Virtual Mini Conference on Graph Theory & Combinatorics

March 12 – 13th (Virtual)

Zoom Link:

https://gatech.zoom.us/j/99844490208?pwd=QVhVQTlrVHV6cHAzRnJQczRJMklYdz09

Meeting ID: 998 4449 0208
Passcode: 141592

Organizers:

Guangming Jing (Augusta University)
Zhiyu Wang (Georgia Institute of Technology)
Xingxing Yu (Georgia Institute of Technology)

Schedule

Saturday, March 12th

Saturday Morning, March 12th		
Time	**Speaker**	**Title**
9:00-9:30	Guantao Chen	Multiplicity of the second-largest eigenvalue of graph
9:35-10:05	Linyuan Lu	Anti-Ramsey number of disjoint rainbow bases in all matroids
10:10-10:40	Songling Shan	Edge coloring graphs with large minimum degree
10:45-11:15	Dan Cranston	Kempe Equivalent List Colorings
Lunch Break		
13:00-13:30	Ryan Martin	Planar Turán number of the 6-cycle
13:35-14:05	Eva Czabarka	On the maximum diameter of k-colorable graphs
14:10-14:40	C.Q. Zhang	Some recent progress about Berge-Fulkerson Conjecture,and the Matching Pair Lemma
14:45-15:15	Yi Zhao	Subgraphs of multipartite graphs
15:20-15:50	Xiaonan Liu	Counting Hamiltonian cycles in planar triangulations
15:55-16:25	Mark Ellingham	Nonorientable embeddings of graphs and digraphs with large faces

Sunday, March 13th

Sunday, March 13th		
Time	**Speaker**	**Title**
9:00-9:30	Anton Bernshteyn	Counting colorings of triangle-free graphs
9:35-10:05	Zixia Song	The Erdős-Lovász Tihany Conjecture
10:10-10:40	Gexin Yu	Some connectivity conditions for graph rigidity
10:45-11:15	Laszlo Székely	Maximum and minimum Wiener index of planar triangulations and quadrangulations
11:20-11:50	Emily Heath	A Strengthening of the Erdős-Szekeres Theorem
End of Conference		

Title & Abstracts

Counting colorings of triangle-free graphs

Anton Bernshteyn

Georgia Institute of Technology

According to a celebrated theorem of Johansson, every triangle-free graph G of maximum degree Δ has chromatic number $O(\Delta/\log\Delta)$. Molloy recently improved this upper bound to $(1+o(1))\Delta/\log\Delta$. In this talk, we will strengthen Molloy's result by establishing an optimal lower bound on the number of proper q-colorings of G when q is at least $(1+o(1))\Delta/\log\Delta$. One of the main ingredients in our argument is a novel proof technique introduced by Rosenfeld. This is joint work with Tyler Brazelton, Ruijia Cao, and Akum Kang.

Multiplicity of the second-largest eigenvalue of graphs

Guantao Chen

Georgia State University

The *multiplicity of the second-largest eigenvalue* of the adjacency matrix $A(G)$ of a connected graph G, denoted by $m(\lambda_2, G)$, is the number of times of the second-largest eigenvalue of $A(G)$ appears. In 2019, Jiang, Tidor, Yao, Zhang and Zhao gave an upper bound on $m(\lambda_2, G)$ for graphs G with bounded degrees, and applied it to solve a longstanding problem on equiangular lines. We showed that if G is a 3-connected planar graph or 2-connected outerplanar graph, then $m(\lambda_2, G) \leq \delta(G)$, where $\delta(G)$ is the minimum degree of G. We further prove that if G is a connected planar graph, then $m(\lambda_2, G) \leq \Delta(G)$; if G is a connected outerplanar graph, then $m(\lambda_2, G) \leq \max\{2, \Delta(G)-1\}$, where $\Delta(G)$ is the maximum degree of G. Moreover, these two upper bounds for connected planar graphs and outerplanar graphs, respectively, are best possible. We will discuss general techniques and specific methods we used in the proofs of these results.

