FEM program structure

Advanced Finite Element Methods (ME46050)

Input: A file with the input data f_{in}

function MAIN(f)

- *– initialization stage*
- $\mathscr{M}, \mathscr{C}, \mathscr{B}, \mathscr{P} \leftarrow$ READINPUT(f_{in}) *– get mesh, equation, materials, BCs, problem type*
- $\{\mathscr{N}, \mathscr{E}\} \leftarrow \mathscr{M}$ *– Nodes and element sets from mesh*
- $n \leftarrow |\mathscr{N}| \times \frac{\text{DOFs}}{\text{node}}$ *– get total number of DOFs*
- $\{\boldsymbol{K}, \boldsymbol{F}\} \leftarrow \{\boldsymbol{0}_{n\times n}, \boldsymbol{0}_{n\times 1}\}$ *– initialize data structures*
- *– assembly stage*
- **for** $e \in \mathscr{E}$ **do**
  - $\boldsymbol{k}_e, \boldsymbol{f}_e \leftarrow$ integrate$(e, \mathscr{N}, \mathscr{C})$ *– obtain local stiffness matrix and force vector*
  - $\boldsymbol{K} \leftarrow \boldsymbol{K} \mathbb{A} \boldsymbol{k}_e$ *– assemble contribution to stiffness matrix*
  - $\boldsymbol{F} \leftarrow \boldsymbol{F} \mathbb{A} \boldsymbol{f}_e$ *– assemble contribution to force vectors*
- *– solution stage*
- $\{\boldsymbol{K}, \boldsymbol{F}\} \leftarrow \mathscr{B}(\boldsymbol{K}, \boldsymbol{F})$ *– apply boundary conditions*
- $\boldsymbol{U} \leftarrow \boldsymbol{K}^{-1}\boldsymbol{F}$ *– solve for unknown field*
- *– output stage*
- $\{\boldsymbol{\sigma}, \boldsymbol{\varepsilon}\} \leftarrow \{\boldsymbol{\sigma}(\boldsymbol{U}), \boldsymbol{\varepsilon}(\boldsymbol{U})\}$ *– get stress and strain*
- $f_{\text{out}} \leftarrow$ WRITEOUTPUT$(\mathscr{M}, \boldsymbol{U}, \boldsymbol{\sigma}, \boldsymbol{\varepsilon})$ *– write results to file*

Input: Element e and constitutive law $\mathscr{C}$

function INTEGRATE($e, \mathscr{N}, \mathscr{C}$)

- $c \leftarrow$ CONNECTIVITY(e) *– get element connectivity*
- $n \leftarrow |c| \times \frac{\text{DOFs}}{\text{node}}$ *– get total number of local DOFs*
- $\{\boldsymbol{k}_e, \boldsymbol{f}_e\} \leftarrow \{\boldsymbol{0}_{n\times n}, \boldsymbol{0}_{n\times 1}\}$ *– initialize data structures*
- $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_i & \dots \end{bmatrix}^{\intercal} \leftarrow \mathscr{N}(c)$ *– get element nodal coordinates*
- *– loop over integration points*
- $\{\boldsymbol{\gamma}, \boldsymbol{\xi}\} \leftarrow$ QUADRATURE(e) *– integration weight and master coordinates*
- **for** $i \leftarrow 1, \dots$ **do**
  - $\left\{\boldsymbol{\varphi}, \frac{d\boldsymbol{\varphi}}{dx}\right\} \leftarrow$ SHAPEFUNCTIONS$(\boldsymbol{\xi}_i)$ *– shape functions and their derivatives*
  - $\left\{\boldsymbol{J}^{-1}, j\right\} \leftarrow$ JACOBIAN$(\boldsymbol{X}, \frac{d\boldsymbol{\varphi}}{dx})$ *– obtain Jacobian inverse and determinant*
  - $\boldsymbol{B} \leftarrow \frac{d\boldsymbol{\varphi}}{dx}\boldsymbol{J}^{-1}$ *– obtain Jacobian inverse and determinant*
  - $D \leftarrow \mathscr{C}(e)$ *– constitutive law*
  - $\boldsymbol{k}_e \leftarrow \boldsymbol{k}_e + \boldsymbol{\gamma}_i\, j\, \boldsymbol{B}^{\intercal} D\, \boldsymbol{B}$ *– update stiffness matrix*
  - $\boldsymbol{f}_e \leftarrow \boldsymbol{f}_e + \boldsymbol{\gamma}_i\, j\, \boldsymbol{\varphi}^{\intercal} b$ *– update force vector*
- **return** $\boldsymbol{k}_e, \boldsymbol{f}_e$