \sum_{i\in\mathcal{N}_g}(p_{i,t-1}^{\mathrm{g}} - p_{i,t}^{\mathrm{g}} - \Delta_i^{\mathrm{d}})\pi_i^{\mathrm{d}} \\
&+ \sum_{\ell\in\mathcal{N}_\ell}(-\bar{G}_\ell - \sum_{i\in\mathcal{N}_g}\Gamma_{\ell,i}^{\mathrm{g}}p_{i,t}^{\mathrm{g}} - \sum_{r\in\mathcal{N}_r}\Gamma_{\ell,r}^{\mathrm{r}}\xi_{r,t} - \sum_{s\in\mathcal{N}_s}\Gamma_{\ell,s}^{\mathrm{ss}}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) + \sum_{d\in\mathcal{N}_d}\Gamma_{\ell,d}^{\mathrm{dm}}p_{d,t}^{\mathrm{dm}})\pi_\ell^{\mathrm{G}-} \\
&+ \sum_{\ell\in\mathcal{N}_\ell}(\sum_{i\in\mathcal{N}_g}\Gamma_{\ell,i}^{\mathrm{g}}p_{i,t}^{\mathrm{g}} + \sum_{r\in\mathcal{N}_r}\Gamma_{\ell,r}^{\mathrm{r}}\xi_{r,t} + \sum_{s\in\mathcal{N}_s}\Gamma_{\ell,s}^{\mathrm{ss}}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) - \sum_{d\in\mathcal{N}_d}\Gamma_{\ell,d}^{\mathrm{dm}}p_{d,t}^{\mathrm{dm}} - \bar{G}_\ell)\pi_\ell^{\mathrm{G}+} \\
&+ (\sum_{i\in\mathcal{N}_g}p_{i,t}^{\mathrm{g}} + \sum_{s\in\mathcal{N}_s}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) + \sum_{r\in\mathcal{N}_r}\xi_{r,t} - \sum_{d\in\mathcal{N}_d}p_{d,t}^{\mathrm{dm}})\pi^{\mathrm{ba}} \\
&+ \sum_{i\in\mathcal{N}_g}(x_{i,t}^{\mathrm{g}}\underline{p}_i^{\mathrm{g}} - p_{i,t}^{\mathrm{g}})\pi_i^{\mathrm{g}-} + \sum_{i\in\mathcal{N}_g}(p_{i,t}^{\mathrm{g}} - x_{i,t}^{\mathrm{g}}\bar{p}_i^{\mathrm{g}})\pi_i^{\mathrm{g}+} \\
&+ \sum_{s\in\mathcal{N}_s}(\underline{E}_s - E_{s,t})\pi_s^{\mathrm{e}-} + \sum_{s\in\mathcal{N}_s}(E_{s,t} - \bar{E}_s)\pi_s^{\mathrm{e}+} \\
&+ \sum_{s\in\mathcal{N}_s}(-p_{s,t}^{\mathrm{sc}})\pi_s^{\mathrm{sc}-} + \sum_{s\in\mathcal{N}_s}(p_{s,t}^{\mathrm{sc}} - \bar{p}_s^{\mathrm{sc}})\pi_s^{\mathrm{sc}+} + \sum_{s\in\mathcal{N}_s}(-p_{s,t}^{\mathrm{sd}})\pi_s^{\mathrm{sd}-} + \sum_{s\in\mathcal{N}_s}(p_{s,t}^{\mathrm{sd}} - \bar{p}_s^{\mathrm{sd}})\pi_s^{\mathrm{sd}+} \\
&+ \sum_{a=1}^{A}\sum_{i\in\mathcal{N}_g}(-C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}}) + \theta_{i,a}^1 p_{i,t}^{\mathrm{g}} + \theta_{i,a}^2 x_{i,t}^{\mathrm{g}})\pi_{i,a}^{\mathrm{C}} + \sum_{i\in\mathcal{N}_g}(p_{i,t}^{\mathrm{g}} - \textstyle\sum_j \vartheta_j p_{i,t}^{\mathrm{g},j})\pi_i^{\mathrm{ccg}}
\end{aligned}
$$

$$+\sum_{s\in\mathcal{N}_s}(E_{s,t}-\sum\nolimits_j \vartheta_j E^j_{s,t})\pi^{\mathrm{cce}}_s+(\tau_t-\sum\nolimits_j \vartheta_j\bar{\mathcal{F}}_{t+1}(p^{\mathrm{g},j}_{i,t},E^j_{s,t}))\pi^{\mathrm{cco}}+(\sum\nolimits_j \vartheta_j-1)\pi^{\mathrm{ccj}}$$

$$+\sum_{r\in\mathcal{N}_r}(\underline{\xi}_{r,t}-\xi_{r,t})\pi^{\mathrm{xi}-}_r+\sum_{r\in\mathcal{N}_r}(\xi_{r,t}-\bar{\xi}_{r,t})\pi^{\mathrm{xi}+}_r \tag{EC.4}$$

