

$$\diamond \text{ 柱坐标散度: } \nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}.$$

$$\diamond \text{ 柱坐标旋度: } \nabla \times \mathbf{A} = \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{\rho} + \left( \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \hat{\phi} + \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi} \right] \hat{z}.$$

$$\diamond \text{ 柱坐标梯度: } \nabla \Phi = \frac{\partial \Phi}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial \Phi}{\partial \phi} \hat{\phi} + \frac{\partial \Phi}{\partial z} \hat{z}.$$

$$\diamond \text{ 柱坐标拉普拉斯: } \Delta \Phi = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \Phi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\partial^2 \Phi}{\partial z^2}.$$

$$\diamond \text{ 球坐标散度: } \nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}.$$

$$\diamond \text{ 球坐标旋度: } \nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] \hat{\mathbf{r}} + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right] \hat{\theta} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \hat{\phi}.$$

$$\diamond \text{ 球坐标梯度: } \nabla \Phi = \frac{\partial \Phi}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial \Phi}{\partial \phi} \hat{\phi}.$$

$$\diamond \text{ 球坐标拉普拉斯: } \Delta \Phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \phi^2}.$$

$$\diamond \nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \Delta \mathbf{A}.$$

$$\diamond \mathbf{j} = nq\mathbf{u}.$$

$$\diamond d\mathbf{F} = I d\boldsymbol{\ell} \times \mathbf{B} = \mathbf{K} dS \times \mathbf{B} = \mathbf{j} dV \times \mathbf{B}.$$

$$\diamond \mathbf{B} = \frac{\mu_0}{4\pi} \oint_L \frac{I d\boldsymbol{\ell} \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} \iint_S \frac{\mathbf{K} dS \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{j} dV \times \mathbf{r}}{r^3}.$$

$$\diamond \text{ 载流圆线圈轴上一点 } B_z = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{\frac{3}{2}}}.$$

$$\diamond \text{ 线/面/体电流系统磁矩 } \mathbf{m} = \frac{1}{2} \oint_L \mathbf{R} \times I d\mathbf{r} = \frac{1}{2} \iint_S \mathbf{R} \times \mathbf{K} dS = \frac{1}{2} \iiint_V \mathbf{R} \times \mathbf{j} dV.$$

$$\diamond \text{ 磁偶极场 } \mathbf{B} = -\frac{\mu_0 \mathbf{m}}{4\pi r^3} + \frac{3\mu_0 \mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{4\pi r^5}, \text{ 电偶极场 } \mathbf{E} = -\frac{\mathbf{p}}{4\pi \epsilon_0 r^3} + \frac{3(\mathbf{p} \cdot \mathbf{r})}{4\pi \epsilon_0 r^5} \mathbf{r}.$$

$$\diamond \text{ 磁矩在远处产生的磁矢势 } \mathbf{A} = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{r}}{r^3}.$$

$$\diamond B_O = \frac{\mu_0 n I}{2} (\cos \beta_2 - \cos \beta_1).$$

$$\diamond \mathbf{A} = \frac{\mu_0}{4\pi} \oint_L \frac{I d\boldsymbol{\ell}}{r} = \frac{\mu_0}{4\pi} \iint_S \frac{\mathbf{K} dS}{r} = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{j} dV}{r}.$$

$$\diamond \text{ 粒子回旋磁矩 } \mu = \frac{mv_\perp^2}{2B}, \text{ 其中 } v_\perp \text{ 为垂直于磁场方向的速度大小.}$$

$$\diamond \text{ 在力场 } \mathbf{F} \text{ 中带电粒子的漂移速度 } \mathbf{v}_F = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}.$$

$$\diamond \text{ 霍尔效应: } U = K \frac{IB}{d}, \text{ 其中 } K = \frac{1}{nq}.$$

$$\diamond \text{ 小载流线圈在磁场中受力 } \mathbf{F} = (\mathbf{m} \cdot \nabla) \mathbf{B}, \text{ 力矩 } \mathbf{L} = \mathbf{m} \times \mathbf{B} + \mathbf{r} \times (\mathbf{m} \cdot \nabla) \mathbf{B}.$$

