NEURAL NETWORKS

Biological neurons

Neurons are interconnected nerve cells

acting as logic gates

Signals arrive

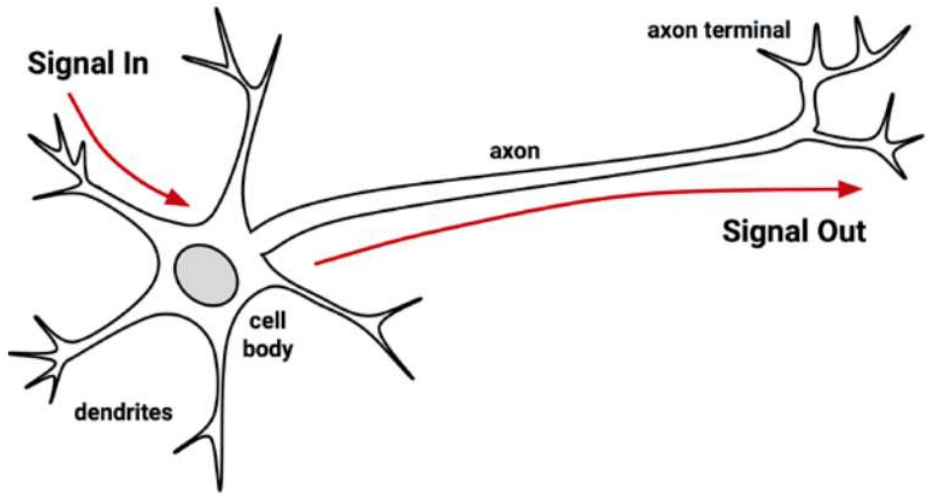
and accumulate

If accumulation

exceeds a

threshold

an output signal is passed on



Artificial neurons

input signals $x_1, \ldots x_k$

output signal y

input signals are

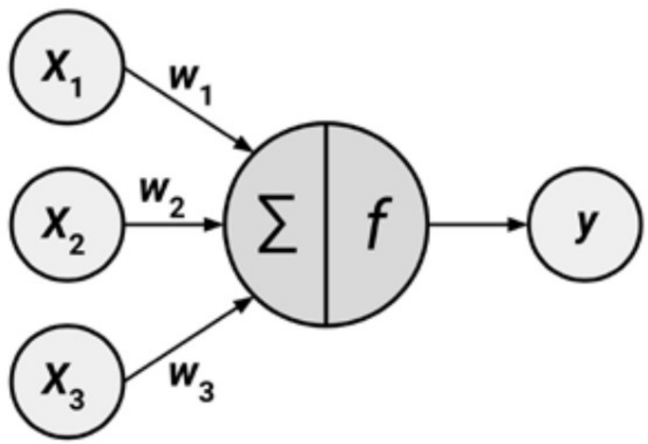
weighted and

summed by the cell

the signal is passed on

using an activation function f

Directed Network



Artificial Neural Network (ANN)