where $\boldsymbol{\pi}_t=[\pi^{\mathrm{e}}_s;\pi^{\mathrm{u}}_i;\pi^{\mathrm{d}}_i;\pi^{\mathrm{G}-}_\ell;\pi^{\mathrm{G}+}_\ell;\pi^{\mathrm{ba}};\pi^{\mathrm{g}-}_i;\pi^{\mathrm{g}+}_i;\pi^{\mathrm{e}-}_s;\pi^{\mathrm{e}+}_s;\pi^{\mathrm{sc}-}_s;\pi^{\mathrm{sc}+}_s;\pi^{\mathrm{sd}-}_s;\pi^{\mathrm{sd}+}_s;\pi^{\mathrm{C}}_{i,a};\pi^{\mathrm{ccg}}_i;$ $\pi^{\mathrm{cce}}_s;\pi^{\mathrm{cco}};\pi^{\mathrm{ccj}};\pi^{\mathrm{xi}-}_r;\pi^{\mathrm{xi}+}_r]$ denotes the dual variable vector for the constraints in (EC.2), $\pi^{\mathrm{u}}_i\geq 0,\pi^{\mathrm{d}}_i\geq 0,\pi^{\mathrm{G}-}_\ell\geq 0,\pi^{\mathrm{G}+}_\ell\geq 0,\pi^{\mathrm{g}-}_i\geq 0,\pi^{\mathrm{g}+}_i\geq 0,\pi^{\mathrm{e}-}_s\geq 0,\pi^{\mathrm{e}+}_s\geq 0,\pi^{\mathrm{sc}-}_s\geq 0,\pi^{\mathrm{sc}+}_s\geq 0,\pi^{\mathrm{sd}-}_s\geq 0,\pi^{\mathrm{sd}+}_s\geq 0,\pi^{\mathrm{C}}_{i,a}\geq 0,\pi^{\mathrm{xi}-}_r\geq 0,\pi^{\mathrm{xi}+}_r\geq 0$, $\hat{\boldsymbol{y}}_t=[C^{\mathrm{g}}_i(p^{\mathrm{g}}_{i,t});p^{\mathrm{g}}_{i,t};E_{s,t};p^{\mathrm{sc}}_{s,t};p^{\mathrm{sd}}_{s,t};\tau_t;\vartheta_j]$ stands for the primal variable vector for the inner problem of (EC.2).

Then, the gradients of $\mathcal{L}(\lambda_t,\boldsymbol{\pi}_t,\hat{\boldsymbol{y}}_t,\boldsymbol{\xi}_t)$ can be calculated as:

$$\nabla\mathcal{L}_{\lambda_t}=\sum_{i\in\mathcal{N}_g}C^{\mathrm{g}}_i(p^{\mathrm{g}}_{i,t})t_\delta+\tau_t-g^*(\boldsymbol{\xi}_t)-\delta,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{e}}_s}=E_{s,t}-E_{s,t-1}-p^{\mathrm{sc}}_{s,t}\alpha^{\mathrm{sc}}t_\delta+p^{\mathrm{sd}}_{s,t}t_\delta/\alpha^{\mathrm{sd}},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{u}}_i}=p^{\mathrm{g}}_{i,t}-p^{\mathrm{g}}_{i,t-1}-\Delta^{\mathrm{u}}_i,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{d}}_i}=p^{\mathrm{g}}_{i,t-1}-p^{\mathrm{g}}_{i,t}-\Delta^{\mathrm{d}}_i,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{G}-}_\ell}=-\bar{G}_\ell-\sum_{i\in\mathcal{N}_g}\Gamma^{\mathrm{g}}_{\ell,i}p^{\mathrm{g}}_{i,t}-\sum_{r\in\mathcal{N}_r}\Gamma^{\mathrm{r}}_{\ell,r}\xi_{r,t}-\sum_{s\in\mathcal{N}_s}\Gamma^{\mathrm{ss}}_{\ell,s}(p^{\mathrm{sd}}_{s,t}-p^{\mathrm{sc}}_{s,t})+\sum_{d\in\mathcal{N}_d}\Gamma^{\mathrm{dm}}_{\ell,d}p^{\mathrm{dm}}_{d,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{G}+}_\ell}=\sum_{i\in\mathcal{N}_g}\Gamma^{\mathrm{g}}_{\ell,i}p^{\mathrm{g}}_{i,t}+\sum_{r\in\mathcal{N}_r}\Gamma^{\mathrm{r}}_{\ell,r}\xi_{r,t}+\sum_{s\in\mathcal{N}_s}\Gamma^{\mathrm{ss}}_{\ell,s}(p^{\mathrm{sd}}_{s,t}-p^{\mathrm{sc}}_{s,t})-\sum_{d\in\mathcal{N}_d}\Gamma^{\mathrm{dm}}_{\ell,d}p^{\mathrm{dm}}_{d,t}-\bar{G}_\ell,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{ba}}}=\sum_{i\in\mathcal{N}_g}p^{\mathrm{g}}_{i,t}+\sum_{s\in\mathcal{N}_s}(p^{\mathrm{sd}}_{s,t}-p^{\mathrm{sc}}_{s,t})+\sum_{r\in\mathcal{N}_r}\xi_{r,t}-\sum_{d\in\mathcal{N}_d}p^{\mathrm{dm}}_{d,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{g}-}_i}=x^{\mathrm{g}}_{i,t}\underline{p}^{\mathrm{g}}_i-p^{\mathrm{g}}_{i,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{g}+}_i}=p^{\mathrm{g}}_{i,t}-x^{\mathrm{g}}_{i,t}\bar{p}^{\mathrm{g}}_i,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{e}-}_s}=\underline{E}_s-E_{s,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{e}+}_s}=E_{s,t}-\bar{E}_s,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{sc}-}_s}=-p^{\mathrm{sc}}_{s,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{sc}+}_s}=p^{\mathrm{sc}}_{s,t}-\bar{p}^{\mathrm{sc}}_s,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{sd}-}_s}=-p^{\mathrm{sd}}_{s,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{sd}+}_s}=p^{\mathrm{sd}}_{s,t}-\bar{p}^{\mathrm{sd}}_s,$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{C}}_{i,a}}=-C^{\mathrm{g}}_i(p^{\mathrm{g}}_{i,t})+\theta^1_{i,a}p^{\mathrm{g}}_{i,t}+\theta^2_{i,a}x^{\mathrm{g}}_{i,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{ccg}}_i}=p^{\mathrm{g}}_{i,t}-\sum\nolimits_j \vartheta_j p^{\mathrm{g},j}_{i,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{cce}}_s}=E_{s,t}-\sum\nolimits_j \vartheta_j E^j_{s,t},$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{cco}}}=\tau_t-\sum\nolimits_j \vartheta_j\bar{\mathcal{F}}_{t+1}(p^{\mathrm{g},j}_{i,t},E^j_{s,t}),$$