Kempe Equivalent List Colorings

Daniel W Cranston
Virginia Commonwealth University

An α, β-Kempe swap in a properly colored graph interchanges the colors on some component of the subgraph induced by colors α and β. Two k-colorings of a graph are k-Kempe equivalent if we can form one from the other by a sequence of Kempe swaps (never using more than k colors). Las Vergnas and Meyniel showed that if a graph is $(k-1)$-degenerate, then each pair of its k-colorings are k-Kempe equivalent. Mohar conjectured the same conclusion for connected k-regular graphs. This was proved for $k = 3$ by Feghali, Johnson, and Paulusma (with a single exception, the 3-prism) and for $k \geq 4$ by Bonamy, Bousquet, Feghali, and Johnson.

In this paper we prove an analogous result for list-coloring. For a list-assignment L and an L-coloring φ, a Kempe swap is called L-valid for φ if performing the Kempe swap yields another L-coloring. Two L-colorings are called L-equivalent if we can form one from the other by a sequence of L-valid Kempe swaps. Let G be a connected k-regular graph with $k \geq 3$ and $G \neq K_{k+1}$. We prove that if L is a k-assignment, then all L-colorings are L-equivalent (again excluding only the 3-prism). When $k \geq 4$, the proof is completely self-contained, implying an alternate proof of the result of Bonamy et al.

Our proofs rely on the following key lemma, which may be of independent interest. Let H be a graph such that for every degree-assignment L_H all L_H-colorings are L_H-equivalent. If G is a connected graph that contains H as an induced subgraph, then for every degree-assignment L_G for G all L_G-colorings are L_G-equivalent. This is joint work with Reem Mahmoud.

On the maximum diameter of k-colorable graphs

Éva Czabarka
University of South Carolina

Czabarka et. al. conjectured that the maximum diameter of k-colorable n-vertex graphs with minimum degree δ is at most $(3 - \frac{2}{k})\frac{n}{\delta} + o(1)$. I will prove that for all $k \geq 3$ that the maximum diameter is at most $(3 - \frac{1}{k-1})\frac{n}{\delta} - 1$, and give a simple proof that for $k \in \{3, 4\}$, it is at most $(3 - \frac{2}{k})\frac{n}{\delta} - 1$, confirming the conjecture for these values. The result for $k = 4$ was proven earlier by Czabarka, Dankelmann and Székely using a different method. Joint work with Stephen J. Smith, and Laszlo A. Székely.

Nonorientable embeddings of graphs and digraphs with large faces

Mark Ellingham

Vanderbilt University

In 1965 Edmonds gave a characterization of when two connected graphs can be embedded in some surface as duals of each other. We show that Edmonds' characterization can be strengthened slightly.

As a corollary, Edmonds observed that every connected graph has a one-face embedding in some surface. Ringel (and independently several others) proved that the surface can be guaranteed to be nonorientable unless the graph is a tree. This result is very well known. Fijavž, Pisanski and Rus showed that an especially nice nonorientable one-face embedding can be constructed when the graph is eulerian. We extend these results to directed embeddings of digraphs, where the boundary of every face must be a directed walk.

Edmonds also observed that every eulerian graph has a two-face embedding in some surface, where both faces are bounded by euler circuits. We call this a *bi-eulerian* embedding. This result is not so well known. We show that the surface can be guaranteed to be nonorientable unless the graph is a cycle. In fact, we prove a more general result on extending a circuit decomposition of a graph to an embedding using an euler circuit. Joint work with Joanna A. Ellis-Monaghan.

A Strengthening of the Erdős-Szekeres Theorem

Emily Heath

Iowa State University

The Erdős–Szekeres Theorem states that any edge-coloring of the ordered complete graph on $rs+1$ vertices with red and blue must contain a red ordered path with r edges or a blue ordered path with s edges. However, not all edges of K_{rs+1} are necessary for this result. We find the unique minimal ordered graph on $rs+1$ vertices with this coloring property and show that any ordered graph with this property must contain our minimal example as a subgraph.