ANN models

the relationship between

a set of input signals

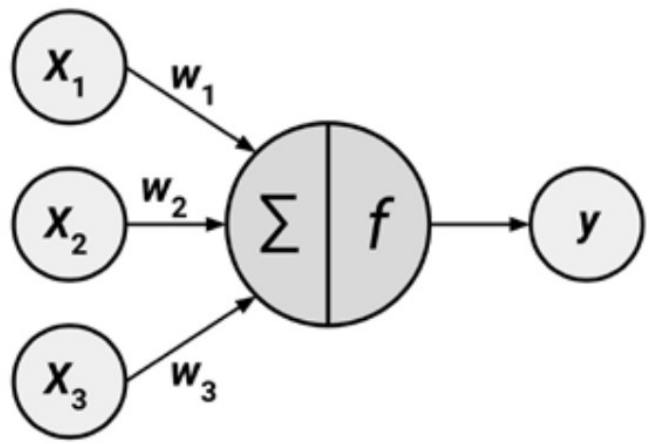
and an output signal

resembling our

understanding of

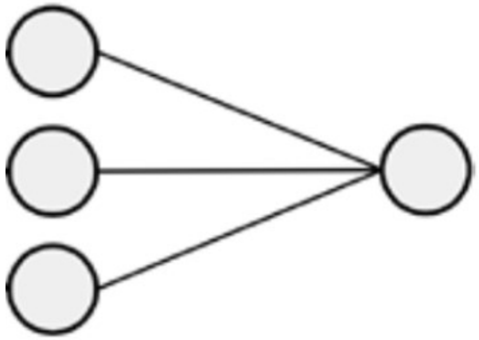
how cells respond to stimuli

from sensory inputs



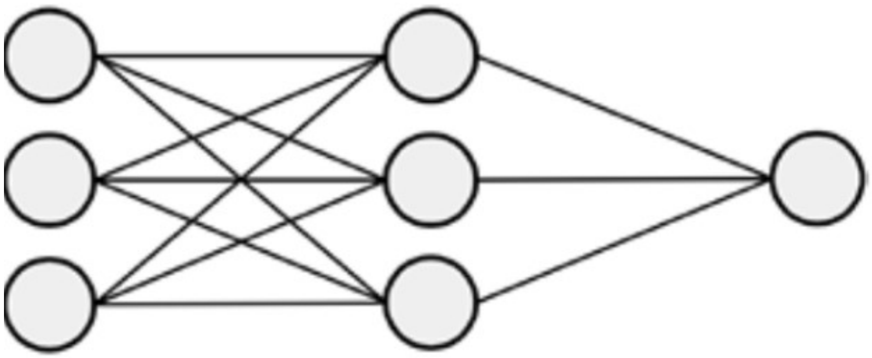
ANN Types

- Single Input Layer ANN



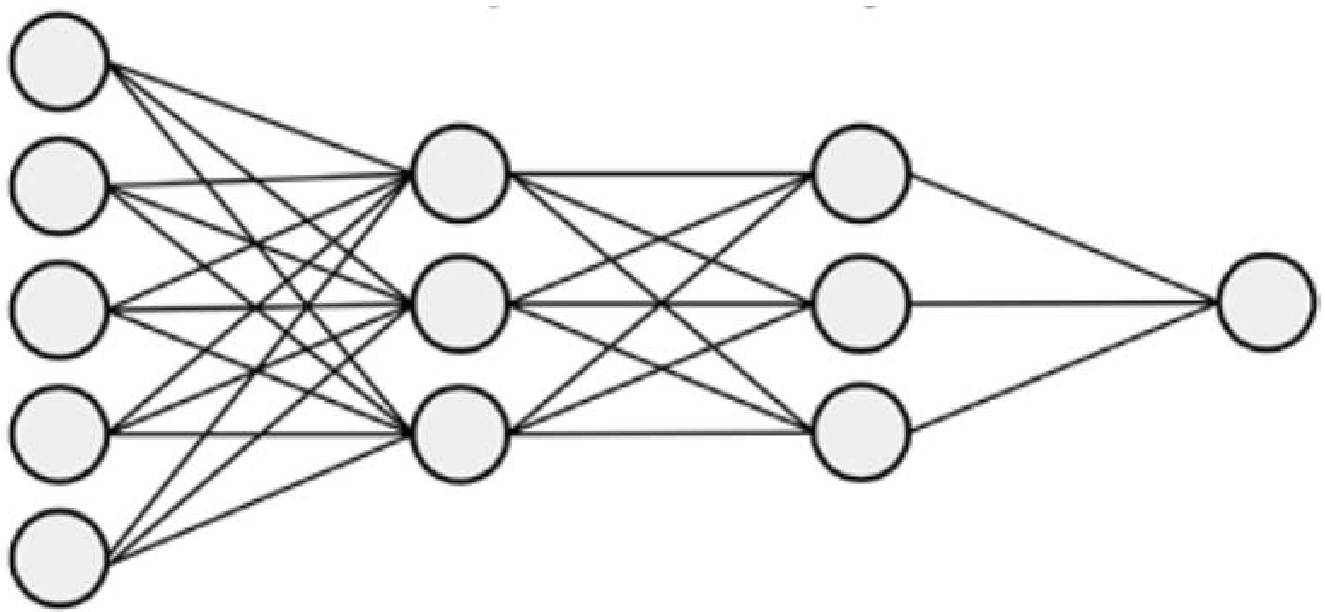
ANN Types

- One input, one hidden layer



ANN Types

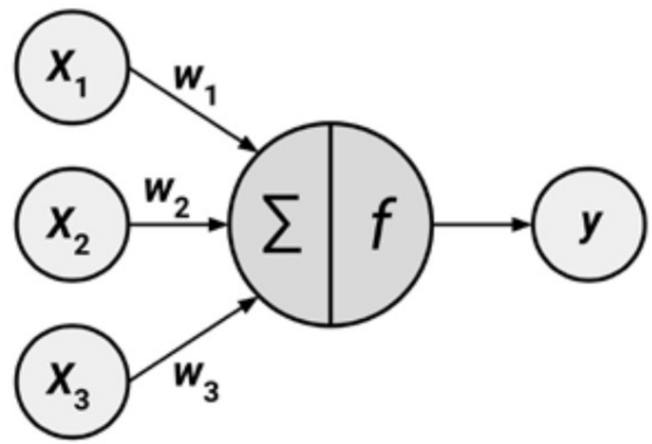
- Multiple hidden layer ANN



Perceptron

Single layer ANN

receives inputs