$$\nabla\mathcal{L}_{\pi^{\mathrm{ccj}}}=\sum\nolimits_j\vartheta_j-1,$$
$$\nabla\mathcal{L}_{\pi_r^{\mathrm{xi}-}}=\underline{\xi}_{r,t}-\xi_{r,t},$$
$$\nabla\mathcal{L}_{\pi_r^{\mathrm{xi}+}}=\xi_{r,t}-\bar{\xi}_{r,t}, \tag{EC.5}$$
$$\nabla\mathcal{L}_{C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}})}=(-1+\lambda_t)t_\delta+\sum_{a=1}^{A}-\pi_{i,a}^{\mathrm{C}},$$
$$\nabla\mathcal{L}_{p_{i,t}^{\mathrm{g}}}=\pi_i^{\mathrm{u}}-\pi_i^{\mathrm{d}}-\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,i}^{\mathrm{g}}\pi_\ell^{\mathrm{G}-}+\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,i}^{\mathrm{g}}\pi_\ell^{\mathrm{G}+}+\pi^{\mathrm{ba}}-\pi_i^{\mathrm{g}-}+\pi_i^{\mathrm{g}+}+\sum_{a=1}^{A}\theta_{i,a}^{1}\pi_{i,a}^{\mathrm{C}}+\pi_i^{\mathrm{ccg}},$$
$$\nabla\mathcal{L}_{E_{s,t}}=\pi_s^{\mathrm{e}}-\pi_s^{\mathrm{e}-}+\pi_s^{\mathrm{e}+}+\pi_s^{\mathrm{cce}},$$
$$\nabla\mathcal{L}_{p_{s,t}^{\mathrm{sc}}}=-\alpha^{\mathrm{sc}}t_\delta\pi_s^{\mathrm{e}}+\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,s}^{\mathrm{ss}}\pi_\ell^{\mathrm{G}-}-\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,s}^{\mathrm{ss}}\pi_\ell^{\mathrm{G}+}-\pi^{\mathrm{ba}}-\pi_s^{\mathrm{sc}-}+\pi_s^{\mathrm{sc}+},$$
$$\nabla\mathcal{L}_{\boldsymbol{p}_{s,t}^{\mathrm{sd}}}=t_\delta/\alpha^{\mathrm{sd}}\pi_s^{\mathrm{e}}-\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,s}^{\mathrm{ss}}\pi_\ell^{\mathrm{G}-}+\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,s}^{\mathrm{ss}}\pi_\ell^{\mathrm{G}+}+\pi^{\mathrm{ba}}-\pi_s^{\mathrm{sd}-}+\pi_s^{\mathrm{sd}+},$$
$$\nabla\mathcal{L}_{\tau_t}=-1+\lambda_t+\pi^{\mathrm{cco}},$$
$$\nabla\mathcal{L}_{\vartheta_j}=-\sum_{i\in\mathcal{N}_g}p_{i,t}^{\mathrm{g},j}\pi_i^{\mathrm{ccg}}-\sum_{s\in\mathcal{N}_s}E_{s,t}^{j}\pi_s^{\mathrm{cce}}-\bar{\mathcal{F}}_{t+1}(p_{i,t}^{\mathrm{g},j},E_{s,t}^{j})\pi^{\mathrm{cco}}+\pi^{\mathrm{ccj}},$$
$$\nabla\mathcal{L}_{\xi_{r,t}}=-\lambda_t\nabla g_{\boldsymbol{\xi}_t,r}^{*}-\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,r}^{\mathrm{r}}\pi_\ell^{\mathrm{G}-}+\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,r}^{\mathrm{r}}\pi_\ell^{\mathrm{G}+}+\pi^{\mathrm{ba}}-\pi_r^{\mathrm{xi}-}+\pi_r^{\mathrm{xi}+}, \tag{EC.6}$$

where $\nabla g^*_{\boldsymbol{\xi}_t}$ represents the gradient of $g^*(\boldsymbol{\xi}_t)$ on $\boldsymbol{\xi}_t$, $\nabla g^*_{\boldsymbol{\xi}_t,r}$ is the r-th component of $\nabla g^*_{\boldsymbol{\xi}_t}$. $g^*(\boldsymbol{\xi}_t)$ and $\nabla g^*_{\boldsymbol{\xi}_t}$ can be obtained by solving (EC.7), which is the dual of the inner problem in (EC.2).