$$\diamond \text{ 麦克斯韦方程组的积分形式: } \begin{cases} \oiint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \rho_0 dV, \\ \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = - \iint_{S_C} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}, \\ \oiint_S \mathbf{B} \cdot d\mathbf{S} = 0, \\ \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = \iint_{S_C} \left( \mathbf{j}_0 + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}. \end{cases} \quad \text{微分形式: } \begin{cases} \nabla \cdot \mathbf{D} = \rho_0, \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{B} = 0, \\ \nabla \times \mathbf{H} = \mathbf{j}_0 + \frac{\partial \mathbf{D}}{\partial t}. \end{cases}$$

$$\diamond \text{ 介质的本构方程: } \mathbf{D} = \epsilon \mathbf{E}, \mathbf{B} = \mu \mathbf{H}, \mathbf{j}_0 = \sigma \mathbf{E}. \text{ 另有 } \mathbf{M} = \chi_m \mathbf{H} \text{ 和 } \mathbf{B} = \mu \mathbf{H}, \text{ 其中 } \mu = \mu_0(1 + \chi_m) = \mu_0 \mu_r.$$

$$\diamond \text{ 边值关系: } \begin{cases} \mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_0, \\ \mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0, \\ \mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0, \\ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_0. \end{cases} \quad \text{另有 } \mathbf{n} \times (\mathbf{M}_2 - \mathbf{M}_1) = \mathbf{K}' \text{ 和 } \oint_L \mathbf{M} \cdot d\boldsymbol{\ell} = \sum I'.$$

◇ 分区均匀线性各向同性介质中的静磁场:

- 介质界面与磁感应线重合: 先计算传导电流在真空中产生的  $\mathbf{B}_0$ , 再利用  $\mathbf{H} = \frac{\mathbf{B}_0}{\mu_0}$  得到  $\mathbf{H}$ , 最后由  $\mathbf{B}_i = \mu_i \mathbf{H}$  分区计算.
- 介质界面与磁感应线垂直: 先计算传导电流在真空中产生的  $\mathbf{B}_0$ , 再待定  $\mathbf{B} = \alpha \mathbf{B}_0$ , 由安培环路定理确定  $\alpha$ .

◇ 磁路定理:  $\mathcal{E}_m = \Phi_B R_m$ , 其中, 磁动势  $\mathcal{E}_m = \sum I_0$ , 磁阻  $R_m = \oint \frac{d\boldsymbol{\ell}}{\mu S}$ , 磁通量  $\Phi_B = BS$ .

◇ 动生电动势  $\mathcal{E}_{\text{动}} = \int_a^b (\mathbf{v} \times \mathbf{B}) \cdot d\boldsymbol{\ell}$ , 感生电动势  $\mathcal{E}_{\text{感}} = - \iint_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$ , 涡旋电场  $\mathbf{E}_{\text{旋}} = - \frac{\partial \mathbf{A}}{\partial t}$ .

◇  $M = k\sqrt{L_1 L_2}$ , 其中耦合系数  $k \in [0, 1]$ ,  $k = 1$  表示理想耦合.

等效自感	顺接	反接
串联	$L_1 + L_2 + 2M$	$L_1 + L_2 - 2M$
并联	$\frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$	$\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

◇ 似稳条件:  $\frac{\ell}{c} \ll \frac{1}{f}$  或  $\ell \ll \frac{c}{f} = \lambda$ .

◇ 单一闭合回路的似稳电路方程:  $e = iR + \frac{q}{C} + L \frac{di}{dt} + M \frac{di'}{dt}$ .

◇ 方程  $\frac{dy}{dx} + p(x)y = q(x)$  满足  $y(x_0) = y_0$  的解为  $y(x) = y_0 \exp\left(-\int_{x_0}^x p(t) dt\right) + \int_{x_0}^x q(s) \exp\left(-\int_s^x p(t) dt\right) ds$ .