Perceptron

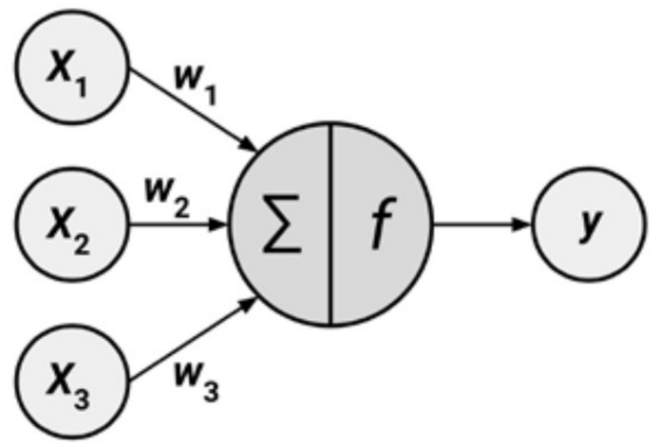
Single layer ANN

receives inputs

combines

inputs with weights

obtain net input



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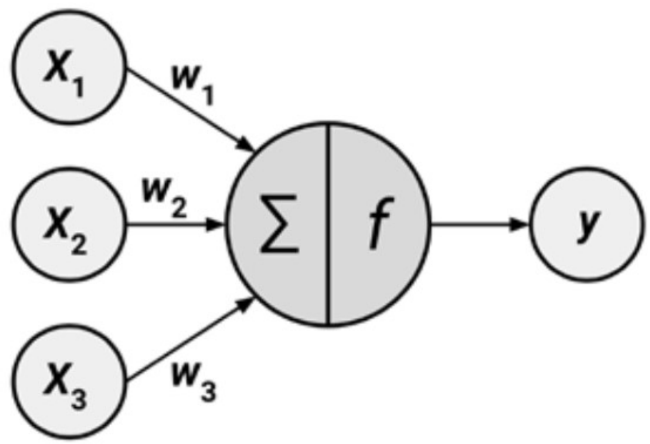
process it



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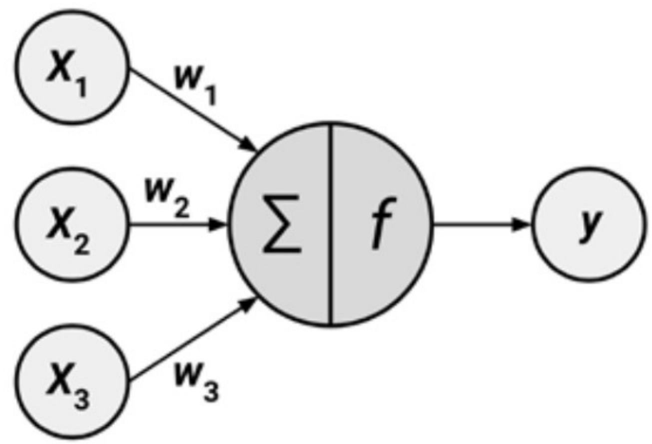

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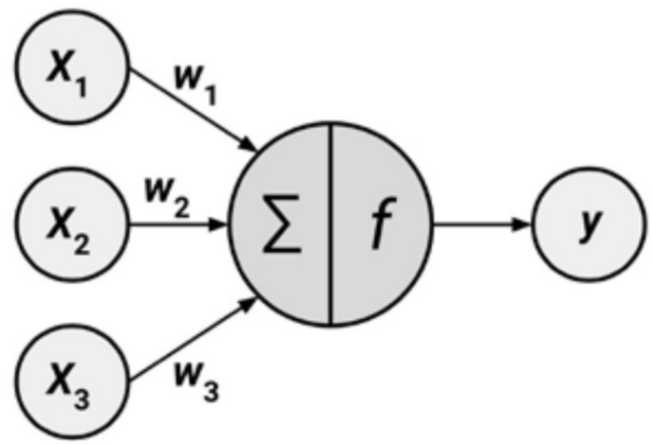

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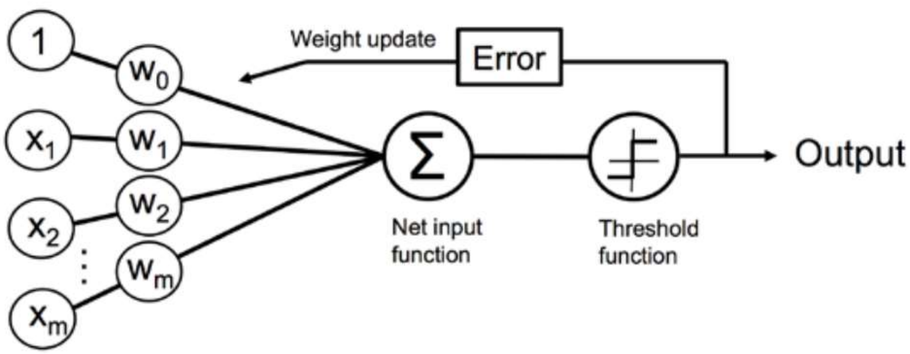

