Naive Bayes

Outline

- Conditional Probability
- Bayes Theorem
- Naive Bayes
- Example

Conditional Probability

$$P[Y = y, X = x] \;=\; P[X = x \,|\, Y = y]\, P[Y = y]$$

Conditional Probability

$$P[Y = y, X = x] = P[X = x \,|\, Y = y]\, P[Y = y]$$

$$= P[Y = y \,|\, X = x]\, P[X = x]$$

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$$P[Y = y \,|\, X = x] = \frac{P[X = x \,|\, Y = y]\, P[Y = y]}{P[X = x]}$$

Bayes Theorem

$$P[Y = y, X = x] = P[X = x \,|\, Y = y]\, P[Y = y]$$

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Example

X1	X2	X3	Y
			YES
			YES
			NO
			YES
			NO
			NO
			NO

New

Bayes Theorem

$$P[Y = y \,|\, X = x] \;=\; \frac{P[X = x \,|\, Y = y]\, P[Y = y]}{P[X = x]}$$

$$P[Y = yes \,|\, X] \;=\; \frac{P[X \,|\, Y = yes]\, P[Y = yes]}{P[X]}$$

Naive Bayes

Assumptions

$$P[Y = yes] \;=\; P[Y = no] = 0.50$$

$$P[X_1 = x_1, X_2 = x_2, X_3 = x_3 \,|\, Y] \;=$$

$$P[X_1 = x_1 \,|\, Y]\; P[X_2 = x_2 \,|\, Y]\; P[X_3 = x_3 \,|\, Y]$$

Naive Bayes - Example

What is the probability that a Red Sports car will be stolen?

Color	Type	Stolen?
Red	Sport	Yes
Red	SUV	No
Red	SUV	Yes
Yellow	Sport	No

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P[Yes | Red, Sport]?

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choose the largest

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$$P[Red, Sport \,|\, yes] = P[Red \,|\, yes]\, P[Sport \,|\, yes]$$

$$= (1)\,(\frac{1}{2}) = \frac{1}{2}$$

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$$= (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$$

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$$P[Red, Sport \,|\, yes] = \frac{1}{2} \qquad \text{predict } yes$$

$$P[Red, Sport \,|\, no] = \frac{1}{4}$$