$$\begin{aligned}
g^*(\boldsymbol{\xi}_t)=\max&\sum_{s\in\mathcal{N}_s}E_{s,t-1}\gamma_s^{\mathrm{e}}+\sum_{i\in\mathcal{N}_g}-(p_{i,t-1}^{\mathrm{g}}+\Delta_i^{\mathrm{u}})\gamma_i^{\mathrm{u}}+\sum_{i\in\mathcal{N}_g}(p_{i,t-1}^{\mathrm{g}}-\Delta_i^{\mathrm{d}})\gamma_i^{\mathrm{d}}\\
&+\sum_{\ell\in\mathcal{N}_\ell}(-\bar{G}_\ell-\sum_{r\in\mathcal{N}_r}\Gamma_{\ell,r}^{\mathrm{r}}\xi_{r,t}+\sum_{d\in\mathcal{N}_d}\Gamma_{\ell,d}^{\mathrm{dm}}p_{d,t}^{\mathrm{dm}})\gamma_\ell^{\mathrm{G}-}\\
&+\sum_{\ell\in\mathcal{N}_\ell}(\sum_{r\in\mathcal{N}_r}\Gamma_{\ell,r}^{\mathrm{r}}\xi_{r,t}-\sum_{d\in\mathcal{N}_d}\Gamma_{\ell,d}^{\mathrm{dm}}p_{d,t}^{\mathrm{dm}}-\bar{G}_\ell)\gamma_\ell^{\mathrm{G}+}\\
&+(-\sum_{r\in\mathcal{N}_r}\xi_{r,t}+\sum_{d\in\mathcal{N}_d}p_{d,t}^{\mathrm{dm}})\gamma^{\mathrm{ba}}+\sum_{i\in\mathcal{N}_g}x_{i,t}^{\mathrm{g}}\underline{p}_i^{\mathrm{g}}\gamma_i^{\mathrm{g}-}-\sum_{i\in\mathcal{N}_g}x_{i,t}^{\mathrm{g}}\bar{p}_i^{\mathrm{g}}\gamma_i^{\mathrm{g}+}\\
&+\sum_{s\in\mathcal{N}_s}\underline{E}_s\gamma_s^{\mathrm{e}-}-\sum_{s\in\mathcal{N}_s}\bar{E}_s\gamma_s^{\mathrm{e}+}-\sum_{s\in\mathcal{N}_s}\bar{p}_s^{\mathrm{sc}}\gamma_s^{\mathrm{sc}+}-\sum_{s\in\mathcal{N}_s}\bar{p}_s^{\mathrm{sd}}\gamma_s^{\mathrm{sd}+}+\sum_{a=1}^{A}\sum_{i\in\mathcal{N}_g}\theta_{i,a}^{2}x_{i,t}^{\mathrm{g}}\gamma_{i,a}^{\mathrm{C}}+\gamma^{\mathrm{ccj}}\\
\text{s.t.}\quad&-t_\delta+\sum_{a=1}^{A}\gamma_{i,a}^{\mathrm{C}}=0,\\
&-\gamma_i^{\mathrm{u}}+\gamma_i^{\mathrm{d}}+\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,i}^{\mathrm{g}}\gamma_\ell^{\mathrm{G}-}-\sum_{\ell\in\mathcal{N}_\ell}\Gamma_{\ell,i}^{\mathrm{g}}\gamma_\ell^{\mathrm{G}+}+\gamma^{\mathrm{ba}}+\gamma_i^{\mathrm{g}-}-\gamma_i^{\mathrm{g}+}-\sum_{a=1}^{A}\theta_{i,a}^{1}\gamma_{i,a}^{\mathrm{C}}+\gamma_i^{\mathrm{ccg}}\le 0,
\end{aligned}$$

$$
\begin{aligned}
&\gamma_s^{\mathrm{e}} + \gamma_s^{\mathrm{e}-} - \gamma_s^{\mathrm{e}+} + \gamma_s^{\mathrm{cce}} \le 0, \\
&-\alpha^{\mathrm{sc}} t_\delta \gamma_s^{\mathrm{e}} - \sum_{\ell \in \mathcal{N}_\ell} \Gamma_{\ell,s}^{\mathrm{ss}} \gamma_\ell^{\mathrm{G}-} + \sum_{\ell \in \mathcal{N}_\ell} \Gamma_{\ell,s}^{\mathrm{ss}} \gamma_\ell^{\mathrm{G}+} - \gamma^{\mathrm{ba}} + \gamma_s^{\mathrm{sc}-} - \gamma_s^{\mathrm{sc}+} \le 0, \\
&t_\delta / \alpha^{\mathrm{sd}} \gamma_s^{\mathrm{e}} + \sum_{\ell \in \mathcal{N}_\ell} \Gamma_{\ell,s}^{\mathrm{ss}} \gamma_\ell^{\mathrm{G}-} - \sum_{\ell \in \mathcal{N}_\ell} \Gamma_{\ell,s}^{\mathrm{ss}} \gamma_\ell^{\mathrm{G}+} + \gamma^{\mathrm{ba}} + \gamma_s^{\mathrm{sd}-} - \gamma_s^{\mathrm{sd}+} \le 0, \\
&-1 + \gamma^{\mathrm{cco}} = 0, \\
&-\sum_{i \in \mathcal{N}_g} p_{i,t}^{\mathrm{g},j} \gamma_i^{\mathrm{ccg}} - \sum_{s \in \mathcal{N}_s} E_{s,t}^{j} \gamma_s^{\mathrm{cce}} - \bar{\mathcal{F}}_{t+1}(p_{i,t}^{\mathrm{g},j}, E_{s,t}^{j}) \gamma^{\mathrm{cco}} + \gamma^{\mathrm{ccj}} \le 0, \\
&\gamma_i^{\mathrm{u}} \ge 0, \gamma_i^{\mathrm{d}} \ge 0, \gamma_\ell^{\mathrm{G}-} \ge 0, \gamma_\ell^{\mathrm{G}+} \ge 0, \gamma_i^{\mathrm{g}-} \ge 0, \gamma_i^{\mathrm{g}+} \ge 0, \gamma_s^{\mathrm{e}-} \ge 0, \gamma_s^{\mathrm{e}+} \ge 0, \\
&\gamma_s^{\mathrm{sc}-} \ge 0, \gamma_s^{\mathrm{sc}+} \ge 0, \gamma_s^{\mathrm{sd}-} \ge 0, \gamma_s^{\mathrm{sd}+} \ge 0, \gamma_{i,a}^{\mathrm{C}} \ge 0.
\end{aligned} \tag{EC.7}
$$