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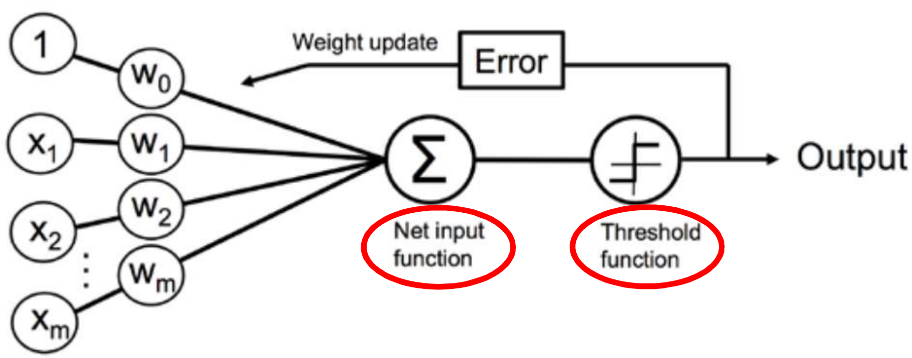

Perceptron

net input function $z = w_1 x_1 + \ldots + w_m x_m$

decision function $\phi(z) = \begin{cases} 1 & if\ z \geq \theta \\ -1 & otherwise \end{cases}$

Perceptron

net input function $$z = w_0 x_0 + w_1 x_1 + \ldots + w_m x_m$$

$$w_0 = -\theta \text{ and } x_0 = 1$$

decision function $$\phi(z) = \begin{cases} 1 & if\ z \geq 0 \\ -1 & otherwise \end{cases}$$

Perceptron

net input function $z = w_0 x_0 + w_1 x_1 + \ldots + w_m x_m = \boldsymbol{w}^T \boldsymbol{x}$

$w_0 = -\theta$ and $x_0 = 1$

decision function
$$\phi(z) = \begin{cases} 1 & if\ z \geq 0 \\ -1 & otherwise \end{cases}$$

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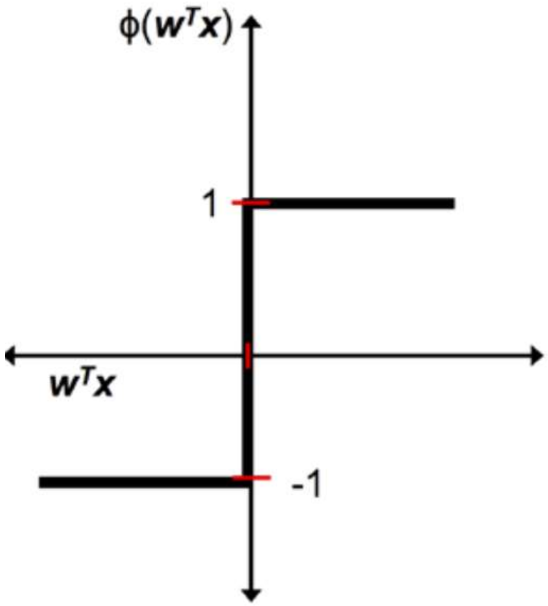
Perceptron