Counting Hamiltonian cycles in planar triangulations

Xiaonan Liu
Georgia Institute of Technology

Hakimi, Schmeichel, and Thomassen [*J. Graph Theory, 1979*] conjectured that every 4-connected planar triangulation G on n vertices has at least $2(n-2)(n-4)$ Hamiltonian cycles, with equality if and only if G is a double wheel. In this paper, we show that every 4-connected planar triangulation on n vertices has $\Omega(n^2)$ Hamiltonian cycles. Moreover, we show that if G is a 4-connected planar triangulation on n vertices and the distance between any two vertices of degree 4 in G is at least 3, then G has $2^{\Omega(n^{1/4})}$ Hamiltonian cycles. Joint work with Zhiyu Wang and Xingxing Yu.

Anti-Ramsey number of disjoint rainbow bases in all matroids

Linyuan Lu
University of South Carolina

Consider a matroid $M = (E, \mathcal{I})$ with its elements of the ground set E colored. A *rainbow basis* is a maximum independent set in which each element receives a different color. The *rank* of a subset S of E, denoted by $r_M(S)$, is the maximum size of an independent set in S. A *flat* F is a maximal set in M with a fixed rank. The *anti-Ramsey* number of t pairwise disjoint rainbow bases in M, denoted by $ar(M,t)$, is defined as the maximum number of colors m such that there exists an m coloring of the ground set E of M which contains no t pairwise disjoint rainbow bases. We determine $ar(M,t)$ for all matroids of rank at least 2: $ar(M,t) = |E|$ if there exists a flat F_0 with $|E| - |F_0| < t(r_M(E) - r_M(F_0))$; and $ar(M,t) = \max_{F: r_M(F) \leq r_M(E)-2}\{|F| + t(r_M(E) - r_M(F) - 1)\}$ otherwise. (Joint work with Andrew Meier.)

Planar Turán number of the 6-cycle

Ryan R. Martin
Iowa State University

Let $\mathrm{ex}_{\mathcal{P}}(n,T,H)$ denote the maximum number of copies of T in an n-vertex planar graph which does not contain H as a subgraph. When $T = K_2$, $\mathrm{ex}_{\mathcal{P}}(n,T,H)$ is the well studied function, the planar Turán number of H, denoted by $\mathrm{ex}_{\mathcal{P}}(n,H)$. The topic of extremal planar graphs was initiated by Dowden (2016). He obtained sharp upper bound for both $\mathrm{ex}_{\mathcal{P}}(n,C_4)$ and $\mathrm{ex}_{\mathcal{P}}(n,C_5)$. Later on, Lan, Shi and Song continued this topic and proved that $\mathrm{ex}_{\mathcal{P}}(n,C_6) \leq \frac{18(n-2)}{7}$. In this paper, we give a sharp upper bound $\mathrm{ex}_{\mathcal{P}}(n,C_6) \leq \frac{5}{2}n - 7$, for all $n \geq 18$, which improves Lan's result. We also pose a conjecture on $\mathrm{ex}_{\mathcal{P}}(n,C_k)$, for $k \geq 7$. Joint work with Debarun Ghosh, Ervin Győri, Addisu Paulos, and Chuanqi Xiao.

Edge coloring graphs with large minimum degree.

Songling Shan
Illinois State University

Let G be a simple graph with maximum degree $\Delta(G)$. A subgraph H of G is overfull if $|E(H)| > \Delta(G)\lfloor|V(H)|/2\rfloor$. Chetwynd and Hilton in 1985 conjectured that a graph G with $\Delta(G) > |V(G)/3|$ has chromatic index $\Delta(G)$ if and only if G contains no overfull subgraph. The best previous results supporting this conjecture have been obtained for regular graphs. For example, Perković and Reed verified the conjecture for large regular graphs G with degree arbitrarily close to $|V(G)|/2$. We provide a similar result for general graphs asymptotically, showing that for any given $0 < \epsilon < 1$, there exists a positive integer n_0 such that the following statement holds: if G is a graph on $2n \geq n_0$ vertices with minimum degree at least $(1 + \epsilon)n$, then G has chromatic index $\Delta(G)$ if and only if G contains no overfull subgraph. Joint work with Michael J. Plantholt.