Thus, the gradient can be obtained by $\nabla g^*_{\boldsymbol{\xi}_t,r} = -\sum_{\ell \in \mathcal{N}_\ell} \Gamma_{\ell,r}^{\mathrm{r}} \gamma_\ell^{\mathrm{G}-} + \sum_{\ell \in \mathcal{N}_\ell} \Gamma_{\ell,r}^{\mathrm{r}} \gamma_\ell^{\mathrm{G}+} - \gamma^{\mathrm{ba}}$.

Denote $\boldsymbol{\varphi}_t = [\lambda_t; \boldsymbol{\pi}_t]$ be the dual variable vector, $\boldsymbol{z}_t = [\hat{\boldsymbol{y}}_t; \boldsymbol{\xi}_t]$ be the primal variable vector. The gradient in PDBO can be represented by (EC.8) and (EC.9).

$$
\begin{aligned}
\nabla \mathcal{L}_{\boldsymbol{\varphi}_t}(\boldsymbol{\varphi}_t; \boldsymbol{z}_t) = [&\nabla \mathcal{L}_{\lambda_t}; \nabla \mathcal{L}_{\pi_s^{\mathrm{e}}}; \nabla \mathcal{L}_{\pi_i^{\mathrm{u}}}; \nabla \mathcal{L}_{\pi_i^{\mathrm{d}}}; \nabla \mathcal{L}_{\pi_\ell^{\mathrm{G}-}}; \nabla \mathcal{L}_{\pi_\ell^{\mathrm{G}+}}; \nabla \mathcal{L}_{\pi^{\mathrm{ba}}}; \nabla \mathcal{L}_{\pi_i^{\mathrm{g}-}}; \nabla \mathcal{L}_{\pi_i^{\mathrm{g}+}}; \\
&\nabla \mathcal{L}_{\pi_s^{\mathrm{e}-}}; \nabla \mathcal{L}_{\pi_s^{\mathrm{e}+}}; \nabla \mathcal{L}_{\pi_s^{\mathrm{sc}-}}; \nabla \mathcal{L}_{\pi_s^{\mathrm{sc}+}}; \nabla \mathcal{L}_{\pi_s^{\mathrm{sd}-}}; \nabla \mathcal{L}_{\pi_s^{\mathrm{sd}+}}; \nabla \mathcal{L}_{\pi_{i,a}^{\mathrm{C}}}; \nabla \mathcal{L}_{\pi_i^{\mathrm{ccg}}}; \\
&\nabla \mathcal{L}_{\pi_s^{\mathrm{cce}}}; \nabla \mathcal{L}_{\pi^{\mathrm{cco}}}; \nabla \mathcal{L}_{\pi^{\mathrm{ccj}}}; \nabla \mathcal{L}_{\pi_r^{\mathrm{xi}-}}; \nabla \mathcal{L}_{\pi_r^{\mathrm{xi}+}}]
\end{aligned} \tag{EC.8}
$$

$$
\nabla \mathcal{L}_{\boldsymbol{z}_t}(\boldsymbol{\varphi}_t; \boldsymbol{z}_t) = [\nabla \mathcal{L}_{C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}})}; \nabla \mathcal{L}_{p_{i,t}^{\mathrm{g}}}; \nabla \mathcal{L}_{E_{s,t}}; \nabla \mathcal{L}_{p_{s,t}^{\mathrm{sc}}}; \nabla \mathcal{L}_{\boldsymbol{p}_{s,t}^{\mathrm{sd}}}; \nabla \mathcal{L}_{\tau_t}; \nabla \mathcal{L}_{\vartheta_j}; \nabla \mathcal{L}_{\xi_{r,t}}] \tag{EC.9}
$$

EC.3. FO-RDDP Steps

Algorithm 2 FO-RDDP Scheme

Require: Maximum iteration number K', time horizon T.