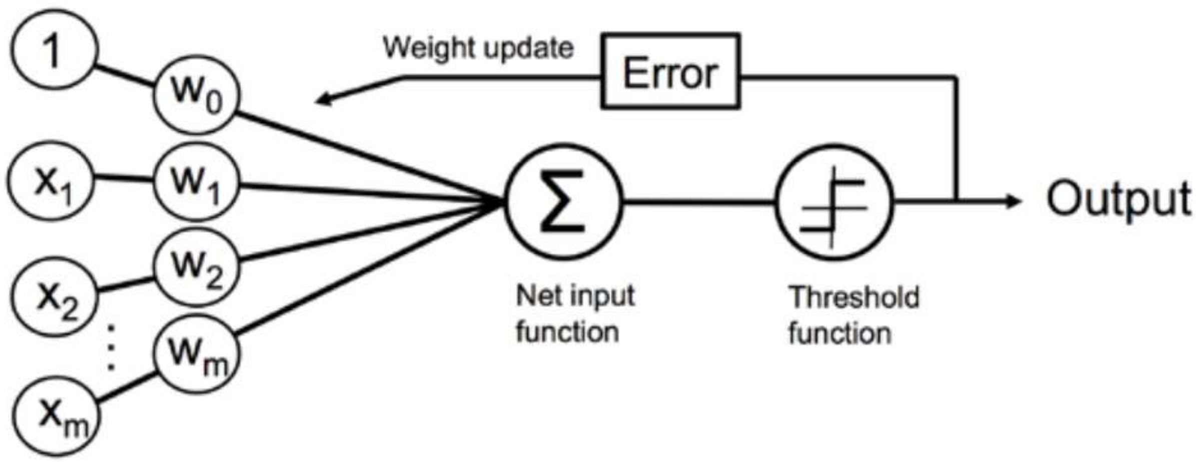
threshold function $\phi(z)$

$$z = \boldsymbol{w}^T \boldsymbol{x}$$

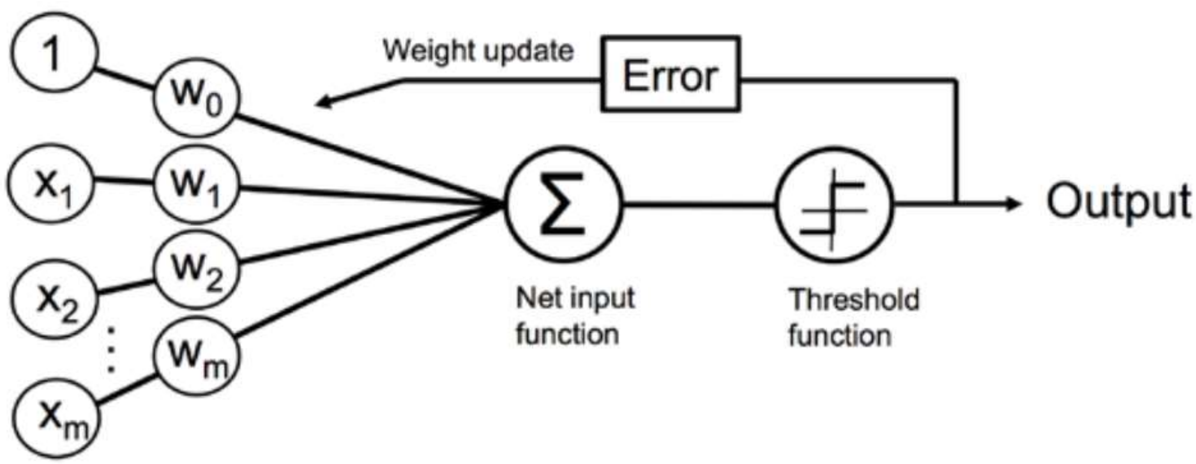
$$\phi(z) = \begin{cases} 1 & if\ z \geq 0 \\ -1 & otherwise \end{cases}$$



Perceptron


1
w_0
x_1
w_1
x_2
w_2
x_m
w_m
Weight update
Error
Σ
Net input function
Threshold function
Output

Perceptron learning rule



Perceptron learning rule

- Randomly initialize the weights $w_1, \ldots, w_k$
- For each row $i = 1, \ldots, n$

  find $\hat{y} = \phi(z)$

  find the error $(y - \hat{y})$

  update the weights $w_j = w_j + \Delta w_j$

  using $\Delta w_j = \lambda(y_i - \hat{y}_i)\, x_{ij}$

  for $j = 1, \ldots, p$
- Repeat N times

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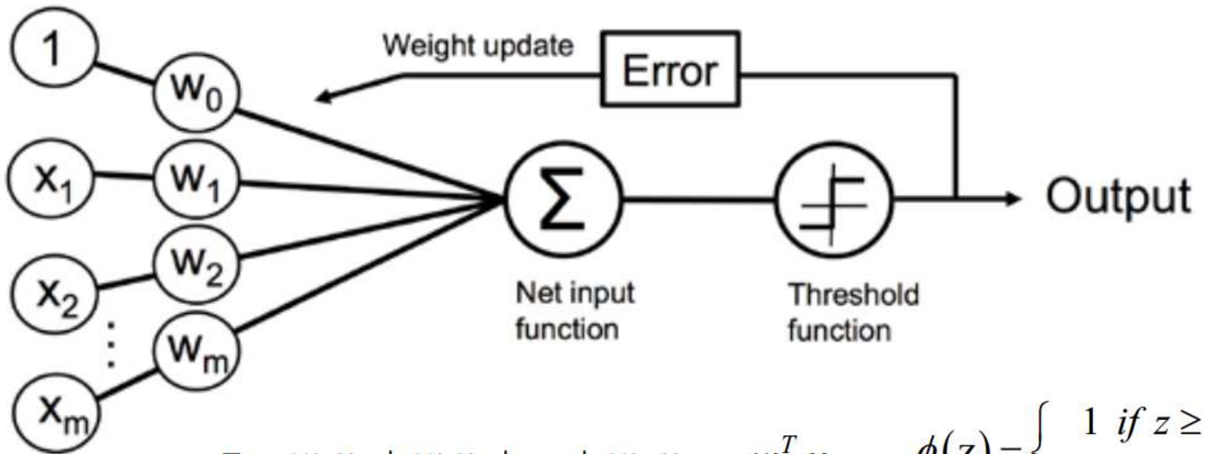
Perceptron learning rule



$$z = w_0 x_0 + w_1 x_1 + \ldots + w_m x_m = \boldsymbol{w}^T \boldsymbol{x}$$

$$\phi(z) = \begin{cases} 1 & if\ z \geq 0 \\ -1 & otherwise \end{cases}$$

Notes

- Perceptron works if data is linearly separable
- Decision function = Threshold function
- iterations N = epochs
- $\hat{y} = \phi(z)$