The Erdős-Lovász Tihany Conjecture.

23 Nov
9:00am
R301

Zi-Xia Song
University of Central Florida

Let $s \geq 2$ and $t \geq 2$ be integers. A graph G is (s, t)-splittable if $V(G)$ can be partitioned into two sets S and T such that $\chi(G[S]) \geq s$ and $\chi(G[T]) \geq t$. The Erdős-Lovász Tihany Conjecture from 1968 states that every graph G with $\omega(G) < \chi(G) = s + t - 1$ is (s, t)-splittable. In this talk we will survey the history of the Erdős-Lovász Tihany Conjecture and the main ideas of recent results.

Maximum and minimum Wiener index of planar triangulations and quadrangulations

Laszlo A. Székely
University of South Carolina

The Wiener index of a connected graph is the sum of the distances between all unordered pairs of vertices. We provide formulae for the minimum Wiener index of simple triangulations and quadrangulations with given connectedness, and provide the extremal structures which attain these values. As a main tool, we provide upper bounds for the maximum degree of highly connected triangulations and quadrangulations. Joint work with Éva Czabarka, Trevor V. Olsen, and Stephen J. Smith.

Some connectivity conditions for graph rigidity

Gexin Yu
College of William and Mary

Abstract: A graph is rigid in R^d if one places the vertices of the graph in the R^d, in general position, there will be no simultaneous continuous motion of all the points, other than Euclidean congruences, that preserves the lengths of all the graph edges. In R^2, Geiringer in 1927, and independently Laman in 1970, gave a nice combinatorial characterization of rigid graphs. Using another characterization, Lovász and Yemini in 1982 showed that every 6-connected graph is rigid. We give two further sufficient connectivity conditions for a graph to be rigid, both in R^2 and R^d for d at least three. This is based on a joint work with Xiaofeng Gu, Wei Meng, Martin Rolek, and Yue Wang, and with Xiaofeng Gu and Runrun Liu.

Some recent progress about Berge-Fulkerson Conjecture, and the Matching Pair Lemma

C.Q. Zhang
West Virginia University

It is conjectured by Berge and Fulkerson that if G is a bridgeless cubic graph, then $2G$ is 6-edge-colorable. It is evident that we are only interested in snarks for this conjecture. This talk will survey some recent progress. Most of these progress are based on a technical lemma (an equivalent statement to the conjecture). The Lemma says: G is Fulkerson colorable if and only if G contains a pair of disjoint matchings M_1, M_2 such that (1) the union of M_1 and M_2 is an even subgraph, (2) for each $i = 1, 2$ and for each component Q of $G - M_i$, either Q is 2-regular or the suppressed cubic graph of Q is 3-edge-colorable. We will also present an analog of this technical lemma for Fan-Raspaud Conjecture, a weak version of Berge-Fulkerson Conjecture. (Fan and Raspaud conjectured that every bridgeless cubic graph contain three perfect matchings such that no edge is covered by all of them).

Subgraphs of multipartite graphs

Yi Zhao
Georgia State University

In 1975 Bollobás, Erdős, and Szemerédi asked the following question: given positive integers n, t, r with $2 \leq t \leq r - 1$, what is the largest minimum degree $\delta(G)$ among all r-partite graphs G with parts of size n and which do not contain a copy of K_{t+1}? The $r = t + 1$ case has attracted a lot of attention and was fully resolved by Haxell and Szabó, and Szabó and Tardos in 2006. In this talk we discuss a recent join work with Lo and Treglown on the $r > t+1$ case of the problem, including an exact solution in the case when $r \equiv -1 \pmod{t}$. We will also discuss Turán numbers in multipartite graphs, which can be viewed as a generalization of Zarankiewicz problem, including a recent work with Han on the Turán number of disjoint triangles in 4-partite graphs.