Ensure: $\underline{\boldsymbol{y}}$, $\underline{\mathcal{F}}_t(\boldsymbol{y}_{t-1})$ and $\bar{\mathcal{F}}_t(\boldsymbol{y}_{t-1})$ for $t = 1, \cdots, T$

1: Set $\underline{\mathcal{F}}_t(\boldsymbol{y}_{t-1}) = \underline{\mathcal{F}}_t^0(\boldsymbol{y}_{t-1})$, $\bar{\mathcal{F}}_t(\boldsymbol{y}_{t-1}) = \bar{\mathcal{F}}_t^0(\boldsymbol{y}_{t-1})$, $\underline{\mathcal{F}}_{T+1}(\boldsymbol{y}_T) = \bar{\mathcal{F}}_{T+1}(\boldsymbol{y}_T) = 0$.

2: **for** $k' = 1$ to K' **do**

3: Stage one problem:
Obtain $\underline{\boldsymbol{y}}_1$ by solving (EC.3) at stage one.
If $\underline{\mathcal{F}}_2(\underline{\boldsymbol{y}}_1) = \bar{\mathcal{F}}_2(\underline{\boldsymbol{y}}_1)$, terminate, otherwise, go on.

4: Forward pass (for $t = 2, \cdots, T$):
Obtain $\boldsymbol{\xi}_t^{\mathrm{fw}}$ by solving $\bar{F}_t(\underline{\boldsymbol{y}}_{t-1})$ in (EC.2) through PDBO listed in Algorithm 1.
Solve $\underline{F}_t(\underline{\boldsymbol{y}}_{t-1}; \boldsymbol{\xi}_t^{\mathrm{fw}})$ in (EC.3) to obtain an optimal solution $\underline{\boldsymbol{y}}_t$.

5: Backward pass (for $t = T, \cdots, 2$):
Obtain $\boldsymbol{\xi}_t^{\mathrm{bw}}$ by solving $\bar{F}_t(\underline{\boldsymbol{y}}_{t-1})$ in (EC.2) through PDBO listed in Algorithm 1.
Calculate value $\bar{F}_t'(\underline{\boldsymbol{y}}_{t-1}, \boldsymbol{\xi}_t^{\mathrm{bw}})$ through (EC.10) with $\boldsymbol{\xi}_t = \boldsymbol{\xi}_t^{\mathrm{bw}}$, obtain a new upper-bound point $\{p_{i,t-1}^{\mathrm{g}}; E_{s,t-1}; \bar{F}_t'(\underline{\boldsymbol{y}}_{t-1}, \boldsymbol{\xi}_t^{\mathrm{bw}})\}$.
Solve the problem of $\underline{F}_t^{\mathrm{dual}}(\underline{\boldsymbol{y}}_{t-1}, \boldsymbol{\xi}_t^{\mathrm{bw}})$ in (EC.11) to obtain a new plane with $\beta_{i,\mathcal{K}+1}^1 = \eta_i^{\mathrm{ramp}}, \beta_{s,\mathcal{K}+1}^2 = \eta_s^{\mathrm{e}}, \beta_{\mathcal{K}+1}^3 = \underline{F}_t^{\mathrm{dual}}(\underline{\boldsymbol{y}}_{t-1}, \boldsymbol{\xi}_t^{\mathrm{bw}}) - \sum_{s\in\mathcal{N}_s} E_{s,t-1}\eta_s^{\mathrm{e}} - \sum_{i\in\mathcal{N}_g} p_{i,t-1}^{\mathrm{g}}\eta_i^{\mathrm{ramp}}$.
Update the lower bounds as $\mathrm{env}\big(\min\{\bar{\mathcal{F}}_t(\boldsymbol{y}_{t-1}), \bar{F}_t'(\underline{\boldsymbol{y}}_{t-1}, \boldsymbol{\xi}_t^{\mathrm{bw}}) + \delta_{\underline{\boldsymbol{y}}_{t-1}}(\boldsymbol{y}_{t-1})\}\big)$.
Update the upper bounds as $\max\{\underline{\mathcal{F}}_t(\boldsymbol{y}_{t-1}), \sum_{i=1}^{N_g} \beta_{i,\mathcal{K}+1}^1 p_{i,t-1}^{\mathrm{g}} + \sum_{s=1}^{N_s} \beta_{s,\mathcal{K}+1}^2 E_{s,t-1} + \beta_{\mathcal{K}+1}^3\}$.

6: **end for**

7: **return** $\underline{\boldsymbol{y}}_1$, $\underline{\mathcal{F}}_t(\boldsymbol{y}_{t-1})$ and $\bar{\mathcal{F}}_t(\boldsymbol{y}_{t-1})$ for $t = 2, \cdots, T$

To start the FO-RDDP method, the initialized $N+1$ candidate points for the upper-bound problem are set as $\{\mu\mathbf{e}_1, \cdots, \mu\mathbf{e}_n, \cdots, \mu\mathbf{e}_N, \mathbf{0}\}$, where $\mu = \sum_{s\in\mathcal{N}_s} \bar{E}_s + \sum_{i\in\mathcal{N}_g} \bar{p}_i^{\mathrm{g}}$, $\{\mathbf{e}_n\}_{n=1}^N$ is the canonical basis of $\mathbb{R}^N$, $N = N_s + N_g$.