ADAptive LInear NEuron (Adaline)

- Another single-layer NN
- net input *z* is transformed using an Activation function

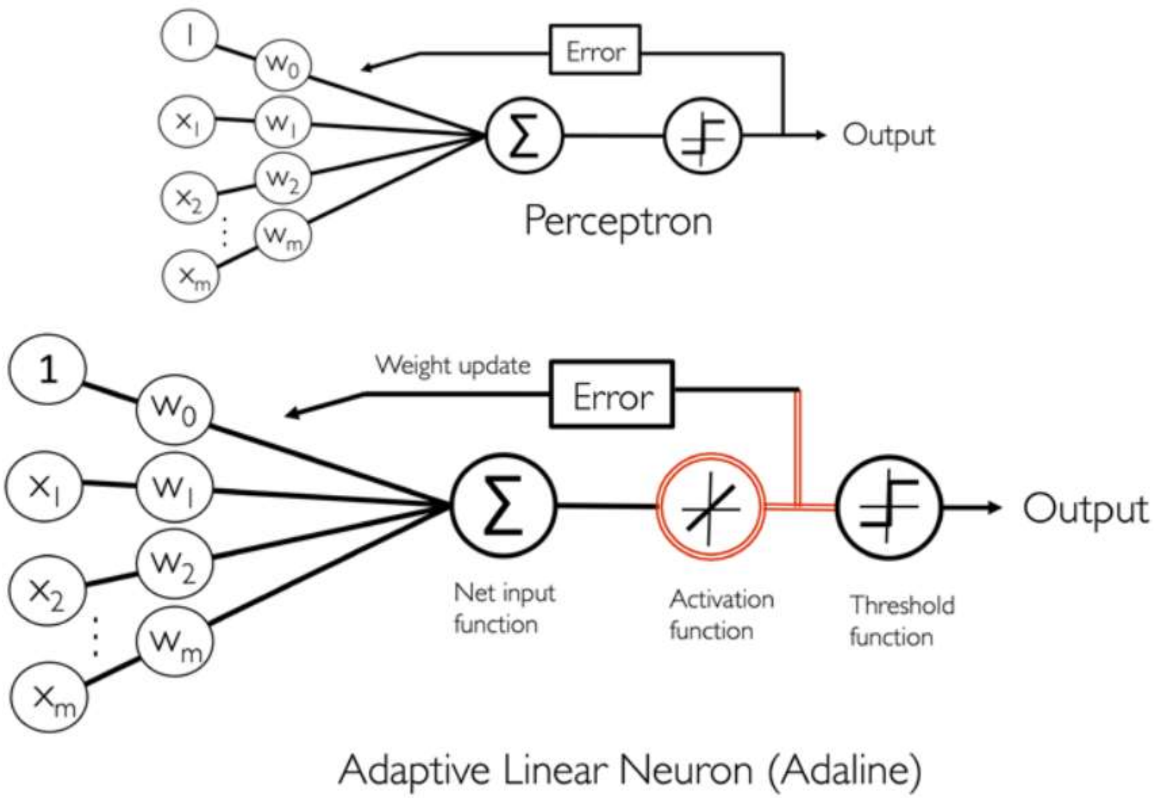
$$z = w_1 x_1 + \ldots + w_m x_m$$

ADAptive LInear NEuron (Adaline)

- Another single-layer NN
- net input *z* is transformed using an Activation function
- For Adaline Activation function is $\phi(z) = z$
- Prediction is given by threshold function

$$\hat{y} = \begin{cases} 1 & \text{if} \quad z > 0 \\ -1 & \text{if} \quad z \leq 0 \end{cases}$$

Adaline vs Perceptron



Adaline stopping condition

- Minimize loss function

$$
\begin{aligned}
J(\boldsymbol{w}) &= \frac{1}{2}\sum_{i=1}^{n}(y_i - \phi(z_i))^2 \\
&= \frac{1}{2}\sum_{i=1}^{n}(y_i - z_i)^2 \\
&= \frac{1}{2}\sum_{i=1}^{n}[y_i - w_1\,x_1 + \cdots + w_p\,x_p]^2
\end{aligned}
$$

quadratic and differentiable

- How to find $w_1, \ldots, w_k$ that minimize the loss?