◇ RL 电路充电  $i(t) = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{R}{L}t}\right)$ , 放电  $i(t) = \frac{\mathcal{E}}{R} e^{-\frac{R}{L}t}$ , 时间常数  $\tau_L = \frac{L}{R}$ .

◇ RC 电路充电  $i(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$ , 放电  $i(t) = -\frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$ , 时间常数  $\tau_C = RC$ .

◇ RLC 电路方程:  $\frac{d^2 q}{dt^2} + 2\beta \frac{dq}{dt} + \omega_0^2 q = \omega_0^2 q_0$ , 其中阻尼因子  $\beta = \frac{R}{2L}$ , 固有频率  $\omega_0 = \frac{1}{\sqrt{LC}}$ ,  $q_0 = C\mathcal{E}$ . 充电时,

- 欠阻尼  $\beta < \omega_0$  时,  $q = q_0 - q_0 e^{-\beta t} \left( \cos \omega t + \frac{\beta}{\omega} \sin \omega t \right)$ , 其中  $\omega = \sqrt{\omega_0^2 - \beta^2}$ .
- 过阻尼  $\beta > \omega_0$  时,  $q = q_0 - \frac{1}{2\gamma} q_0 e^{-\beta t} [(\beta + \gamma)e^{\gamma t} - (\beta - \gamma)e^{-\gamma t}]$ , 其中  $\gamma = \sqrt{\beta^2 - \omega_0^2}$ .
- 临界阻尼  $\beta = \omega_0$  时,  $q = q_0 - q_0(1 + \beta t)e^{-\beta t}$ .

放电时,  $q' = q_0 - q$ .

◇ 一个载流线圈的磁能  $W_m = \frac{1}{2} LI^2 = \frac{1}{2} I \Phi_m$ ,  $N$  个载流线圈系统的磁能  $W_m = \frac{1}{2} \sum_{i,k=1, i \neq k}^N M_{ik} I_i I_k + \frac{1}{2} \sum_{i=1}^N L_i I_i^2$ .

◇ 电流环磁矩在外磁场中的磁能为  $\mathbf{m} \cdot \mathbf{B}$ , 小磁针/磁铁/磁材料/带电粒子圆周运动产生的磁矩的磁势能为  $-\mathbf{m} \cdot \mathbf{B}$ .

◇  $N$  个载流线圈在外场中的磁能  $W_m = \sum_{k=1}^N I_k \iint_{S_k} \mathbf{B}(\mathbf{r}) \cdot d\mathbf{S}$ .

◇ 磁能密度  $w_m = \frac{1}{2} \mathbf{B} \cdot \mathbf{H}$ , 宏观磁能密度  $\frac{1}{2} \mu_0 H^2$ , 磁化功密度  $\frac{1}{2} \mu_0 \mathbf{M} \cdot \mathbf{H}$ .

◇ 平面电磁波  $\mathbf{E}, \mathbf{H}, \mathbf{k}$  满足右手正交关系,  $\varepsilon E^2 = \mu H^2$ , 传播速度  $v = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon \mu}} = \sqrt{\frac{\varepsilon_0 \mu_0}{\varepsilon \mu}} c$ , 折射率  $n = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r}$ .

◇ 电磁场的能量密度  $w = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} + \frac{1}{2} \mathbf{B} \cdot \mathbf{H}$ , 能流密度/坡印廷矢量  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ , 动量密度  $\mathbf{g} = \mathbf{D} \times \mathbf{B}$ , 角动量密度  $\mathbf{l} = \mathbf{r} \times \mathbf{g}$ .

◇ 平面电磁波  $\mathbf{S} = w\mathbf{v}$ ,  $\mathbf{g} = \frac{1}{v^2} \mathbf{S}$ , 真空中波的强度  $I := \langle S \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2c\mu_0} = \frac{cB_0^2}{2\mu_0}$ .

◇ 反射系数 = 反射光能流密度/入射光能流密度, 光压 (平均光压强)  $\bar{p} = (1 + R)\bar{w}$ .