The candidate points are calculated in the forward pass, and the bounds are updated in the backward pass. In the backward pass, the upper-bound value needs to be computed by (EC.10) after $\boldsymbol{\xi}_t^{\mathrm{bw}}$ is obtained. The lower bounds are updated by solving (EC.11).

$$
\begin{aligned}
\bar{F}_t'(\boldsymbol{y}_{t-1}, \boldsymbol{\xi}_t) = \min_{\boldsymbol{y}_t} \sum_{i\in\mathcal{N}_g} C_i^{\mathrm{g}}(p_{i,t}^{\mathrm{g}}) t_\delta + \tau_t \\
\text{s.t.}\ \ E_{s,t} = E_{s,t-1} + p_{s,t}^{\mathrm{sc}}\alpha^{\mathrm{sc}} t_\delta - p_{s,t}^{\mathrm{sd}} t_\delta/\alpha^{\mathrm{sd}},\ s\in\mathcal{N}_s, \\
p_{i,t}^{\mathrm{g}} - p_{i,t-1}^{\mathrm{g}} \le \Delta_i^{\mathrm{u}},\ i\in\mathcal{N}_g, \\
p_{i,t-1}^{\mathrm{g}} - p_{i,t}^{\mathrm{g}} \le \Delta_i^{\mathrm{d}},\ i\in\mathcal{N}_g, \\
-\bar{G}_\ell \le \sum_{i\in\mathcal{N}_g} \Gamma_{\ell,i}^{\mathrm{g}} p_{i,t}^{\mathrm{g}} + \sum_{r\in\mathcal{N}_r} \Gamma_{\ell,r}^{\mathrm{r}} \xi_{r,t} + \sum_{s\in\mathcal{N}_s} \Gamma_{\ell,s}^{\mathrm{ss}}(p_{s,t}^{\mathrm{sd}} - p_{s,t}^{\mathrm{sc}}) - \sum_{d\in\mathcal{N}_d} \Gamma_{\ell,d}^{\mathrm{dm}} p_{d,t}^{\mathrm{dm}} \le \bar{G}_\ell, \ell\in\mathcal{N}_\ell,
\end{aligned}
$$

$$
\begin{aligned}
&\sum_{i\in\mathcal{N}_g} p^{\mathrm{g}}_{i,t} + \sum_{s\in\mathcal{N}_s}(p^{\mathrm{sd}}_{s,t} - p^{\mathrm{sc}}_{s,t}) + \sum_{r\in\mathcal{N}_r}\xi_{r,t} = \sum_{d\in\mathcal{N}_d} p^{\mathrm{dm}}_{d,t},\\
&\quad x^{\mathrm{g}}_{i,t}\underline{p}^{\mathrm{g}}_i \le p^{\mathrm{g}}_{i,t} \le x^{\mathrm{g}}_{i,t}\bar{p}^{\mathrm{g}}_i,\ i\in\mathcal{N}_g,\\
&\quad \underline{E}_s \le E_{s,t} \le \bar{E}_s,\ s\in\mathcal{N}_s,\\
&\quad 0 \le p^{\mathrm{sc}}_{s,t} \le \bar{p}^{\mathrm{sc}}_s,\ s\in\mathcal{N}_s,\\
&\quad 0 \le p^{\mathrm{sd}}_{s,t} \le \bar{p}^{\mathrm{sd}}_s,\ s\in\mathcal{N}_s,\\
&\quad C^{\mathrm{g}}_i(p^{\mathrm{g}}_{i,t}) \ge \theta^1_{i,a}p^{\mathrm{g}}_{i,t} + \theta^2_{i,a}x^{\mathrm{g}}_{i,t},\ a=1,\cdots,A.\\
&\quad p^{\mathrm{g}}_{i,t} = \sum\nolimits_j \vartheta_j p^{\mathrm{g},j}_{i,t},\ i\in\mathcal{N}_g,\\
&\quad E_{s,t} = \sum\nolimits_j \vartheta_j E^j_{s,t},\ s\in\mathcal{N}_s,\\
&\quad \tau_t - \sum\nolimits_j \vartheta_j \bar{\mathcal{F}}_{t+1}(p^{\mathrm{g},j}_{i,t}, E^j_{s,t}) = 0,\\
&\quad \vartheta_j \ge 0, \sum\nolimits_j \vartheta_j = 1,\ j=1,\cdots,J_t.
\end{aligned}
\tag{EC.10}
$$