Adaline stopping condition

loss function $$J(\boldsymbol{w}) = \frac{1}{2}\sum_{i=1}^{n}[y_i - w_1 x_1 + \cdots + w_p x_p]^2$$

learning algorithm

- Randomly initialize the weights $w_1, \ldots, w_k$
- Update the weights using gradient descent

$$w_j = w_j + \Delta w_j$$

Gradient descent

loss function $$J(\boldsymbol{w}) = \frac{1}{2}\sum_{i=1}^{n}[y_i - w_1 x_1 + \cdots + w_p x_p]^2$$

gradient vector $\nabla \boldsymbol{J}(\boldsymbol{w})$ is vector of partial derivatives

- Randomly initialize the weights $w_1, \ldots, w_k$
- Update the weights using gradient descent

$$w_j = w_j + \Delta w_j$$

- where $\Delta \boldsymbol{w} = -\lambda \nabla \boldsymbol{J}(\boldsymbol{w})$

Gradient descent

- The partial derivatives are

$$\frac{\partial J}{\partial w_j} = -\sum_{i=1}^{n} (y_i - \phi(z_i))\, x_{ij}$$

- The weight change for feature *j* is

$$\begin{aligned}\Delta w_j &= -\lambda \frac{\partial J}{\partial w_j} \\ &= \lambda \sum_{i=1}^{n} (y_i - \phi(z_i))\, x_{ij}\end{aligned}$$

Notes

- Adaline iterates until loss function is minimized

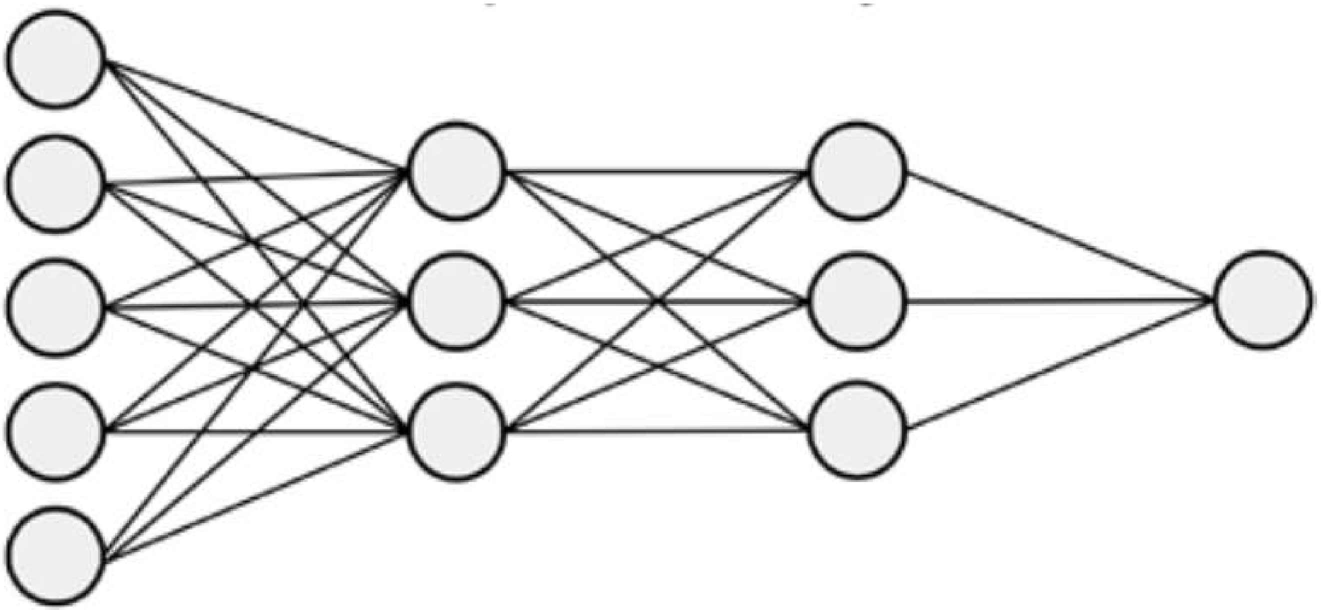
  (until convergence)