$$
\begin{aligned}
&\underline{F}^{\mathrm{dual}}_t(\boldsymbol{y}_{t-1},\boldsymbol{\xi}_t) = \max \sum_{s\in\mathcal{N}_s} E_{s,t-1}\eta^{\mathrm{e}}_s + \sum_{i\in\mathcal{N}_g} p^{\mathrm{g}}_{i,t-1}\eta^{\mathrm{ramp}}_i - \sum_{i\in\mathcal{N}_g}\Delta^{\mathrm{d}}_i\eta^{\mathrm{d}}_i - \sum_{i\in\mathcal{N}_g}\Delta^{\mathrm{u}}_i\eta^{\mathrm{u}}_i\\
&+ \sum_{\ell\in\mathcal{N}_\ell}(-\bar{G}_\ell - \sum_{r\in\mathcal{N}_r}\Gamma^{\mathrm{r}}_{\ell,r}\xi_{r,t} + \sum_{d\in\mathcal{N}_d}\Gamma^{\mathrm{dm}}_{\ell,d}p^{\mathrm{dm}}_{d,t})\eta^{\mathrm{G}-}_\ell\\
&+ \sum_{\ell\in\mathcal{N}_\ell}(\sum_{r\in\mathcal{N}_r}\Gamma^{\mathrm{r}}_{\ell,r}\xi_{r,t} - \sum_{d\in\mathcal{N}_d}\Gamma^{\mathrm{dm}}_{\ell,d}p^{\mathrm{dm}}_{d,t} - \bar{G}_\ell)\eta^{\mathrm{G}+}_\ell + (-\sum_{r\in\mathcal{N}_r}\xi_{r,t} + \sum_{d\in\mathcal{N}_d}p^{\mathrm{dm}}_{d,t})\eta^{\mathrm{ba}}\\
&+ \sum_{i\in\mathcal{N}_g} x^{\mathrm{g}}_{i,t}\underline{p}^{\mathrm{g}}_i\eta^{\mathrm{g}-}_i - \sum_{i\in\mathcal{N}_g} x^{\mathrm{g}}_{i,t}\bar{p}^{\mathrm{g}}_i\eta^{\mathrm{g}+}_i + \sum_{s\in\mathcal{N}_s}\underline{E}_s\eta^{\mathrm{e}-}_s - \sum_{s\in\mathcal{N}_s}\bar{E}_s\eta^{\mathrm{e}+}_s\\
&- \sum_{s\in\mathcal{N}_s}\bar{p}^{\mathrm{sc}}_s\eta^{\mathrm{sc}+}_s - \sum_{s\in\mathcal{N}_s}\bar{p}^{\mathrm{sd}}_s\eta^{\mathrm{sd}+}_s + \sum_{a=1}^{A}\sum_{i\in\mathcal{N}_g}\theta^2_{i,a}x^{\mathrm{g}}_{i,t}\eta^{\mathrm{C}}_{i,a} + \sum_{\kappa=1}^{\mathcal{K}}\beta^3_\kappa\eta^{\mathrm{obj}}_\kappa\\
&\text{s.t.}\quad -t_\delta + \sum_{a=1}^{A}\eta^{\mathrm{C}}_{i,a} = 0,\\
&\eta^{\mathrm{ramp}}_i + \sum_{\ell\in\mathcal{N}_\ell}\Gamma^{\mathrm{g}}_{\ell,i}\eta^{\mathrm{G}-}_\ell - \sum_{\ell\in\mathcal{N}_\ell}\Gamma^{\mathrm{g}}_{\ell,i}\eta^{\mathrm{G}+}_\ell + \eta^{\mathrm{ba}} + \eta^{\mathrm{g}-}_i - \eta^{\mathrm{g}+}_i - \sum_{a=1}^{A}\theta^1_{i,a}\eta^{\mathrm{C}}_{i,a} - \sum_{\kappa=1}^{\mathcal{K}}\beta^1_{i,\kappa}\eta^{\mathrm{obj}}_\kappa \le 0,\\
&-\eta^{\mathrm{ramp}}_i + \eta^{\mathrm{d}}_i - \eta^{\mathrm{u}}_i = 0\\
&\eta^{\mathrm{e}}_s + \eta^{\mathrm{e}-}_s - \eta^{\mathrm{e}+}_s - \sum_{\kappa=1}^{\mathcal{K}}\beta^2_{s,\kappa}\eta^{\mathrm{obj}}_\kappa \le 0,\\
&-\alpha^{\mathrm{sc}}t_\delta\eta^{\mathrm{e}}_s - \sum_{\ell\in\mathcal{N}_\ell}\Gamma^{\mathrm{ss}}_{\ell,s}\eta^{\mathrm{G}-}_\ell + \sum_{\ell\in\mathcal{N}_\ell}\Gamma^{\mathrm{ss}}_{\ell,s}\eta^{\mathrm{G}+}_\ell - \eta^{\mathrm{ba}} + \eta^{\mathrm{sc}-}_s - \eta^{\mathrm{sc}+}_s \le 0,\\
&t_\delta/\alpha^{\mathrm{sd}}\eta^{\mathrm{e}}_s + \sum_{\ell\in\mathcal{N}_\ell}\Gamma^{\mathrm{ss}}_{\ell,s}\eta^{\mathrm{G}-}_\ell - \sum_{\ell\in\mathcal{N}_\ell}\Gamma^{\mathrm{ss}}_{\ell,s}\eta^{\mathrm{G}+}_\ell + \eta^{\mathrm{ba}} + \eta^{\mathrm{sd}-}_s - \eta^{\mathrm{sd}+}_s \le 0,
\end{aligned}
$$

$$
\begin{aligned}
&-1+\sum_{\kappa=1}^{\mathcal{K}} \eta_{\kappa}^{\mathrm{obj}}=0, \\
&\eta_i^{\mathrm{d}} \geq 0, \eta_i^{\mathrm{u}} \geq 0, \eta_{\ell}^{\mathrm{G}-} \geq 0, \eta_{\ell}^{\mathrm{G}+} \geq 0, \eta_i^{\mathrm{g}-} \geq 0, \eta_i^{\mathrm{g}+} \geq 0, \eta_s^{\mathrm{e}-} \geq 0, \eta_s^{\mathrm{e}+} \geq 0, \\
&\eta_s^{\mathrm{sc}-} \geq 0, \eta_s^{\mathrm{sc}+} \geq 0, \eta_s^{\mathrm{sd}-} \geq 0, \eta_s^{\mathrm{sd}+} \geq 0, \eta_{i,a}^{\mathrm{C}} \geq 0, \eta_{\kappa}^{\mathrm{obj}} \geq 0.
\end{aligned} \tag{EC.11}
$$