- Logistic regression is Adaline network with activation

$$\phi(z) = \frac{1}{1 + e^{-z}}$$

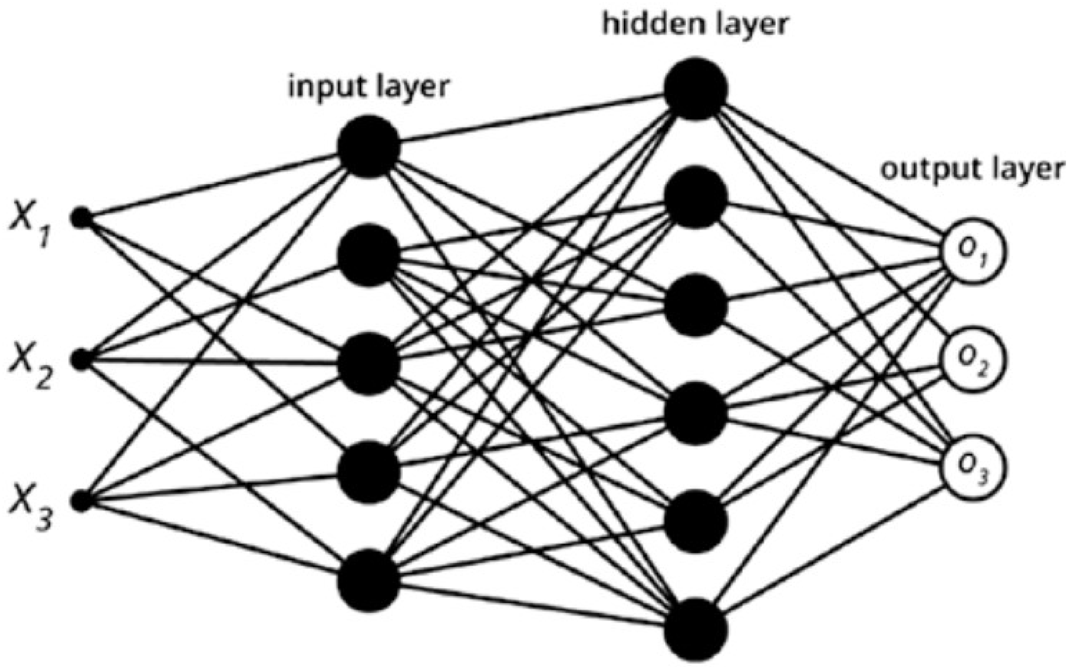
Multilayer NN

- One-node output layer



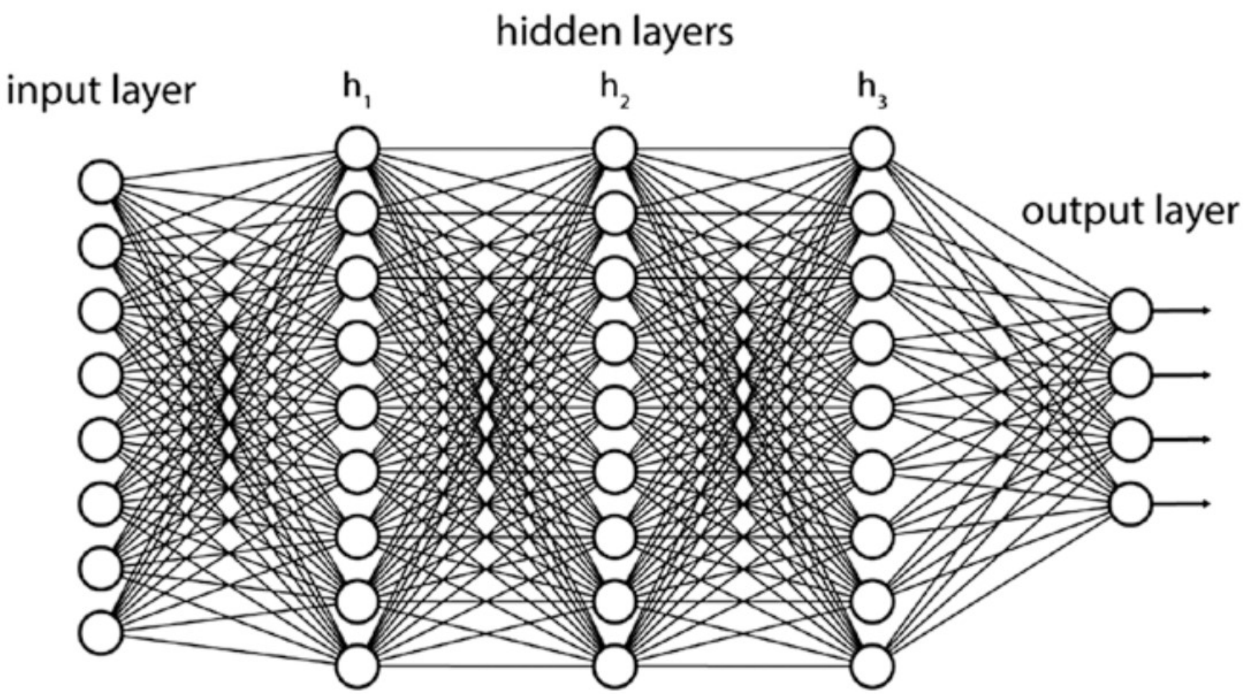
Multilayer NN

- Three-node output layer



Deep Neural Network

- Three hidden and a four-node output layer



Notes

- NN layer is called Dense if all nodes are connected with neighbor layers
- NN is called Multilayer Perceptron (MLP) if all layers are dense
- NN is called Deep if the number of hidden layers is large (usually 8 or more)

Notes

- For small datasets use a small number of hidden layers otherwise risk of overfitting
- May consider increasing the number of hidden layers with the dataset size
- Fine tune number of epochs, number of layers, learning rate, etc.

Other Neural Networks

- Convolutional Neural Networks (convnets)
- Recurrent Neural Networks